An Optimization Technique for the Solution of an Enhanced Warehouse Location Problem

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Abstract: - Companies always have to secure that the distribution of their actual products meets the demand of their customers in an efficient way, they sometimes are forced to replan the distribution network. First this paper elaborates on considering the importance of considering the delivery time and a zone-oriented model for structuring a distribution network under consideration of time. In this way it is an enhanced version of the warehouse location problem, which is regarded to be a complex issue. Therefore in the second step the development of a technique to solve this model by using Simulated Annealing is described.
The described project is promoted by the Deutsche Forschungsgemeinschaft (Lu 373/18-2).

Keywords: - Warehouse location problem, zone based, time, distribution strategy, Simulated Annealing

1 Introduction

In order that companies nowadays can operate successfully, the distribution of their products has to meet the demand of their customers and has to be executed in an efficient way. To secure this, the structure of the distribution is tested from time to time regarding the quality. Sometimes the distribution of the goods has to be new planned or has to be reorganized. Reasons for restructuring of the distribution are [1]:

- The company wants to open up to a new geographical region for their products [2]. The company has grown so that their distribution structure does not offer enough capacity anymore.
- The distribution network is inapplicable to fulfil the customer demands such as Just-in-time delivery [3].
- The old distribution structure does not match with the present company structur, e.g. after a merger or the purchase of another company [4].

This task becomes more important considering that the distribution can make up to 30% of the turnover of a company, depending on the branch [5, 6].

2 Problem Formulation

Goods distribution or physical distribution in general are logistical activities, which help to bridge space and time in order to bring goods from the place of production to the different customers [7, 8]. The distribution of goods can be realized in different ways. For example it can be considered in one production chain of different companies or also from production to the ultimate consumer. Likewise it is not defined who undertakes the distribution of the goods. This could be the producing company, the recipient or an external service provider. All kinds of goods distribution have in common that it can be characterized by a distribution system. The distribution system is composed of the elements storage, transportation and order processing of which the relation to each other can be presented by a distribution structure and by a distribution strategy [9].

This research project examines and develops methods and techniques to optimize the distribution structure, which describes the physical composition of the distribution system. The distribution structure can be characterized by four elements:

- the number of warehouse levels,
- the number of warehouses per level,
- the geographical location of the warehouses, and
- the delivery area of the individual warehouses.

These are decision criterions which have to be fixed by planning the distribution network. The object of this project is to develop a technique to plan cost minimized distribution structures considering the delivery time.

2.1 Description of the classical problems

Corresponding to literature this is a very complex problem, which has been examined in a basic approach quite often. Some classical problems are [10, 11]:

- the Location Problem,
- the Multifacility Location Problem,

- the Location Allocation Problem, and
- the Warehouse Location Problem.

Beside these classical problems a series of different special cases have been examined, for example the WLP with more than one warehouse on the highest warehouse level. However the existing models for the considered problems show a number of weak points.

Due to the contact to further different companies which could not be examined in earlier projects it became obvious that there were fundamental weaknesses in the modelling of distribution structures according to the classical models. This led to this research project in which a zone-oriented distribution model has been developed under consideration of delivery time. The zone-oriented model with respect to time represents an extension to the most sufficient Warehouse Location Problem in the following respect:

- The original Warehouse Location Problem only regards costs for transportation and storage. In reality most of the distributors have to consider the delivery time, because it becomes a more crucial factor for both suppliers and customers, for example in connection with the delivery of perishable goods and spare parts.
- The transportation costs are often zone-oriented, therefore they do not depend on the distance as the crow flies or the street distance between two geographical points in form of a continuous function, but on specific costs for the transportation between each two regions. The new model considers such a zone-based transportation cost structure.

2.2 The enhanced problem

In literature there are two different basic approaches of planning the places of warehouses, the discrete and the continuous [10, 11].

2.2.1 The discrete approach

In the discrete approach a preselection has been taken place. The result is a finite set of geographical places. This set of geographical places represents a planar network, of which only the vertices are considered as potential warehouse places. With the optimization the set of vertices respectively places which in the sense of the formulation of the objective function should serve best, are determined and also in which function, for example as an entrepot, a regional warehouse or a central warehouse. In this way it is a postion determination in a network.

2.2.2 The continuous approach

In the continuous approach a whole area is considered, in which the geographical places, in which the warehouses can be settled, is freely elegible and not limited to a finite set of points. Therefore it is a position determination in the plane.

