Solving a Multiple Criteria Decision-Making Problem Under Uncertainty Using Protrade and @RISK 4.0

ALEJANDRA DUENAS, KIM BEST, AND NEIL MORT
Department of Automatic Control and Systems Engineering
The University of Sheffield
Mappin Street, Sheffield S1 3JD
UNITED KINGDOM

Abstract: Despite the number of approaches established for Multiple Objective Optimisation Problems, few of them have been developed for the decision making process. Moreover methods that incorporate uncertainty in the decision making process are scarce. This paper describes an interactive method to handle the Decision-Maker's preferences using a Genetic Algorithm. The Genetic Algorithm uses the Probabilistic Tradeoff Development method (PROTRADE) to describe the fitness function based on normalised objectives and introducing uncertainty in the decision variables as well as in the problem constraints. Using real-valued representation the algorithm is applied to a case study for the problem of multiple land uses. The results of this algorithm are compared with those found using a risk analysis program (@RISK 4.0 and RISKOptimiser).

Key-Words: risk analysis, multiple criteria decision-making, genetic algorithms, multiple objective problems, probabilistic tradeoff development method, @RISK, RISKOptimiser

1 Introduction

Decisions are made every day. What product to produce, where to invest, which vendor to use, which car or house to buy, what price to charge, the list is endless. The decisions are probably based on whatever data one has at hand, such as, historical costs, competitors' prices, vendor estimates, etc., but how often is it that there is full, complete information? Demand fluctuates, prices change, costs rise. The wrong decision can be made if all possible scenarios are not accounted for. Making the right decision means performing risk analysis.

Therefore, to change the deterministic world approach to a realistic one, risk must be included in the criteria of the problems. This, as well as the assessment of the impact of a risk or uncertain variable on the outcome of a decision, is investigated in this paper.

There are many definitions for the word risk, but according to Palisade Corporation it is the potential for realisation of unwanted, adverse consequences to human life, health, property, or the environment. In other words it is the probability of occurrence of an undesirable outcome.

Risk Analysis, according to Palisade Corporation, in a broad sense, is any method, qualitative and/or quantitative, used to assess the impacts of risk on decisions. A myriad of Risk Analysis methods are used that blend both qualitative and quantitative techniques. Given a better understanding of the possible outcomes that could occur, the goal of any of these methods is to help the decision-maker choose a course of action.

Most risk analysis methods are performed through simulations [1]. When problems exhibit significant uncertainty, which is generally quite difficult to deal with analytically, simulation is particularly useful.

Furthermore most problems in practice consider the optimisation of several objectives simultaneously [2], [3]. These problems are termed multiobjective optimisation problems (MOP) and one of their most important characteristics is that a large set of solutions is acceptable (these solutions are considered equivalent). Genetic algorithms (GA) have been used to solve several optimisation problems including MOPs. Generally, GAs are stochastic algorithms based on natural evolution principles, that perform a search starting from an initial population and the application of certain genetic operators to find an optimal solution.

Multiple criteria decision-making (MCDM**)** can be understood as the support system used to help the decision-maker (DM) to solve a decision problem. A decision problem normally includes attributes, objectives, goals and criteria [4]. Then it is possible to consider that MOPs are a subset of MCDM problems.

In this paper a MCDM problem under uncertainty (multiple use land reclamation problem) is solved using a probabilistic tradeoff development method

(PROTRADE) with a genetic algorithm (GA). To measure the performance of the GA and PROTRADE this problem was also solved using a risk analysis program called @RISK 4.0 and RISKOptimiser, in such a way to compare the results obtained.

2 @RISK and RISKOptimiser

@RISK is a decision and risk analysis program which due to its flexibility of application and capacity to handle complex inputs and large data sets has become very popular in the manufacturing industry [5]. It is based on a technique called Monte Carlo simulation, and it allows DMs to explore the range of possible outcomes for any decision by using probability distribution functions or ranges of possible values to represent uncertain factors in spreadsheet models. @RISK randomly samples from the probability distribution functions and records the resulting outcomes, during a simulation. The result is a distribution of possible outcomes, and the probabilities of each outcome occurring. This not only tells what could happen, but how likely it is to happen, and therefore, assists the decision-maker in making his/her decision by helping them recognise that some outcomes are more likely to occur than others, and should therefore be given more weight in their evaluation. This enables the users/decision-makers to look at literally thousands of scenarios. All this added information sounds like it might complicate decisions, but in fact, one of simulation's greatest strengths is its power of communication. The results of @RISK are given graphically which illustrates the risk is faced in any situation. It is easy to understand this graphical presentation of results.

