Genetic Algorithm and Singular Value Decomposition in the Design of Fuzzy Systems for the Modelling of Explosive Cutting Process

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Abstract: -Genetic Algorithm (GA) and Singular Value decomposition (SVD) are deployed for optimal domains selection of antecedents and consequents, respectively, of associated fuzzy sets in fuzzy systems which are used for modelling of explosive cutting process of plates by shaped charges. The aim of such modelling is to show how the depth of penetration varies with the variation of important parameters namely, apex angle, standoff, liner thickness, and mass of charge. It is demonstrated that SVD can be effectively used to fine tuning of such fuzzy models obtained either from direct matching method or designed by GA method. Such application of SVD will highly improve the performance of genetically designed fuzzy systems to model the very complex process of explosive cutting of plates by shaped charges.

Key-Words:- Fuzzy systems, Explosive Cutting Process, Genetic Algorithms, SVD

1 Introduction

Modelling of processes and system identification using input-output data have always attracted many research efforts. In fact, system identification techniques are applied in many fields in order to model and predict the behaviours of unknown and/or very complex systems based on given input-output data [1]. Theoretically, in order to model a system, it is required to understand the explicit mathematical input-output relationship precisely. Such explicit mathematical modelling is, however, very difficult and is not readily tractable in poorly understood systems. Alternatively, soft-computing methods [2], which concern computation in imprecise environment, have gained significant attention. The main components of soft computing, namely, fuzzy-logic, neural network, and genetic algorithm have shown great ability in solving complex non-linear system identification and control problems. Several research efforts have been expended to use evolutionary methods as effective tools for system identification [3-6]. Among these methodologies, fuzzy rule-based systems have been an active research field for their unique ability to build models based on experimental data. The concept of fuzzy sets which deal with uncertain or vague information, paved the way for applying them to real and complex tasks [7]. Indeed, fuzzy-logic, coupled with rule-based systems, has the ability of modelling of the approximate and imprecise reasoning processes which is common in human thinking or human problem solving. This results a policy which can be accordingly evaluated mathematically by using fuzzy set theory. Therefore, fuzzy systems as universal approximator [8-10] can be effectively employed to perform input-output mapping. Such fuzzy systems can be iteratively designed using different evolutionary search methods [11-13] or using hybrid learning rule in ANFIS [14]. In fact, these fuzzy systems are trained by examples (X_i, y_i) (*i=1, 2, …, M*) in terms of input-output pairs. Recently, a combination of orthogonal transformation and back propagation methods has been proposed to train a candidate fuzzy model and to remove its unnecessary fuzzy rules [15]. In some other very recent works, it is also shown that Singular Value Decomposition (SVD) can be used to enhance the performance of both fuzzy and GMDH-type neural networks models obtained using some simple heuristic approaches[16-17].

Explosive cutting of plates using shaped charges is one of the processes in mechanical engineering in which the physical interactions of various involved parameters are rather complex [18]. In fact, during the last few decades the use of explosives as a source of energy has found many applications in engineering. The main difference between explosives, magnetomotive forces, impact and any other source of energy is that a very large amount of energy is made available to do work in a very short period of time. Explosives are now used in such diverse fields as welding, bulk cladding of plates, forming, sizing, powder compaction, hardening, and cutting. The use of explosives is not due to solely to their speed, but also that sometimes there may be no other way of achieving the same results as in explosive welding of dissimilar metals. In cutting metals using linear shaped charge, an explosive charge with a metallic liner is placed at a certain distance from the metal part. The cutting action is the consequence of the development of a very high-speed jet of molten metal produced by the collapse of the liner. A linear shaped charge consists of long metal liner backed with an explosive charge as shown in Fig.1.

