Robust Procedure for Flow Coefficient Calculation

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Abstract: In synthesis of pneumatic driving systems, various parameters are applied to describe flow performance characteristics of elements. The Standard ISO6358 [1] offers two notions: the sonic conductance, *C* and the critical ratio of static pressures, *b*. In analysis there is applied the flow coefficient, μ, which defines a correspondence of the real flow to the isentropic flow. For an effective CAD systems which assists the analysis and the synthesis, it is required a fast and accurate procedure of conversion *C* and *b* into μ. In the present paper disadvantageous of two known computational techniques are discussed when used for CAD, and what more there are proposed new algorithms, which utilise the artificial neural networks (ANN) approach. Encouraging results of preliminary tests are presented.

Key–Words: pneumatics, artificial neural network, computer aided design, flow coefficient

1 Introduction

When designing the power subsystem of a pneumatic driving system, two goals may be recognised [2]: the synthesis and the analysis. In the first case, some parameters are to be defined which are necessary to choose an element from catalogues. In the analysis, dynamic properties are to be evaluated and a conclusion is made on whether the specifications are satisfied by a system. For the both tasks, the flow properties of pneumatic elements are necessary.

Typically the flow properties are described by standard parameters, which are:

- Flow coefficient K_V,
- Nominal mass (or volumetric) rate Q_n,
- Flow characteristics,
- Coefficients *C* and *b,*
- Coefficient μ,
- Armature nominal diameter d_k.

Analysis of these parameters, as well as of the methods of determination of their values, in the context of CAD systems requirements, implies some conclusions, as follows. The *C* and *b* are better matched for the synthesis, whereas the coefficient μ is more useful for the analysis. Thus, for CAD systems it is needed an effective calculation method for transformation of *C* and *b* into μ.

2 Definitions

Definitions, the way of determination, as well as applications of *C* and *b* are defined in ISO 6358 [1].

The sonic conductance *C* is defined as the ratio of the mass flow *m* through the element to the product of the input gas pressure p_1 and its density ρ_N, in the standard conditions (p_N=100000 Pa, T_N=293.15 K), for the critical flow

$$C = \frac{\dot{m}}{p_1 \cdot \rho_N} \quad \text{at} \quad T_0 = T_N \tag{1}$$

The critical pressure ratio *b* is the maximal value of p_2/p_1, at which the critical flow occurs. This is a situation, when the flow in some sub–space of an element is equal to the local sound velocity. It occurs when the inlet pressure p_1 is sufficiently high in comparison to the outlet pressure p_2. The mass flow is then proportional to the inlet pressure p_1 and inversely proportional to the root square of the inlet stream temperature T_1, and does not depend upon the outlet pressure p_2.

The flow coefficient is defined as the ratio of the real to the theoretical flow rate [4]:

$$\mu = \frac{\dot{m}}{\dot{m}_s} \tag{2}$$

The theoretical flow rate is interpreted as an isentropic stream flow rate:

$$\dot{m}_s = f \cdot \frac{p_0}{\sqrt{T_0}} \cdot \sqrt{\frac{2 \cdot k}{(k-1) \cdot R}} \cdot \varphi(\varepsilon) \tag{3}$$

where ε stands for the pressure rate p_{zew}/p_0, and the function of the ratio is defined as follows:

$$\varphi(\varepsilon) = \begin{cases} 0.2588 & \text{for...} \varepsilon \le 0.5282 \\ \sqrt{\varepsilon^{\frac{2}{k}} - \varepsilon^{\frac{k+1}{k}}} & \text{for...} \varepsilon > 0.5282 \end{cases} \tag{4}$$

where k is the adiabatic exponent.

It may be noted, that in the definition of the sonic conductance C, the pressure means its static value, whereas in the definition of the coefficient μ the pressure is defined as the total value, which takes into account the velocity of gas.

3 Problem Formulation

The analysis of the preceding definitions, and the results of the experimental data from computations let conclude [5], that in order to determine μ it is not sufficient to know C and b. The last depends also on the maximal diameter d of the element and on the instantaneous ratio of pressures. Relatively to the location of the point b to the point E (0.5282), the characteristic of μ may have two typical shapes (Fig. 1 and 2).

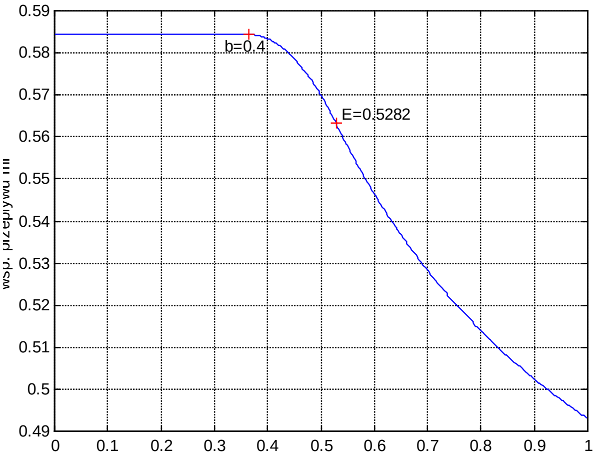

Fig 1 Exemplary relation of the coefficient μ versus the pressures ratio ε (for b less than E)

The target of the present work is to elaborate an efficient and robust procedure to calculate a value of μ, provided given are: the sound conductivity, the critical pressure ratio, the maximal diameter d and the instantaneous ratio of total pressures at the inlet and outlet p_{zew}/p_0.

