DETERMINING PARETO OPTIMAL CONTROLLER PARAMETER SETS OF AIRCRAFT CONTROL SYSTEMS USING GENETIC ALGORITHM

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Abstract: Change of parameters, such as dynamic pressure, Mach number, etc., have a significant influence on the dynamic properties of aircraft, and consideration of every necessary point in the flight envelope is important for the design of a flight control system. Because of this characteristic of the design problem one of the efficient design methods is the multi-model approach. A genetic algorithm is coded to optimize a vector of performance indices of each model in multi model. The results in this paper showed that the genetic algorithm is convenient for determining pareto optimal controller parameter sets.

Key-Words: Flight control, genetic algorithm, pareto optimal, multiobjective optimization.

1 Introduction

Aircraft control problems are usually more complicated than those of other vehicles. Because the aircraft have six degrees of freedom: Three associated with angular motion about the aircraft's center of gravity and three associated with the translation of the center of gravity. [1]

The model of a conventional aircraft characterizing its motion dynamics is a set of nonlinear differential equations. The first step in solving the stability problem is modeling. The second step is linearization of those equations to obtain a set of linear differential equations with constant coefficients. This linearized set represents the motion dynamics about the operating point of interest. Considering the straight-symmetric wings-level flights within a given flight envelope there are infinitely many operating points, since the flight envelope consists of infinitely many altitude-velocity pairs. In practice, designers sample altitude-velocity pairs at sufficiently many points (generally four points for aircraft examples) in the flight envelope.[2]

The stability and control problem of the aircraft may be summarized as computing suitable feedback coefficients from motion sensors to the deflectional surfaces of the aircraft such that flight condition is preserved under external disturbances.

Clearly this set of coefficients works for all the sampled points in the flight envelope. Since coefficients of the nonlinear differential equations, and consequently that of the linearized differential equations, are continuous functions of both altitude and velocity, it is concluded that the coefficients work good for any altitude-velocity pairs in the flight envelope. This method is called multi-model approach.

In aircraft control problem it is also necessary to ensure certain design objectives for all points in the flight envelope. Hence a multi model approach together with vector optimization is best suited for an aircraft control system design. On the other hand if we consider different models of an aircraft obtained at different points of a flight envelope and design objectives that should be optimized for each model, difficulty of a control system design for an aircraft can be easily seen.

In this study, it will be shown that excessive calculation load and the difficulty of finding controller parameters that make system objectives optimal (called pareto-optimal points) for an aircraft flight control problem can be overcame by genetic algorithm.

2 Multi-Model Control Approach to Aircraft Control Problem

The problem of control system design is stated with explicit uncertainty bounds for physical parameters in the plant model and performance bounds as design objectives. A finite number of typical plant parameter values is used to define a multi-model problem[3]. The plant dynamics is not uniquely given, but it is described using multiple candidates of dynamical systems or multiple models [4,5].

As it has already been stated mathematical model of an aircraft is nonlinear. This nonlinear model may be linearized for small deviations from stationary flight with constant altitude h and velocity v. Then linear model depends on these operating conditions. However, these operating conditions may vary with flight conditions or environmental changes in flight envelope. A linearized aircraft dynamics is described by a state space model,

$$\begin{aligned} \mathbf{x} &= \mathbf{A}(\mathbf{q})\mathbf{x} + \mathbf{B}(\mathbf{q})\mathbf{u} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} \end{aligned} \quad (1)$$

where $\mathbf{q} = \begin{bmatrix} v & h \end{bmatrix}^T$ is the vector of uncertain plant parameters. Assume the state variables in $\mathbf{x}$ are chosen such that the output matrix $\mathbf{C}$ does not depend on $\mathbf{q}$.

The coefficients of the closed loop characteristic polynomial are functions of both the plant parameters $\mathbf{q}$ and the controller parameters $\mathbf{k}$.[3]

$$D(s,\mathbf{q},\mathbf{k}) = d_0(\mathbf{q},\mathbf{k}) + d_1(\mathbf{q},\mathbf{k})s + \ldots + d_{n-1}(\mathbf{q},\mathbf{k})s^{n-1} + s^n \quad (2)$$

A typical robustness problem is then: Find a $\mathbf{k}$ such that the roots of $D(s,\mathbf{q},\mathbf{k})$ have negative real parts and ensures all desired objectives for all $\mathbf{q} \in \Omega$. Where Ω is the set of possible flight conditions in the flight envelope. In aircraft control systems design only a finite set of models for $\mathbf{q}_1, \mathbf{q}_2, ..., \mathbf{q}_N$ are available. In this case the set of all stability regions in the subspaces for $\mathbf{q} = \mathbf{q}_1$, $\mathbf{q} = \mathbf{q}_2, \ldots, \mathbf{q} = \mathbf{q}_N$ projected into one $\mathbf{K}$-space. This is the multi model approach [3]. After the set of all stabilizing $\mathbf{k}$ of all stability regions in the subspaces $\mathbf{q}_1, \mathbf{q}_2, ..., \mathbf{q}_N$ was determined, the second step is determining subset of this stabilizing $\mathbf{k}$ set which optimize vectorial performance criteria [3].

