A FUZZY MATHEMATICS APPROACH TO MODELING EMERGENT HOLONIC STRUCTURES

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Abstract: - This paper proposes fuzzy entropy minimization as an emergent property of holonic structures. When applied in the context of multi-agent systems this emergent property leads to automatic clustering of the agents into holonic organizations. When modeling enterprises as software agents the property turns into an inherent characteristic of holonic virtual organizations that enables clustering the best collaborative partners and/or resources at all levels of a holonic (virtual) enterprise. Applicability of the method to on-line reconfiguration of dynamic virtual organizations is proven by a simulation example.

Key-words: - Holonic structure; emergent virtual organization; multi-agent systems; fuzzy entropy minimization; metamorphic architecture; dynamic re-configuration.

1 Introduction: Emergent Holonic Structures

A holonic structure is a holarchy of collaborative entities, where the entity is regarded as a holon. (Here the term entity is used in a broad, generic manner: entity, system, 'thing', agent). The term holon was coined by Artur Koestler to denominate entities that exhibit simultaneously both autonomy and cooperation capabilities which demand balance of the contradictory forces that define each of these properties on a behavioral level. One main characteristic of a holon is its multiple granularity manifested through replication into self-similar structures at multi-resolution levels. Such a heterarchical decomposition turns out into a nested hierarchy of fractal entities – named holarchy. A holonic entity has three levels of granularity, Fig. 1 [1]:

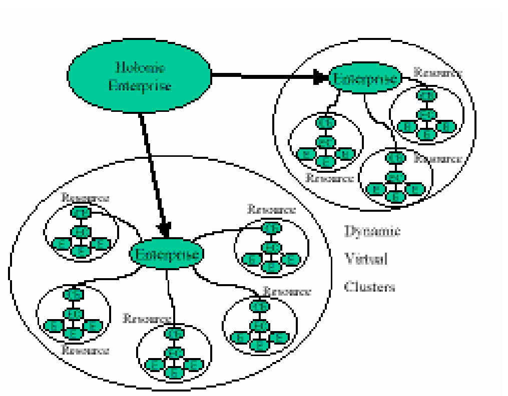

Fig.1 Dynamic Clustering Pattern in the Holonic Enterprise

1.1 Global inter-entity collaborative level

At this level several holon-entitys cluster into a collaborative holarchy to produce a product or service. The clustering criteria support maximal synergy and efficiency. Traditionally this level was regarded as a mostly static chain of customers and suppliers through which the workflow and information was moving from the end customer that required the product to the end supplier tat delivered it. In the holonic entity the supply chain paradigm is replaced by the collaborative holarhy paradigm (Fig. 1). With each collaborative partner modeled as an agent that encapsulates those abstractions relevant to the particular cooperation, a dynamic virtual cluster (Fig. 1) emerges that can be configured on-line according to the collaborative goals (e.g. by finding the best partners for the collaboration). Such a dynamic collaborative holarchy can cope with unexpected disturbances (e.g. replace a collaborative partner that can not deliver within the deadline) through on-line re-configuration of the open system it represents. It provides on-line order distribution across the available partners as well as deployment mechanisms that ensure real-time order error reporting and on-demand order tracking.

1.2 Intra-entity level

Once each entity has undertaken responsibility for the assigned part of the work, it has to organize in turn its own internal resources to deliver on time according to the coordination requirements of the collaborative cluster. Planning and dynamic scheduling of resources at this level enable functional reconfiguration and flexibility via (re)selecting functional units, (re)assigning their locations, and (re)defining their interconnections (e.g., rerouting around a broken machine, changing the functions of a multi-functional machine). This is achieved through a replication of the dynamic virtual clustering mechanism having now each resource within the entity cloned as an agent that abstracts those functional characteristics relevant to the specific task assigned by the collaborative holarchy to the partner. Re-configuration of schedules to cope with new orders or unexpected disturbances (e.g. when a machine breaks) is enabled through re-clustering of the agents representing the actual resources of the entity, Fig. 2. The main criteria for resource (re)allocation when (re)configuring the schedules are related to cost minimization achieved via multi-criteria optimization.