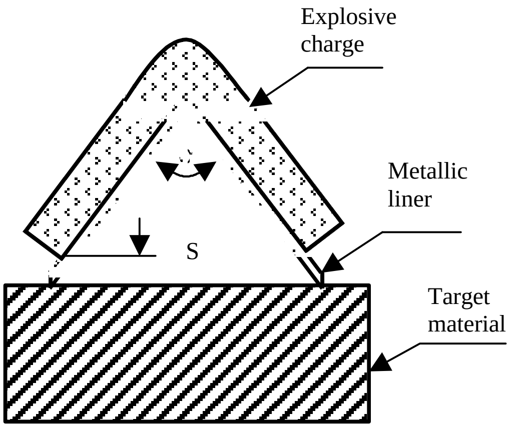

Fig. 1: A linear shape charge: S=Standoff distance; α= Apex angle

In this paper, it is shown that optimally-designed fuzzy systems using GA and SVD can effectively model the depth of penetration as a function of four important input parameters in explosive cutting process, namely, the apex angle, the standoff, the liner thickness, and the mass of charge. In this way, firstly, a simple heuristic method for designing fuzzy systems from input-output data pairs is considered [19]. Secondly, it is shown that genetic algorithms [20] can be effectively used to modify the initially symmetrical fuzzy partitioning of antecedents of the associated fuzzy sets so that the performance of such fuzzy model is enhanced. Thirdly, SVD can now be employed to optimally modify the consequents part of the associated fuzzy sets of the model with symmetric triangular membership functions in antecedents [16] and/or the one with genetically evolved asymmetric fuzzy partitioning of the antecedents. In this way, it is demonstrated that such application of SVD will highly improve the performance of the fuzzy models. In particular, it is shown that GA and SVD can effectively design and tune the fuzzy systems for the modelling the data that obtained from high-energy rate explosive cutting process. Finally, the behaviour of such optimally fuzzy models can be further improved by optimising a set of weighting factor applied to the fuzzy rules.

2 Modelling Using Fuzzy Systems

Fuzzy systems that consist of a set of fuzzy IF-THEN rules can be used in modelling in order to map inputs to outputs. The formal definition of the identification problem is to find a function $\hat{f}$ so that it can be approximately used instead of the actual one, f, in order to predict output $\hat{y}$ for a given input vector $X=(x_1, x_2, x_3, ..., x_n)$ as close as possible to its actual output y. Therefore, given *M* observations of multi-input-single-output data pairs so that

$$y_i = f(x_{i1}, x_{i2}, x_{i3}, ..., x_{in}) \quad (i=1,2,\ldots,M), \qquad (1)$$

it is now possible to build a look-up table to be used to train a fuzzy system to predict the output values $\hat{y}_i$ for any given input vector $X=(x_{i1}, x_{i2}, x_{i3}, ..., x_{in})$, that is

$$\hat{y}_i = \hat{f}(x_{i1}, x_{i2}, x_{i3}, ..., x_{in}) \qquad (i=1,2,\ldots,M). \quad (2)$$

The problem is now to determine a fuzzy system so that the difference between the actual output and the predicted one is minimised, that is

$$\sum_{i=1}^{M} [\hat{f}(x_{i1}, x_{i2}, x_{i3}, ..., x_{in}) - y_i]^2 \rightarrow \min. \qquad (3)$$

In this way, a set of linguistic fuzzy IF-THEN rules is designed to approximate f by $\hat{f}$ using *M* observations of n-input-single-output data pairs (X_i, y_i) (*i=1, 2, …, M*). The fuzzy rules embodied in such fuzzy models can be conveniently expressed using the following generic form

$$Rule_l : IF \quad x_1 \ is\ A_l^{(j_1)}\ AND\ x_2\ is\ A_l^{(j_2)}\ AND, \ ..., x_n\ is\ A_l^{(j_n)}\ THEN\ y\ is\ B_l^{(k)} \qquad (4)$$

in which $j_i \in \{1,2,...,r\}$ and $k \in \{1,2,...,s\}$. The entire fuzzy sets in x_i and y_i spaces are given as