For each model of the multi model problem $(\mathbf{A}_j, \mathbf{B}_j)$, j=1,2,…,N, a vector of performance indices

$$\mathbf{g}(\mathbf{k}) = \begin{bmatrix} g_1(\mathbf{k}) \\ g_2(\mathbf{k}) \\ . \\ . \\ g_m(\mathbf{k}) \end{bmatrix} \quad (3)$$

must be formed (that is N times). A pareto optimal value of $\mathbf{g}$ is wanted. If the set of Pareto optimal solutions for a given plant is known, the most desired solution in this set can be selected. [6]

2.1 Multiobjective Optimization

In single-criterion optimization, we seek the best (highest or lowest) value of the well-defined objective (utility or cost) function. In multiobjective (or vector valued) optimization the notion of optimality is not all obvious. If we refuse to compare apples to oranges then we must come up with a different definition of optimality, one that respects the integrity of each of separate criteria. The concept of Pareto Optimality helps us to do this. [7]

A Pareto-optimal point is a point in a set that does not have any better point different from itself in its open neighborhood. Hence, a set of Pareto-optimal points P can be defined as [6]

$$P := \left\{ x \middle| \exists U(x) : B(x) \cap U(x) = \{x\} \right\} \quad (4)$$

Where $U(x)$ represents an open neighborhood of x, $B(x)$ is a set of better performance vectors of x. Pareto-optimal solutions can be found by genetic algorithms [7].

3 Genetic Algorithms

Genetic Algorithms (GAs) are global numerical optimization methods, patterned after the natural processes of genetic recombination and evolution.

The GA used in this paper known as the simple genetic algorithm. In this algorithm, the three-operator GA with only minor deviations from the original is used.

An initial population of binary strings is created randomly. Each of these strings represents one possible solution to the search problem. Next the solution strings are converted into their decimal equivalents and each candidate solution is tested in this environment. The fitness of each candidate is evaluated through some appropriate measure. The algorithm is driven towards maximizing this fitness measure. Application of the GA to an optimal control problem entails minimizing the selected performance index. After the fitness of the entire population has been determined, it must be determined whether or not the termination criterion has been satisfied. If the criterion is not satisfied then we continue with the three genetic operators: reproduction, crossover and mutation.[8]

Fitness-proportionate reproduction is effected through the simulated spin of a weighted roulette wheel. The roulette wheel is biased with the fitnesses of each of the solution candidates. The wheel is spun N times where N is the number of strings in the population. Copying strings according to their fitness values means that strings with a higher value have a higher probability of contributing one or more off spring in the next generation[7]. This operation yields a new population of strings that reflect the fitnesses of the previous generation's fit candidates. The next operation, crossover, is performed on two strings at a time that are selected from the population at random. Crossover involves choosing a random position in the two strings and swapping the bits that occur after this position. The resulting crossover yields two new strings means the strings are part of the new generation [8]. The crossover rate specifies the number of strings which are effected crossover operator.

The mechanics of reproduction and crossover are suprisingly simple, involving random number generation , string copies , and some partial string exchanges.[7]

The final genetic operator in the algorithm is mutation. Mutation is performed sparingly, typically every 100-1000 bit transfers from crossover, and it involves selecting a string at random as well as a bit position at random and changing it from 1 to 0 or vice-versa. After mutation, the new generation is completed and the procedure begins again with fitness evaluation of the population [8].

4 An Application to Longitudinal Flight Control

In this paper, we consider pitch orientation control system for short-period approximation of an aircraft.

In [9], it has been shown that a simple genetic algorithm can be used to find a controller parameter for pitch orientation control system for short-period approximation, as shown Fig.1, with a single performance criterion. But for the study presented here, two criteria were taken and a genetic algorithm was coded in PASCAL programming language to determine the pareto-optimal solutions of controller parameters for an aircraft.