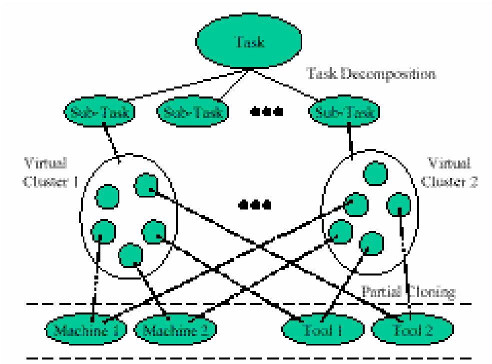

Fig.2 Task Distribution Pattern at the Intra-Enterprise level

1.3 Machine (physical agent) level

This level is concerned with the distributed control of the physical machines that actually perform the work. To

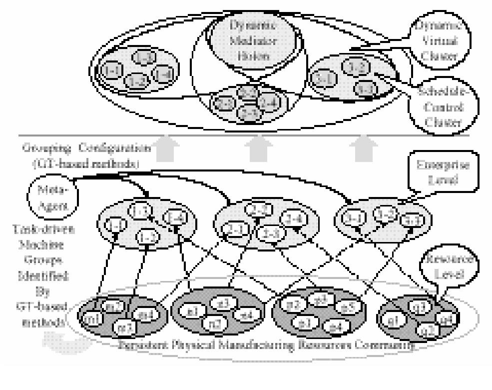

Fig.3 Physical and Logical Levels of a Holonic Entity

enable agile manufacturing through the deployment of self-reconfiguring, intelligent distributed automation elements each machine is cloned as an agent that abstracts a major role in the structural organization of any holonic entity is played by the mediator agent (Fig. 4). In the sequel we will prove that by embedding fuzzy entropy minimization within the mediator agent at the logical level - a perfect holonic structure at the physical level – is reached.

2 A Fuzzy Model for Holonic Structures

2.1 The Approach

A multi-agent system (MAS) is regarded as a dynamical system in which agents exchange information organized through reasoning into knowledge about the assigned goal [2]. Optimal knowledge corresponds to an optimal level of information organization and distribution among the agents. It seems natural to consider the entropy as a measure of the degree of order in the information spread across the multi-agent system [3]. This information is usually uncertain, requiring several ways of modeling to cope with the different aspects of the uncertainty. Fuzzy set theory offers an adequate framework [4] that requires the use of generalized fuzzy entropy [5].

One can envision the agents in the MAS as being under the influence of an information "field" which drives the inter-agent interactions towards achieving "equilibrium" with other agents with respect to this entropy [4]. The generalized fuzzy entropy is the measure of the "potential" of this field and equilibrium for the agents under this influence corresponds to an optimal organization of the information across the MAS with respect to the assigned goal's achievement. When the goal of the MAS changes (due to unexpected events, such as need to change a partner, machine break-down, etc.) the equilibrium point changes as well inducing new re-distribution of information among the agents with new emerging agent interactions. This mechanism enabling dynamic system re-configuration with re-distribution of priorities is the essence of the emergent dynamic holonic structure. In the next sub-sections of this section, we will prove that when the agents clustering into a holonic structure the MAS reaches equilibrium, which ensures optimal accomplishment the assigned goal (task).

2.2 Vagueness Modeling in MAS – The Problem

It is already well known that among the other uncertainty facets, *vagueness* deals with information that is *inconsistent* [6]. In the context of MAS, this means that the clear distinction between a possible plan reaching the imposed goal and a plan leading, on the contrary, to a very different goal is hardly distinguishable. We call *partition* the clustering configuration in which the union of all clusters is identical to the agent set when clusters are *not* overlapping. If the clusters overlap (i.e. some agents are simultaneously in two different clusters) the clustering configuration is called a *cover*. We define a *plan* as being the succession of all states through which the MAS transitions until it reaches its goal. Each state of the MAS is described by a certain clustering configuration covering the agents set.