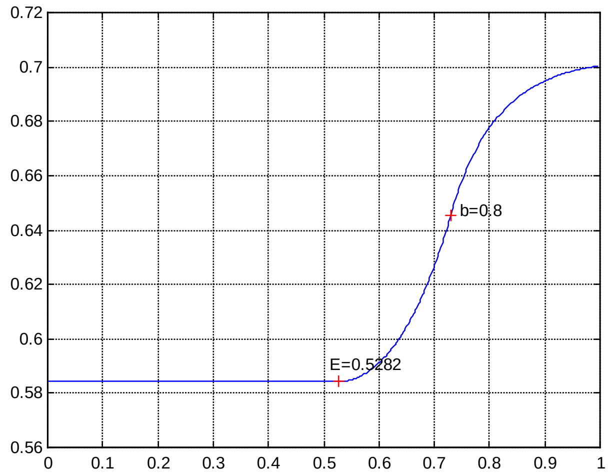

Fig. 2 Exemplary relation of the coefficient μ versus the pressures ratio ε (for b greater than E)

4 Numerical Calculation Procedures

The first proposal of the procedure was published by Iwaszko [5] in 1999. But this algorithm contains the implicit relation for the ratio of the total pressures ε. The procedure itself comprises five steps:

1. Determination of the maximal value of the Mach number, *Ma;*
2. Iterative determination the set of values of the flow coefficient as a function of *Ma*, which is randomly chosen from the range of zero to its maximal value;
3. Determination of total pressures values;
4. Approximation of the data set;
5. Computation a value of the flow coefficient for the defined value of the pressures ratios.

The foregoing procedure has two substantial disadvantageous: 1) a random choice of the *Ma* numbers makes it necessary to dense the points because it does not provide a uniform distribution of the points of approximation; and 2) due to the iteration technique its time effectiveness is limited.

Personal discussions with the Author yield a new version of the algorithm [6], which unfortunately again has some drawbacks.

When looking on the characteristics of the flow coefficient versus to the pressure ratio, one may recognise three typical sub–domains (Fig. 1 and 2). Application of the modified version of the algorithm makes necessary to solve an equation (5), which is non–linear in respect to σ (auxiliary parameter), in a broad values scope of remaining parameters.

$$\left(\sigma_{sp}^{\frac{2}{k}}-\sigma_{sp}^{\frac{k+1}{k}}\right)\cdot A^2\cdot(1-b)^2-(1-2\cdot b)\cdot\sigma_{sp}^2+ -2\cdot b\cdot\frac{p_{zew}}{p_0}\cdot\sigma+\left(\frac{p_{zew}}{p_0}\right)^2=0 \tag{5}$$

where:

$$A=\frac{0.25\cdot\Pi\cdot\dfrac{1}{\rho_N}\cdot\sqrt{\dfrac{2\cdot k\cdot R\cdot T_N}{k-1}}}{\dfrac{C}{d^2}} \tag{6}$$

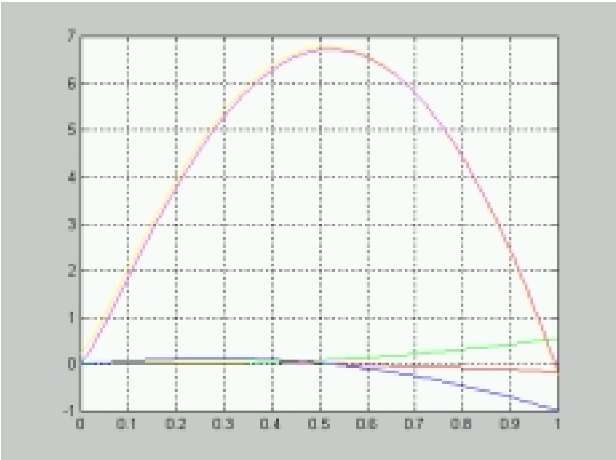

Fig. 3. Exemplary graphical interpretation of the equation (5)

But it turned out not a simple job. The auxiliary parameter σ may be computed from equation (5) iteractively rather than directly. In the case of the small declination of the curve (see Fig. 3) the decision of stopping the procedure may not be unique, thus the algorithm is unstable. So the numerical procedures are time–consuming (due to the necessary iterations), not reliable and not accurate (because of troublesome type of the equation).