Both approaches have these aspects in common:

- the customers are pointwise spread over the considered area
- the producers are pointwise spread over the considered area
- the transportation costs between two places depend on the distance between these places.

In reality the requirements look different, so that the problems with the above approach are not illustratable. Examples for this are:

1) The number of customers is so huge, that the calculation time for the optimization on the computers grows enormously. This can lead to the situation that the particular companies have to work area-wide. Examples for this can be the mail delivery or the collection of waste material by disposal companies.

2) The customers are not individually locatable, since there is no tight and permanent relationship between customer and company, but only geographical areas can be contained in which the needs emerge and which the particular company has to cover. Examples for this can be the emergency doctor.

3) The transportation costs often depend not directly on the distance between two places but on a zone-oriented transportation structure, in which the transportation costs depend on the specific transportation cost rate for respectively two zones.

Intention of the research project is the development of efficient ways for structuring the distribution of goods with the help of a time and zone-oriented approach.

3 CONCEPTION OF A TIME- AND ZONE-ORIENTED APPROACH FOR STRUCTURING MULTILEVEL DISTRIBUTION SYSTEMS WITH CONSIDERATION OF A DISTRIBUTION STRATEGY

This led to the development of a zone-oriented network model, which can be described as follows:

- The surveyed area in which the goods are supposed to be distributed is divided into a finite set of disjoint zones which cover the whole area.
- In each zone there is exactly one customer. If in reality there exist more than one customer in one zone the characteristics of all customers in one zone will be subsumed to one customer. If there is no customer in one zone the demand of this customer regarding each product will be zero.
- In each zone there is exactly one warehouse. If in reality there exist more than one warehouse in one

zone, the characteristics of all warehouses will be subsumed to one warehouse. If there is no warehouse in one zone, the handling of the goods of this warehouse regarding each product will be zero.

- In each zone there is exactly one producer. If in reality there exist more than one producer in one zone, the characteristics of all producers in one zone will be subsumed to one producer. If there is no producer in one zone, the lot size of this producer regarding each product will be zero.
- Transports of all products from each zone to another can take place, especially within one zone. If no transport should take place between two zones, the transport quantity will be zero.
- The transportation costs depend on the transport quantity, as well as on the starting and ending zone of the particular transports.

3.1 Consideration of the time

Often not only the warehouse and transportation costs are deciding for the determination of places but also further factors. That way the factor time becomes more and more important. This factor should be considered in the zone-oriented network model and this can be done in two ways:

1) For the distribution structure which has to be calculated, a fixed time is determined, in which all customers have to be supplied. Furthermore it is stated how many kilometres a transport vehicle can cover within one hour on average. Afterwards the maximum distance which may not be exceeded in order to supply the particular customer in time is calculated from the given delivery time and the average transport speed. Eventually a restriction is defined which says that no customer is supposed to be further away than the calculated maximum distance to the warehouse to which it belongs to. A distribution structure, which does not fulfil this restriction, is in this sense not allowed.

2) Penalty costs represent a method in which certain restrictions, in this case time restrictions, can be treated variable, by not taking them as fixed constraints which restrict the space for valid solutions but as additional costs. The idea is to penalize solutions with a marginal outcast against the time restriction less than those which infringe the time restriction more. The stronger the break of the restriction is, the higher the penalty costs, which are taken directly into the objective function, will be. In this way it is possible and permitted to generate good solutions in the optimization technique for each particular problem which represent substantial less warehouse and transport costs than others if accepted that the time restriction might possibly be violated.

In this research project, where the difficulty of structuring the goods distribution is considered, it is the aim to secure that each customer can be supplied within one in advance determined and fixed time span, by one of the warehouses of the company, which wants to optimize its distribution structure. The time needed to transport something from a warehouse to a customer can be calculated using a time matrix in which the transport time of each zone pair is held on. The time matrix is assumed to be known. With the help of the needed transport time and the provided delivery time it can be pinpointed if it is possible to provide the customers in the different zones on time. If no timely delivery is possible the time difference is calculated and the penalty costs will be determined with the help of a polynomial. Also the value of the goods which will be delivered with a delay, will be taken into the polynomial, so that this way a weighting of the particular zones is undertaken. The exact formula of the penalty costs can be described as follows:

3.2 Variables

a^P, b^P, c^P = polynomial coefficients of the penalty costs

$\Delta t(x, y)$ = needed delay time to deliver goods from zone x to y in comparison to the guaranteed delivery time