If the information spread across the MAS is vague, one can construct only a collection of *source-plans* (i.e. sets of clustering configurations considered as sources for plans) associated with a specific global goal. There are two main differences between a *plan* and a *source-plan*. First, in a plan, the occurrence of the clustering configurations in time is clearly specified, whereas in a source-plan it is usually unknown. Secondly, in a plan, the configurations may be repeating while the source-plan includes only different configurations that can be extracted to construct a plan, following some strategy.

Starting from this uncertain information, the problem is to provide fuzzy models of MAS, useful in selecting *the least uncertain (the least vague) source-plan*.

2.3 Mathematical Formulation of the Problem

Denote by $\mathsf{A}_N = \{a_n\}_{n\in\overline{1,N}}$ the set of $N \geq 1$ agents that belong to the MAS. Based only on the initial uncertain information, one can build a family $\mathsf{P} = \{\mathsf{P}_k\}_{k\in\overline{1,K}}$, containing $K \geq 1$ collections of clustering configurations, for a preset global goal. Each P_k ($k \in \overline{1,K}$) can be referred to as a *source-plan* in the sense that it can be a source of partitions for a MAS plan. Thus, a source-plan is expressed as a collection of $M_k \geq 1$ different clustering configurations covering A_N, possible to occur during the MAS evolution towards its goal: $\mathsf{P}_k = \{P_{k,m}\}_{m\in\overline{1,M_k}}$. The only available information about P_k is the *degree of occurrence* associated to each of its configurations, $P_{k,m}$, which can be assigned as a number $\alpha_{k,m} \in [0,1]$. Thus, the corresponding degrees of occurrence are members of a two-dimension family $\{\alpha_{k,m}\}_{k\in\overline{1,K};m\in\overline{1,M_k}}$, which, as previously stated, quantifies all the available information about MAS.

In this framework, we aim to construct a measure of uncertainty, V (from "vagueness"), fuzzy-type, real-valued, defined on the set of all source-plans of A_N and optimize it in order to select the *least vague* source-plan from the family $\mathsf{P} = \{\mathsf{P}_k\}_{k\in\overline{1,K}}$:

$$\mathsf{P}_{k_0} = \underset{k\in\overline{1,K}}{\arg\operatorname{opt}} V(\mathsf{P}_k) \qquad (1)$$

where $k_o \in \overline{1,K}$.

The cost function V required in problem (1) will be constructed by using a *measure of fuzziness* [6]. We present hereafter the steps of this construction.

3 Emergence of Holonic Structures

3.1 Constructing fuzzy relations between agents

We model agent interactions through fuzzy relations considering that two agents are in relation if they exchange information. As two agents exchanging information are as well in the same cluster one can describe the clustering configurations using these fuzzy relations. The family of fuzzy relations, $\{\mathsf{R}_k\}_{k\in\overline{1,K}}$, between the agents of MAS (A_N) is built using the numbers $\{\alpha_{k,m}\}_{k\in\overline{1,K};m\in\overline{1,M_k}}$ and the family of source-plans $\{\mathsf{P}_k\}_{k\in\overline{1,K}}$. Consider $k \in \overline{1,K}$ and $m \in \overline{1,M_k}$ arbitrarily fixed. In construction of the fuzzy relation R_k, one starts from the observation that associating agents in clusters is very similar to grouping them into *compatibility* or *equivalence classes*, given a (binary) crisp relation between them. The compatibility properties of reflexivity and symmetry are fulfilled for covers (overlapped clusters), whereas the equivalence conditions of compatibility and transitivity stand for partitions. The corresponding crisp relation denoted by $R_{k,m}$, can be described by the statement: *two agents are related if they belong to the same cluster*. The facts that a and b are, respectively are not in the relation $R_{k,m}$ (where $a,b \in \mathsf{A}_N$) are expressed by "$aR_{k,m}b$" and "$a\neg R_{k,m}b$". The relation $R_{k,m}$ can also be described by means of a $N \times N$ matrix $H_{k,m} \in \Re^{N\times N}$ - the *characteristic matrix* - with elements ($H_{k,m}[i,j]$) being only 0 or 1, depending on whether the agents are or not in the same cluster. (Here, $\Re$ is the real numbers set.) Thus:

$$H_{k,m}[i,j] \stackrel{def}{=} \begin{cases} 1 \ , \ a_i R_{k,m} a_j \\ 0 \ , \ a_i \neg R_{k,m} a_j \end{cases} \tag{2}$$

$\forall i,j \in \overline{1,N}$.