These obstacles produce a motivation to device a new algorithm, which should be faster, more reliable and more accurate.

5 Neural Computing

Having applied an earlier experience [7] the authors have decided to use the Artificial Neural Networks technique for calculation of the flow coefficient μ.

The first algorithm [5] was used to produce a 80 000 vectors set, which mapped the relation of the flow coefficient μ and the sonic conductance C, the critical pressures ratio b, the diameter d and the pressure ratio p_{zew}/p. The practice scope of data for values of C, b and d was covered. The set was divided into two sub–sets: a learning and a testing one.

There were examined many feed–forward nets: with one and with two hidden layers, and with various numbers of neurons. Eventually one structure was chosen and a full learning process was completed. A satisfying statistics was obtained: a good convergence and a small value of MSE (*Mean Square Error*) (Fig. 4).

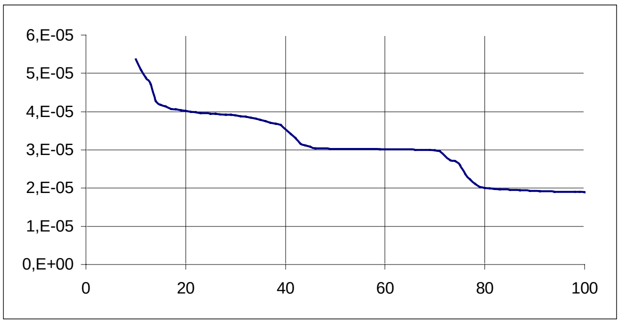

Fig. 4. Convergence of the learning process (MSE versus the epoch number: 10 to 100)

Thus results may be classified as satisfying (Fig. 5). In majority of tests we got the absolute error smaller then 0.007 (the relative error below 1%). Greater errors occurred for greater values of the flow coefficient μ. But it does not effect the general evaluation of the ANN, because there were small number of such events, and in the practice there are no pneumatic elements of the flow coefficient value $\mu \geq 0.75$.

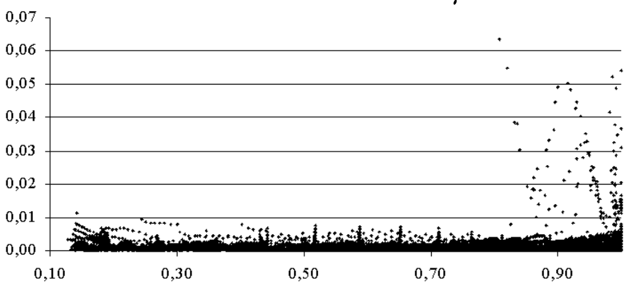

Fig. 5. The absolute error versus the flow coefficient μ

Such wholesome approach to the neural calculations is not a unique possible. As the typical is a division of the sonic flow conductance into three sub–regions, so it is possible to device a separate ANN for each one and one additional net to decide which region should be explored. This concept makes easier to synthesise optimal structures of the nets and increases the accuracy of calculations. The algorithm is as follows:

```
INPUT(C, b, d, pzew/p0)
IF b > 0.5288
THEN b= 0.5288
IF b > pzew/p0
THEN ANN1 (μ)
ELSE IF pzew/p0 < 0.5288
THEN ANN2 (μ)
ELSE ANN3 (μ)
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Once the improved numerical algorithm by Iwaszko was published, the application of ANN for calculations appeared not to be rational. But later a new hybrid approach was proposed, where numeric procedures are combined with ANNs.

The first hybrid procedure is similar to the preceding one with the three ANNs, with the logical conditions and ANN1 and ANN2 are substituted by numerical algorithms. The ANN3 remains as the only one neural part. Such approach provides very fast computations and a fairly good accuracy. Table 1 shows the distribution of the absolute error along the testing process of ANN3. Notable is, that these are preliminary results, and that all errors of greater values occur for larger values of the flow coefficient.

The second proposed hybrid procedure, in fact is a modification of the first one. Here the ANN is used to solve the equation (5). This solution may increase the accuracy, but the time of calculations remains the same. This concept yet was not tested.

Table 1. Errors of ANN3

Error range	Number of points
0.07 – 0.08	1
0.06 – 0.07	0
0.05 – 0.06	2
0.04 – 0.05	6
0.03 – 0.04	10
0.02 – 0.03	17
0.01 – 0.02	99
0.00 – 0.01	4996
SUM	5131

mean error 0.0092

6 Conclusions

There are two alternatives for conversion of C and b into μ. The first is fully iterative and time consuming, the other one entails the necessity to solve the Eqn. (5). This task is not trivial in CAD systems, because the equation is not compatible for numeric computations. This has motivated the authors for the presented work.

A satisfactory proposal of application of ANN procedures is presented. The ANN yields effective, enough accurate and reliable computing technique.

We believe that the current version may be modified in respect of the configuration and of the computing system, as well.

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