$L_A(x)$ = zone of the entrepot of zone x

$C^P(x)$ = value of the products of zone x

3.3 Penalty costs of zone x

$$K^P(x) = \begin{cases} a^P \cdot C^P(x) \cdot \Delta t(x, L_A(x))^2 + b^P \cdot C^P(x) \cdot \Delta t(x, L_A(x)) + c^P & \text{for} \quad \Delta t(x, L_A(x)) > 0 \\ 0 & \text{for} \quad \Delta t(x, L_A(x)) < 0 \end{cases} \tag{1}$$

In literature parallels can be found in the linear programming [12, 13, 14]. There is also the possibility to take additional factors into consideration using restrictions. And there is also a number of works about the consideration of these restrictions in the objective function by implementing the Lagrange-Relaxation [15, 16, 17]. But in this context the restrictions are limited to linear functions. This restriction is not undertaken in this project.

3.4 Consideration of the distribution strategy

The distribution is made up of the realization of the distribution structure and of the distribution strategy. In the distribution strategy it is stated at which point of time the managing clerk of a warehouse orders what quantum of a product. In order to consider this distribution strategy, there are two possibilities:

1) By calculating the value of the objective function reality is simulated. That means, that a period is taken out, the customer demands are simulated in a realistic way, as well as the stock quantity of the warehouse and the additional delivery from the superior warehouses.

2) The impacts of the delivery strategy on the warehouse stock, the standard of the service and their impacts on the distribution are calculated and are taken into the model. This can be realized by determining by the the service level of the warehouse. In the service level it is stated the possibility that the customers demand is fulfilled by the particular warehouse. If the directly superior warehouse can not deliver, the proximate superior warehouse delivers. This can be pictured numerically in the mathematical model with the help of probabilities in terms of the service level.

In earlier projects, possibility number 1 has been realized. But different problems and disadvantages occured in the implementation. On the one hand for the simulation runs a number of data is needed. Especially all customer demands over a longer period of time have to be recorded contemporary. But in reality companies usually only record the outward stock movement which often does not match with the customer demands, for example because the warehouse did not have certain products available and that way could either not deliver at all or, if possible and acceptable for the customer, could bring an equal item to the customer. That means that there are problems with the data gathering because of a higher effort for the company. On the other hand the simulation runs, which are undertaken with each calculation of the objective function during the optimization technique, are time-consuming.

Because of these reasons possibility 2 is realized in this research project. The used definiton of the service level is attached with the value and represents the value percentage of the delivered goods of one warehouse in relation to the whole order value [18, 19].

3.5 IMPLEMENTATION OF COMPANY INSPECTIONS

In the context of company inspections data have been collected and extensive talks have been conducted with different firms. During these talks the above mentioned first basic conception of the model for the optimization of distribution structure has been critically analyzed and discussed. Because of the collected data and the discussions the model has been revised and was adjusted according to the real conditions. The results of this work step will be shortly presented below.

3.5.1 Transferability of the model on further questions

The conception is independent of the direction of the material flow.This means in particular, that it is not relevant for the model if it is used for the distribution or the collection of goods. Therefore this model and the thereon constructing optimization technique of distribution structures can not only be used to solve problems in distribution, but also to the collection of material. This takes succesfully place in the limits of the company inspection and on the validating of the model after the computational implementation by optimizing reloading places for the communal waste disposal. Crucial by this modified practice is the interpretation of the different storage stage that was adapted to the new situation. By this way, the field of applications of the optimizing technique for distribution structures has been increased enormously.

3.5.2 The way to consider time

The both possibilities of the consideration of the factor time by restrictions or by penalty costs have been described above. Whilst the talks during the company inspection, different company situations where elaborated. According to this the different roles of time in the distribution of goods, the two possibilities to take time into account in the optimization of distribution structures have been analyzed and discussed critically. As a result, it has been decided to consider time in form of penalty costs in the objective function. The reason therefore lies in the substantial variability and the better possibilities of basic approach to the particular problem statements. In the following this will be shown with the help of some characteristic examples:

1) In different areas of the spare parts supply in the sector of mechanical engineering and electrical engineering the competition is very big and the spare part provision is on a high level. The logistics and the timely supply is important in these areas. For logistics service enterprises benchmarks like the delivery service and the therewith associated possibility of short-dated supply of spare parts is an important and essential feature of quality to be competitive. According to this the maximal distance between a customer and the superordinated storage becomes a restriction that must not be exceeded. Therefore the presentation of time by means of restrictions is suitable. The same effect can be achieved by successively increasing the penalty costs.