This matrix is symmetric (obviously, if $aR_{k,m}b$, then $bR_{k,m}a$) and with unitary diagonal (since every agent is in the same cluster with itself). It allows us to completely specify only the configuration $P_{k,m}$, as proves the following result (see the proof in Appendix):

<u>Theorem 1.</u> *Let $P = \{A_1, A_2, ..., A_M\}$ be a clustering configuration of the agents set A_N (where A_m is a cluster, $\forall m \in \overline{1,M}$): $\mathsf{A}_N = \bigcup_{m=1}^{M} A_m$. Construct the following matrix $H \in \{0,1\}^{N\times N}$:*

$$H[i,j] \stackrel{def}{=} \begin{cases} 1 \ , \ \exists m \in \overline{1,M} \text{ so that } \{a_i, a_j\} \subseteq A_m \\ 0 \ , \ otherwise \end{cases},$$

$\forall i,j \in \overline{1,N}$.

Then P is uniquely determined by H.

This result shows that the relation $R_{k,m}$ defined by the agents' inclusion in the same cluster is uniquely assigned to the clustering configuration $P_{k,m}$ (no other configuration can be described by $R_{k,m}$). Thus, each crisp relation $R_{k,m}$ can be uniquely associated to the degree of occurrence assigned to its configuration: $\alpha_{k,m}$. Together, they can define a so-called *α-sharp-cut* of the fuzzy relation R_k, by using the equality ($=$) instead of inequality ($\geq$) in the classical definition of *α- cut*. Therefore, the crisp relation $R_{k,m}$ is a α-sharp-cut of R_k, defined for $\alpha_{k,m}$.

Consequently, we can construct an elementary fuzzy (binary) relation $\mathsf{R}_{k,m}$ whose membership matrix is expressed as the product between the characteristic matrix $H_{k,m}$,defined by (2), and the degree of occurrence $\alpha_{k,m}$, that is: $\alpha_{k,m}H_{k,m}$. This fuzzy set of $\mathsf{A}_N \times \mathsf{A}_N$ is also uniquely associated to $P_{k,m}$.

If $k \in \overline{1,K}$ is kept fixed, but m varies in the range $\overline{1,M_k}$, then a family of fuzzy elementary relations is generated: $\{\mathsf{R}_{k,m}\}_{m\in\overline{1,M_k}}$. Naturally, R_k is then defined as the fuzzy union:

$$\mathsf{R}_k \stackrel{def}{=} \bigcup_{m=1}^{M_k} \mathsf{R}_{k,m} \tag{3}$$

<u>Theorem 2</u>. *Let Q and R be two binary fuzzy relations and $\mathsf{M_Q}$, respectively $\mathsf{M_R}$ their $N \times N$ membership matrices. Denote by C the composition: $\mathsf{C} = \mathsf{Q} \circ \mathsf{R}$. Then $\mathsf{M_C} = \mathsf{M_Q} \circ \mathsf{M_R}$ (fuzzy product) and:*

1. *If Q and R are reflexive relations, C is also reflexive.*
2. *If Q and R are symmetric relations, C is also symmetric.*
3. *If $\mathsf{Q} = \mathsf{R}$ and R is a transitive relation, C is also transitive.*

It is very important to preserve the proximity property of relation R_k by composition with itself. For more details see [7].