$$A^{(i)} = \{A^{(1)}, A^{(2)}, A^{(3)}, ..., A^{(r)}\} \qquad (5a)$$

and

$$B^{(i)} = \{B^{(1)}, B^{(2)}, B^{(3)}, ..., B^{(s)}\}. \qquad (5b)$$

These entire fuzzy sets are first assumed symmetric triangular defined on the domains $[-a_i, +b_i]$ (*i=1,2,…,n*) and $[-g, +q]$, respectively. In this way, the domains are appropriately selected so that all the fuzzy sets are complete; that is for any $x_i \in [-a_i, +b_i]$ there exist $A^{(j)}$ in equation (5a) such that the degree of membership function in non-zero, $m_{A^{(j)}}(x_i) \neq 0$. Each fuzzy set $A^{(j)}$ in which $j \in \{1,2,...,r\}$ is represented by triangular membership functions in the form

$$m_{A^{(j)}}(x) = triangle(x; a_j, b_j, c_j) \quad (6)$$

where a_j, b_j, c_j, are adjustable parameters in antecedents. It is evident that that the number of such parameters involved in the antecedents of fuzzy models can be readily calculated as *3(nr-2)*, where *n* is the dimension of input vector and *r* is the number of fuzzy sets in each antecedent After defining fuzzy sets to completely cover the input and output spaces, it is now possible to generate one rule from each input-output data pairs [19]. However, it can be readily observed that more than one rule is possible for each input-output data pairs. It is evident that rules with the same antecedents but different consequents are reduced to the one having the highest degree. Genetic algorithm and singular value decomposition are then applied to fine-tuning of the both antecedents and the consequents fuzzy partitioning, respectively. The fuzzy rule expressed in equation (4) is a fuzzy relation in $U \times V$ in which $A^{(i)}$ and B are fuzzy sets in U_i and V so that $U = U_1 \times U_2 \times U_3 \times ... \times U_n$ and $Rule = A^{(j_1)} \times A^{(j_2)} \times A^{(j_3)} \times ... \times A^{(j_n)} \to B$. It is evident that the input vector $X = (x_1, x_2, x_3, ..., x_n)^T \in U$ and $y \in V$. Using Mamdani algebraic product implication, the degree of such local fuzzy IF-THEN rule can be evaluated in the form

$$m_{Rule_l} = m_U(x_1, x_2, ..., x_n) \bullet m_{B_l^{(k)}}(y) \quad (7)$$

where

$U = A_l^{(j_1)} \times A_l^{(j_2)} \times ... \times A_l^{(j_n)}$ and

$$m_U(x_1, x_2, ..., x_n) = \prod_{i=1}^{n} m_{A_l^{(i)}}(x_i) . \quad (8)$$

In these equations $m_{A_l^{(j_i)}}$ and $m_{B_l^{(k)}}$ represent the degree of membership of input x_i and output y regarding their l_{th} fuzzy rule's linguistic value, $A_l^{(j_i)}$ and $B_l^{(k)}$, respectively. If the fuzzy set $B_l^{(k)}$ is normal with centre $\hat{y}_l$, then using singleton fuzzifier, product inference engine, and centre average defuzzifier leads to the fuzzy system in the form

$$f(X) = \frac{\sum_{l=1}^{N} \bar{y}_{Rule_l} \left(\prod_{i=1}^{n} m_{A_l^{(j_i)}}(x_i) \right)}{\sum_{l=1}^{N} \left(\prod_{i=1}^{n} m_{A_l^{(j_i)}}(x_i) \right)} \quad (9)$$

when a certain set containing N fuzzy rules in the form of equation (4) is available. It is then clear that in equation (9) $\bar{y}_{Rule_l} \in \{\bar{y}_1, \bar{y}_2, ..., \bar{y}_s\}$ where s is the total number of fuzzy linguistic terms of the consequence. Since some of rules' consequences may be the same, equation (9) can be alternatively represented in the following linear regression form