2) In the supply of the retail industry with high quality articles of the consumer goods industry, especially in the area of the food industry, the factor time also is important for the logistics service enterprises. In contrast to the above mentioned situation of the spare part provision, it is sufficient to deliver the goods in 24 hours. If there are fulfilled particular minimum standards, the factor time plays a minor role compared with the securing of the freshness of food. Therefor the factor time is to take into account in a variable way and to be included into the consideration of the cost reduction. For example if a customer is geographically far away from the other customers and the producers, there must be created a stock near to this single customer under consideration of time by means of restrictions, although it is economically not useful. By using penalty costs it is allowed that the superordinated storage is far away from this customer so that it is impossible to deliver this customer in the right time under normal circumstances. Instead of this penalty costs are accepted. They can show an increased effort, for example for an urgent transport.

3) In other areas of economy the factor time plays a minor role to render the logistical service to the customer. E.g. for the communal waste disposal it is sufficient if the removal of the waste is conducted on the fixed day. In these cases the reducing of costs is important. This cost-savings in the example of the disposal economy can be computed directly by the time saved by the collecting vehicles, which do the transport between the clients and the container reloading point. Thereby there are reducing of costs dependent from the saved time, which must be subtracted from the costs. This can not be presented by means of restrictions. However, due to the great variability of the penalty costs it is possible to show up cost-savings with help of negative penalty costs. In this manner the storage and transportation costs are reduced about the corresponding height of the cost-savings.

Beyond the great possibilities of adaptation on the described problem there are further evidences especially for a successful use of this method in different areas of operation research, that the use of penalty costs is an appropriate method for consideration of time [20, 21].

3.5.3 IMPLEMENTATION OF A PROTOTYPE

In order to solve the described model a technique was developed and implemented. Thereby Simulated Annealing has been used as an optimization method of the artificial intelligence. The method of Simulated Annealing is kept very general in consideration of the starting problem and the form of the implementation. Therein, on the one hand, the universal applicability of this technique on the different optimization problems, is founded. On the other hand this requires a higher effort in the development of the particular optimization technique and the adaptation on the here given problem of the structuring of goods distribution systems. That is why in the following the procedure of the program of the prototype, which will be implemented in this setting of this research project, is outlined.

An in the program saved goods distribution structure is called individual. At first a simple goods distribution structure, as a starting individual, is originated. This can be done in the way, that, for example, all customers are supplied by one central warehouse. This individual is evaluated with the help of an objective function. In the following, this individual has to be modified in the way that a new individual is generated. This operation is named mutation. In these mutations it is important to pay attention that the considered individual can in perpetuity adopt any imaginable form of the goods distribution structure. This leads to two problems:

1) A number of different mutation possibilities has to exist which help to generate every allowed goods distribution structure from the starting individual. This leads to the development of the following mutations:

- generation of a new warehouse,
- elimination of an existing warehouse,
- displacement of a warehouse in zones,
- reassignment of a zone to another warehouse, and
- reassignment of a warehouse in the warehouse hierarchy.

2) In order to secure that the complete tolerable solution space within the scope of a program pass is scanned, it has to be secured that, with the help of a suitable selection algorithm, all kinds of mutations are respectively often invoked.

When the mutation is selected and performed, a new individual is available, which is evaluated with the help of the objective function and is compared with the old individual. If the new individual is better, the new individual will be used for further calculations. If the old individual is better another selection algorithm helps us to decide with which individual it is better to continue the calculations. This selection algorithm presents another problem. Because a number of process parameters, like the analog to the above described temperature, are taken in. The position of these parameters influences decisive the convergence velocity and the quality of the procedure. By implementing the process you can see that the mathematical model gives up to now the opportunity to fulfill all required conditions. That way it can be isolated to the particular problem of the enterprise, for example regarding the wanted number of store levels. That way the process meets the demand for a possibility of a close to reality presentation of practical problems.

4 Conclusion

This paper describes how to solve an enhanced warehouse location problem under consideration of time. First the problem and its practical relevance were described. Thereby the classical discrete and contuinuous approaches were described and the zone-oriented concept explained. Additional to this the consideration of delivery time and the distribution strategy were discussed. The different ways how to consider time by restrictions and according to the Lagrangian relaxation were analysed. Finally some important aspects for the computer aided implementation of such a described procedure.

Beyond this it has been made possible to make different means of transport mathematical comparable. Consequently the problem can be enlarged by including the intermodal transport in the next step. This represents the basis, to develop a process for structuring worldwide distribution systems including intermodality.

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