So far, a bijective map (according to **Theorem 1**) between $\mathsf{P} = \{\mathsf{P}_k\}_{k\in\overline{1,K}}$ and $\mathsf{R} = \{\mathsf{R}_k\}_{k\in\overline{1,K}}$, say T, was constructed:

$$T(\mathsf{P}_k) = \mathsf{R}_k \ , \ \forall k \in \overline{1,K} \tag{4}$$

3.2 The Measure of Fuzziness

The next step aims to construct a measure of fuzziness over the fuzzy relations on $\mathsf{A}_N \times \mathsf{A}_N$, that will be used to select the "minimally fuzzy" relation within the set $\mathsf{R} = \{\mathsf{R}_k\}_{k\in\overline{1,K}}$.

One important class consists of measures that evaluate "the fuzziness" of a fuzzy set by taking into consideration both the set and its (fuzzy) complement. From this large class, we have selected the *Shannon measure*, derived from the generalized Shannon's function:

$$\begin{bmatrix} S:[0,1]^M \;\; \to \Re_+ \\ (x_1,\dots,x_M) \mapsto S(x) \stackrel{def}{=} -\sum_{m=1}^{M}\left[x_m \log_2 x_m + (1-x_m)\log_2(1-x_m)\right]. \end{bmatrix}$$

If the argument of this function is a probability distribution, it is referred to as *Shannon entropy*. If the argument is a membership function defining a fuzzy set, it is refereed to as *(Shannon) fuzzy entropy*. Denote the fuzzy entropy by S_μ. Then S_μ is expressed for all $k \in \overline{1,K}$ by [7]:

$$S_\mu(\mathsf{R}_k) = -\sum_{i=1}^{N}\sum_{j=1}^{N}\mathsf{M}_k[i,j]\log_2 \mathsf{M}_k[i,j] - \sum_{i=1}^{N}\sum_{j=1}^{N}\left[1-\mathsf{M}_k[i,j]\right]\log_2\left[1-\mathsf{M}_k[i,j]\right].$$

Moreover, a *force driving towards knowledge* can be determined [4], by computing the gradient of Shannon fuzzy entropy. It is interesting to remark that the amplitude of this force (its norm), expressed as:

$$\left\|\nabla S_\mu(\mathsf{R}_k)\right\| = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N}\left[\log_2 \frac{1-\mathsf{M}_k[i,j]}{\mathsf{M}_k[i,j]}\right]^2} \tag{5}$$

increases very rapidly in the vicinity of any "perfect knowledge" point.

3.3 The Uncertainty Measure

Although a unique maximum of Shannon fuzzy entropy exists, as proven by (5), we are searching for one of its minima. The required measure of uncertainty, V, is obtained by composing S_μ in with T in (4), that is: $V = S_\mu \circ T$. Notice that V is not a measure of fuzziness, because its definition domain is the set of source-plans (crisp sets) and not the set of fuzzy relations between agents (fuzzy sets). But, since T is a bijection, the optimization problem (1) is equivalent with:

$$\mathsf{P}_{k_0} = T^{-1}(\arg\min_{k\in\overline{1,K}} S_\mu(\mathsf{P}_k)) \tag{6}$$

where $k_o \in \overline{1,K}$.

3.4 Emergence of Holonic Clusters

Once one pair $(\mathsf{P}_{k_0}, \mathsf{R}_{k_0})$ has been selected by solving the problem (6) (multiple choices could be possible, since multiple minima are available), a corresponding source-plan should be identified. Two choices are possible: (8)

- List all the configurations of P_{k_0} (by extracting, eventually, those configurations for which the occurrence degree vanished in R_{k_0}):
  $\mathsf{P}_{k_0} = \{P_{k_0,1}, P_{k_0,2}, \dots, P_{k_0,M_{k_0}}\}$.
- Construct other source-plans by using not P_{k_0}, but R_{k_0}.

(9)

There is a reason for the second option. Usually, the initial available information about MAS is so vague that it is impossible to construct even consistent source-plans. This is the case, for example, when all we can set are the degrees of occurrence corresponding to clusters created only by couples of agents (as we will see in the case study, Section 4). But, it is suitable to identify at least a source-plan for problem solving. (10)

The main idea in constructing different source-plans is to evaluate the α-cuts of R_{k_0} and to arrange them in decreasing order of their membership.