$$f(X) = \sum_{k=1}^{s} p_k(X)\, \bar{y}_k + D \quad (10)$$

where D is the difference between $f(X)$ and corresponding actual output, y, and

$$p_k(X) = \frac{\sum_{t=1}^{n_k} \left(\prod_{i=1}^{n} m_{A_t^{(j_i)}}(x_i) \right)}{\sum_{l=1}^{N} \left(\prod_{i=1}^{n} m_{A_l^{(j_i)}}(x_i) \right)} \qquad \text{if } n_k \geq 1$$

$$p_k(X) = 0. \qquad \text{if } n_k = 0. \quad (11)$$

In equation (11), n_k is the number of the similar $\bar{y}_k \in \{\bar{y}_1, \bar{y}_2, ..., \bar{y}_s\}$ which appear when the numerator of equation (9) is expanded for $\bar{y}_{Rule_l}$ It is therefore evident that equation (10) can be readily expressed in a matrix form for a given *M* input-output data pairs (X_i, y_i) (*i=1, 2, …, M*) in the form

$$Y = P\, \bar{Y}_{Rule} + D \quad (12)$$

where $Y = [y_1, y_2, ..., y_M]^T \in \Re^M$ and $P = [p_1, p_2, ..., p_s]^T \in \Re^{M \times S}$ with $p_i = [p_1, p_2, ..., p_M]^T \in \Re^M$. Such firing strength matrix P and associated fuzzy consequences are obtained when input and output spaces are partitioned into certain number of fuzzy sets and, in addition, the related fuzzy rules are extracted based on direct matching [21]. It is evident that the number of available training data pairs is usually larger than the fuzzy linguistic terms of the consequence, that is $M \geq S$. This situation turns the equation (12) into a least squares estimation process in terms of unknowns, $\bar{y}_{Rule}$, so that the difference D is minimized. The governing normal equations can be expressed in the form

$$\bar{Y}_{Rule} = (P^T P)^{-1} P^T Y \quad . \tag{13}$$

Such modification of output fuzzy partitioning leads to better approximation of given data pairs in terms of minimization of difference vector D. However, such direct solution from normal equations is rather susceptible to round off error and, more importantly, to the singularity of these equations.

3 Application of Genetic Algorithm to the Design of Fuzzy Models

The incorporation of genetic algorithm into the design of such fuzzy models starts by representing the *3(nr-2)* real-value parameters of $\{a_j, b_j, c_j\}$ as a string of concatenated sub-strings of binary digits. Thus, each such sub-string represents the fuzzy partitioning of antecedents of fuzzy rules embodied in such fuzzy models in binary coded form. The fitness, Φ, of each entire string of binary digits which represents a fuzzy system to model the explosive cutting process is readily evaluated in the form of

$$\Phi = 1/E \tag{14}$$

where E is the objective function given by equation (3) and is minimized through an evolutionary process by maximization the fitness, Φ. The evolutionary process starts by randomly generating an initial population of binary strings each as a candidate solution. Then, using the standard genetic operations of tournament selection, crossover, and mutation [20], entire populations of binary string are caused to improve gradually. In this way, fuzzy models of explosive cutting process with progressively increasing fitness, Φ, are produced until no further significant improvement is achievable.

4 Application of Singular Value Decomposition to the Design of Fuzzy Models

Singular Value Decomposition (SVD) is the method for solving most linear least squares problems that some singularities may exist in the normal equations. The SVD of a matrix, $P \in \Re^{M \times S}$, is a factorisation of the matrix into the product of three matrices, a column-orthogonal matrix $U \in \Re^{M \times S}$, a diagonal matrix $W \in \Re^{S \times S}$ with non-negative elements (singular values), and an orthogonal matrix $V \in \Re^{S \times S}$ such that

$$P = U\, W\, V^T \quad . \tag{15}$$

The most popular technique for computing the SVD was originally proposed in [22]. The problem of optimal selection of $\bar{y}_{Rule}$ in equations (9) (12) is firstly reduced to finding the modified inversion of diagonal matrix W [23] in which the reciprocals of zero or near zero singulars (according to a threshold) are set to zero. Then, such optimal $\bar{y}_{Rule}$ are obtained using the following relation