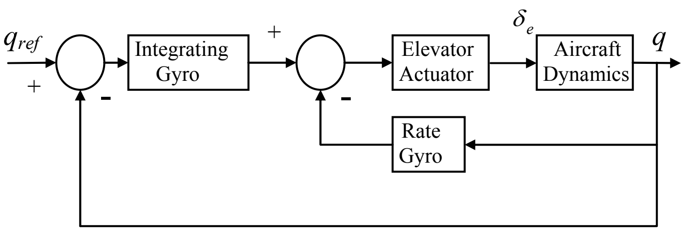

Fig.1 Pitch orientation control system block diagram

The aircraft dynamics block shown in Fig.1 is defined as $\frac{q(s)}{\delta_E(s)}$. Where, q is the pitch angular velocity and δ_E is the displacement of the elevator. In [10], this dynamics for short-period approximation is given by,

$$\frac{(U_0 M_{\delta_E} + Z_{\delta_E} M_\alpha)s + (M_\alpha Z_{\delta_E} - Z_\alpha M_{\delta_E})}{U_0 s^2 - (Z_\alpha + U_0 M_q + U_0 M_\alpha)s + M_q Z_\alpha - U_0 M_\alpha} \quad (5)$$

Rate gyro transfer function is just a gain with a value of 1.5 and integrating gyro transfer function is $\frac{20}{s+20}$.

The goal of the genetic algorithm is to determine the value of K_1 which is the integrating gyro gain shown in Fig.1. This value must be closely ensured by desired damping ratio and minimum settling time at the four flight dynamics in the longitudinal flight envelope. The performance index vector for this problem is defined like in the Eq (3) as,

$$\mathbf{g}(\mathbf{K}_1) = \begin{bmatrix} \left|\zeta_{sp}(\mathbf{K}_1) - \zeta_{sp_{desired}}\right| \\ \\ t_s(\mathbf{K}_1) \end{bmatrix} \quad (6)$$

Where ζ_{sp} is the damping ratio and t_s is the settling time of short period closed loop system shown in Fig.1. The values $\mathbf{K}_1$ making minimum of this vector is the pareto optimal solution set of $\mathbf{K}_1$.

The genetic algorithm program steps to find pareto optimal integrating gyro gain ($\mathbf{K}_1$) set are given below:

1-) Read following parameter values from the file,
   a) flight condition parameters and stability derivatives for four flight conditions.
   b) the genetic algorithm parameters; population and generation size, crossover and mutation rate, parameter resolution,
   c) desired values of performance criteria for the short period closed loop dynamics ($\zeta_{sp_{desired}}$).

2-) Calculate the aircraft dynamics using Eqn.(5) and closed loop dynamics at each flight condition.

3-) For each flight condition, find the range of $\mathbf{K}_{1_i}$, i=1,2,3,4 which stabilizes the closed loop system. The intersection of the stabilizing $\mathbf{K}_{1_i}$ intervals gives the range of stability for all sampled flight conditions.

4-) Do following steps at each flight condition:
   a) Generate an initial population of $\mathbf{K}_1$ in its stability range.
   b) Select dominated and nondominated individuals. The 80 percent of the non dominated individuals are copied to the next generation. Rest of the population is randomly selected.
   c) Apply crossover and mutation operation to selected individuals.
   d) Repeat steps b and c until the generation number reaches specified generation.
   e) Memorize the obtained $\mathbf{K}_1$ interval which are the pareto optimal solution set.

5-) Find intersection of memorized $\mathbf{K}_1$ intervals. If there exist an intersection, this ensures pareto optimal solution set of the vectorial performance index at the four flight conditions and also all of the longitudinal flight envelope.

In this paper, we have chosen an aircraft called BRAVO (a twin-engined, jet fighter aircraft) to apply the multi-model approach and also genetic algorithm. The flight condition parameters and stability derivative values were obtained from [1].

Genetic algorithm parameters are selected as:

Population size: 70
Generation size: 150
Crossover rate: 0.75
Mutation rate: 0.01

Desired damping ratio: $\zeta_{sp_{desired}} = 0.4$

Results obtained for all flight conditions at initial, 100th and 150th generations are given by plotting 70 individuals' $|\zeta - 0.4|$ values against the settling time (t_s) in Fig. 2 to 4 respectively. At initial population (Fig 2), 70 individuals are selected randomly selected from the stabilizing $\mathbf{K}_1$ interval. As it can be easily seen from Fig. 2 that pareto optimal points are the ones obtained for values $|\zeta - 0.4| \le 0.15$ and $t_s \le 0.5$. On generation 100, the indivuduals are mostly focused at this pareto optimal region (Fig 3) and finaly on generation 150 (Fig. 4) all individuals are pareto optimal. The intersection of $\mathbf{K}_1$ values obtained at all flight conditions for the 150th generation individuals has been found as

$$0.11 \le K_1 \le 6.29 \qquad (6)$$

This interval is the pareto optimal $\mathbf{K}_1$ interval. Intersection of corresponding damping ratio and settling time intervals has been found as