The α-cuts of R_{k_0} are the crisp relations $R_{k_0,\alpha}$, for degrees of membership $\alpha \in [0,1]$. The characteristic matrix elements of $R_{k_0,\alpha}$ are defined by:

$$H_{k_0,\alpha}[i,j] \stackrel{def}{=} \begin{cases} 1 \text{ , if } M_{k_0}[i,j] \geq \alpha \\ 0 \text{ , otherwise} \end{cases} \tag{7}$$

$\forall i,j \in \overline{1,N}$.

According to **Theorem 1**, each matrix $H_{k_0,\alpha}$ in (7) generates a unique clustering configuration of agents over A_N. Thus, two categories of source-plans emerge: *equivalence* or *holonic source-plans* (when R_{k_0} is a similarity relation) and *compatibility source-plans* (when R_{k_0} is only a proximity relation).

- When the associated fuzzy relation R_{k_0} is a *similarity* one, then an interesting property of the MAS is revealed: clusters are associated in order to form new clusters, as in a "clusters within clusters" holonic-like paradigm [2]. Moreover, a (unique) similarity relation Q_{k_0} can be constructed starting from the proximity relation R_{k_0}, by computing its *transitive closure*, following the procedure described at Step 2. A. Thus, the potential holonic structure of MAS can be revealed, even when it seems to evolve in a non-holonic manner.

- When R_{k_0} is only a *proximity* relation, tolerance (compatibility) classes can be constructed as collections of eventually overlapping clusters (covers). This time, the fact that clusters could be overlapping (i.e. one or more agents can belong to different clusters simultaneously) reveals the capacity of some agents to play multiple roles by being involved in several tasks at the same time.

4 Conclusions

This paper has proven that fuzzy entropy minimization is the mechanism that organizes structures into holonic entities. An immediate area of application is the automatic reconfiguration of a failed structure with recovery of the holonic properties. In the context of the holonic enterprise – the main applicability is in finding the most appropriate collaborative partners in a virtual organization as well as recovery if one part of the holonic system breaks, by reorganizing the distributed system to achieve maximal efficiency.

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References

[1] Ulieru, M. et.al. (2001) Holonic Enterprise as an Information Ecosystem, *Proc. Workshop on "Holons – Autonomous and Cooperative Agents for the Industry"*, Autonomous Agents International Conference, Montreal, Canada, May 2-6, 2001, pp. 3-20.

[2] Mihaela Ulieru and S. Ramakhrishnan, An Approach to the Modelling of Multi-Agent Systems as Fuzzy Dynamical Systems, *Advances in Artificial Intelligence and Engineering Cybernetics, Vol. V: Multi-Agent Systems/Space-Time Logic/Neural Networks* (George Lasker, Ed.), IIAS-68-99, ISBN 0921836619, **1999**.

[3] Werner E (1996); Logical Foundations of Distributed Artificial Intelligence, *Foundations of Distributed Artificial Intelligence* (eds. O'Hare G.M.P. and Jennings N.R.) (1996) John Wiley & Sons Interscience.

[4] Subramanian, R. and Mihaela Ulieru "Behavioral analysis of multi-agent systems by dynamic modeling with application to technical and social systems", *InterSymp'99, International Conference of the Institute for Advance Studies in Systems Research, Informatics and Cybernetics,* Baden-Baden, Germany, August 2-7, 1999.

[5] Zimmermann, H-J, – *Fuzzy Set Theory And Its Applications*, Kluwer Academic (1991).

[6] Klir, G. and Folger, Tina, *Fuzzy sets, Uncertainty, and Information*, Prentice Hall, (1988).

[7] Stefanoiu, D., Ulieru, Mihaela and Norrie, D., Multi-Agent Systems Planning by Ambiguity Minimization, *Symposium on Computational Intelligence (CI'2000),* ISA'2000 Congress, Wollongong, Australia, December 2000.