$$\bar{Y}_{Rule} = V\, [diag(1/w_j)]\, U^T\, Y \quad . \tag{16}$$

Such procedure which involves the direct matching extraction fuzzy rules together with SVD approach of finding the optimal fuzzy consequences, $\bar{y}_{Rule}$, is noniterative and, therefore, can be effectively employed to build fuzzy models with the presumed symmetric triangular fuzzy partitioning of antecedents or with the asymmetric triangular one obtained using genetic algorithm application discussed before.

5 Fuzzy Modelling of Explosive Cutting Process of Plates by Shaped Charges

The parameters of interest in this multi-input single-output system that affect the magnitude of the depth of penetration are the liner material, the explosive material, the liner shape, the apex angle, the liner thickness, the explosive weight, and distribution, and the standoff distance. Among these parameters, the liner and explosive material together with the liner shape have been kept fixed. The liner material, explosive material, and the liner shape have been selected as Copper/Polymer, SX2, and 'V' shape, respectively. Accordingly, there has been a total number of 43 input-output experimental data considering 4 input parameters, namely, the apex angle, the standoff, the liner thickness, and the explosive weight, in four different groups, which has been thoroughly explained in detail in [16]. In order to model the 4-input-single-output set of data, a fuzzy system with 5 linguistic terms in each antecedent and 35 linguistic terms in consequence was considered, that is, r=5 and s=35. Then by applying the direct matching fuzzy rule extraction method, a set consisting of 29 rules was constructed. In order to tune the obtained fuzzy system, SVD and GA are then applied separately to the consequents and antecedents fuzzy partitioning, respectively. The results of these singular values in four different cases are depicted in Fig. 2. It is clearly evident from this figure that a significant amount of singular values are zero (or nearly zero according to threshold) and thus leads to an optimal selection of the consequents using equation (16).

Fig.3 shows the behaviour of the fuzzy model when its rules were extracted using direct matching with symmetric partitioning. Such behaviour has been improved significantly, as shown in Fig. 4, when the obtained model was further tuned using singular value decomposition of the constructed firing matrix.

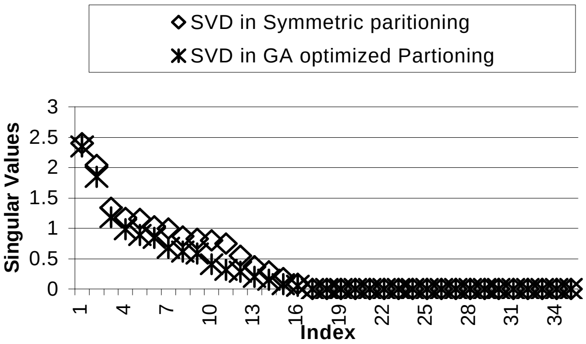

Figure 2: Distribution of singular values of firing strength matrix

Fig. 5 shows the behaviour of the fuzzy model when its rules were extracted using direct matching with asymmetric fuzzy partitioning designed by GA. Such behaviour has been again improved significantly, as shown in Fig. 6, when the obtained model was further tuned using singular value decomposition of the constructed firing matrix. Finally, the fuzzy model designed by GA and SVD methods was further fine tuned using an optimised set of weighting factors for each rules and its very good performance is depicted in Fig. 7.