$$0.4432 \le \zeta \le 0.5169$$

$$0.2863 \le t_s \le 0.3812$$

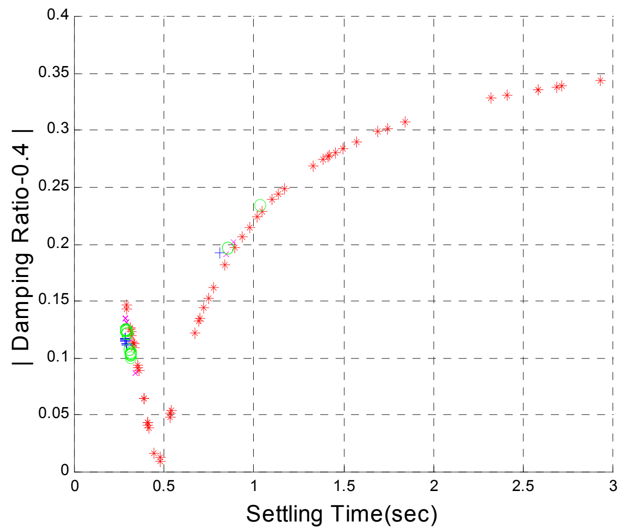

Fig. 2 Initial Population

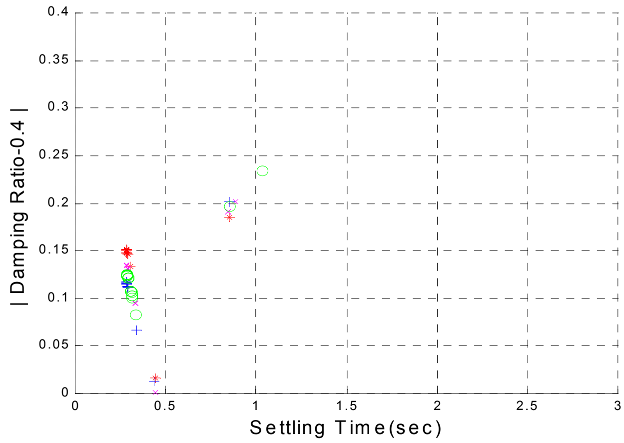

Fig. 3 Generation 100

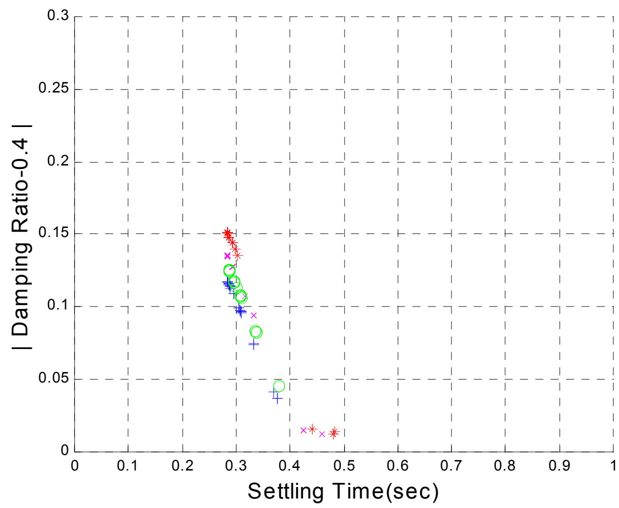

Fig.4 Generation 150

Let us choose $K_1 = 6$ from the interval given in (6). Then damping ratio and settling time of the system obtained for this value of K_1, at each flight condition are:

Flight Condition 1: $\zeta = 0.483$ and $t_s = 0.363$
Flight Condition 2: $\zeta = 0.446$ and $t_s = 0.364$
Flight Condition 3: $\zeta = 0.465$ and $t_s = 0.365$
Flight Condition 4: $\zeta = 0.455$ and $t_s = 0.367$

Unit step response of the system at each flight condition is given in Fig. 5.

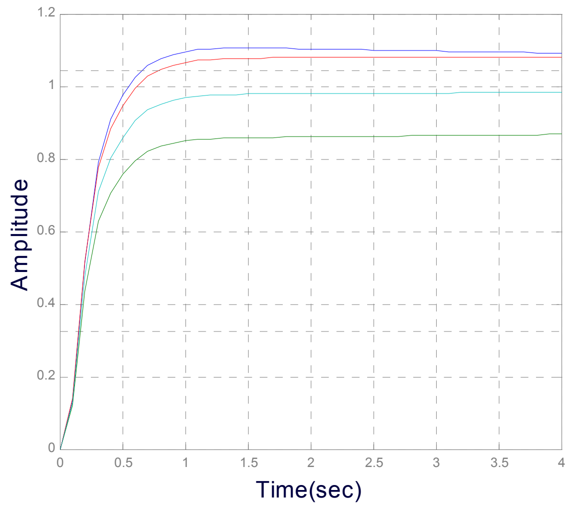

Fig. 5 Unit step response for $K_1 = 6$

5 Conclusion

A vector of two performance indices is optimized by a genetic algorithm of an aircraft, called BRAVO, pitch orientation control system for short-period approximation. Results show that genetic algorithm is an efficient optimization algorithm for the multi-model control approach.

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