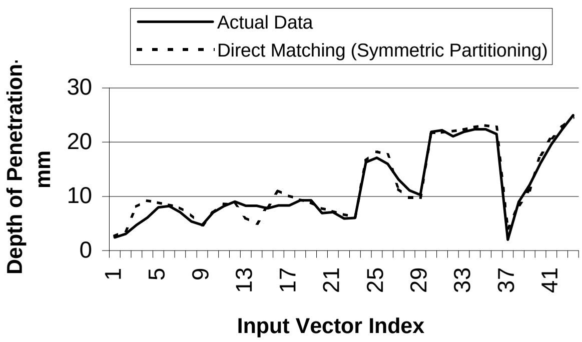

Figure 3: Variation of penetration with input data samples (direct matching fuzzy rules extraction)

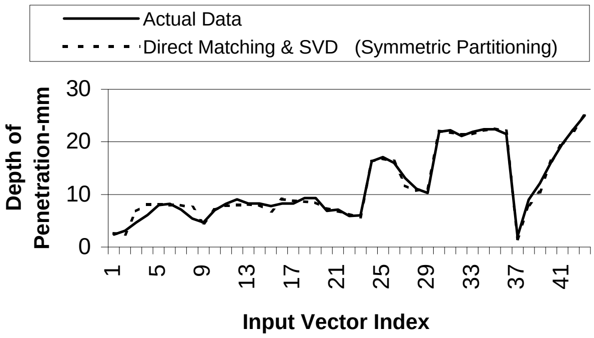

Figure 4: Variation of penetration with input data samples (tuned- fuzzy model by SVD)

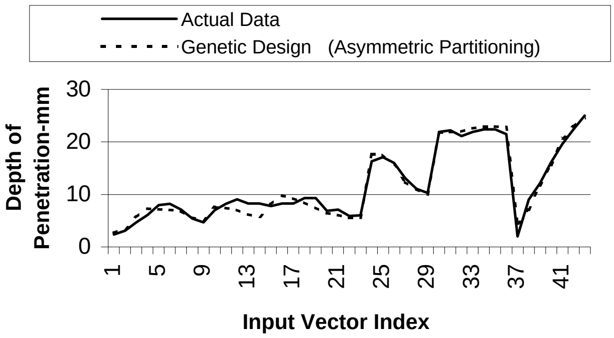

Figure 5: Variation of penetration with input data samples (Genetically partitioned)

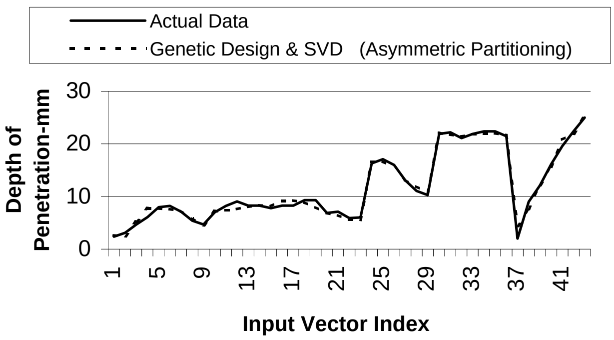

Figure 6: Variation of penetration with input data samples (Genetically partitioned & tuned by SVD)

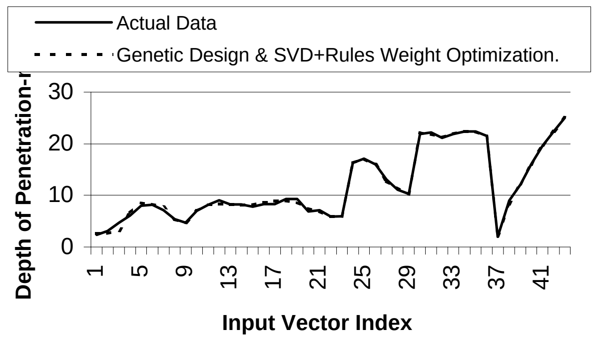

Figure 7: Variation of penetration with input data samples (Genetically partitioned, tuned by SVD with fuzzy rules' weights optimization)

6 Conclusion

Fuzzy systems have been successfully used for the modelling of very complex process of explosive cutting of plates by shaped charges by direct matching and GAs. In this way, it has been shown that fuzzy systems provide effective means to model the depth of penetration according to different input. Moreover, it has been shown that singular value decomposition can significantly improve the performance of such fuzzy systems that can be constructed by either direct matching alone or by direct matching and GAs application.

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