RISKOptimiser achieves the optimisation of @RISK models. This is a stochastic optimisation add-in for Microsoft Excel. It combines the genetic algorithm technology of Evolver (another decision optimisation tool developed by Palisade Corporation, [1], [5]) with the Monte Carlo simulation engine of @RISK to optimise models that include uncertain "stochastic" factors. There is no other package available that has the solving power of RISKOptimiser. It performs optimisation under uncertainty, finding the best combination of parameters while accounting for random, uncontrolled factors. RISKOptimiser runs multiple simulations, each time using genetic algorithms to find a better set of parameters to optimise simulation results. It has countless applications in finance, operations research, and any field where optimisation problems involve uncertainty.

3 Protrade Method

Goicoechea et al. [6] developed a multiobjective stochastic method called Probabilistic Tradeoff Development (PROTRADE). This method is used basically to solve nonlinear problems considering the DM´s preferences (progressive articulation of preferences) and is capable of handling risk. The PROTRADE method consists of the formulation of surrogate and multiple attribute utility functions. The construction of these utility functions leads to a direct application of this method in GAs, where the functions are translated directly to the fitness function.

This is a 12-step method defined as follows:

Step 1 A vector of objective functions is defined using the coefficients expected values:

$$\begin{aligned} &\mathbf{z}(\mathbf{x}) = [z_1(\mathbf{x}), z_2(\mathbf{x}), \ldots, z_p(\mathbf{x})], \\ &g_q(\mathbf{x}) \le 0 \quad \textit{where} \quad q \in I[1, Q] \\ &\mathbf{x} > 0, \end{aligned} \tag{1}$$

$$z_i(\mathbf{x}) = \sum_{j=1}^{n} c_{ij} x_j, \; z_i(\mathbf{x}) = E[z_i(\mathbf{x})],$$

Step 2 Vectors $\mathbf{U}_1$ and $\mathbf{M}$ are defined, having the maximum and minimum values of the objective functions respectively:

$$z_i(\mathbf{x}_i^*) = \max \quad z_i(\mathbf{x}), \qquad i \in I[1, p]$$

$$\mathbf{U}_1 = \begin{bmatrix} z_1(\mathbf{x}_1^*) \\ z_2(\mathbf{x}_2^*) \\ \vdots \\ z_p(\mathbf{x}_q^*) \end{bmatrix} \qquad \mathbf{M} = \begin{bmatrix} z_{1\min} \\ z_{2\min} \\ \vdots \\ z_{3\min} \end{bmatrix} \tag{2}$$

To find the maximum values it is necessary to maximise each objective function separately, subject to constraints $g_q(\mathbf{x}) \le 0$.

Step 3 An initial surrogate function is formulated:

$$F(\mathbf{x}) = \sum_{i=1}^{p} G_i(\mathbf{x}) \tag{3}$$

where

$$G_i = \frac{z_i(\mathbf{x}) - z_{i\min}}{z_i(\mathbf{x}_i^*) - z_{i\min}} \tag{4}$$

where, $z_i(\boldsymbol{x})$ is the value of objective function i, i = 1,2,..,n; z_{imin} is the minimum value obtained when objective i is subjected to the constraints; and $z_i(\boldsymbol{x}^*)$ is the maximum value obtained when objective i is subjected to the constraints.

Step 4 An initial solution $\mathbf{x}_1$ is obtained maximising $F(\mathbf{x})$, subject to constraints $g_q(\mathbf{x}) \leq 0$. This solution is used to generate a goal vector $\mathbf{G}_1$:

$$\mathbf{G}_1 = \begin{bmatrix} G_1(\mathbf{x}_1) \\ G_2(\mathbf{x}_1) \\ \vdots \\ G_q(\mathbf{x}_1) \end{bmatrix} \quad (5)$$

Step 5 A multidimensional utility function is defined, in this case Goicoechea et al. (1979) proposed a multiplicative form:

$$1 + ku(\mathbf{G}) = \prod_{i=1}^{p} [1 + k\, k_i u_i(G_i)] \quad (6)$$

this function is used to reflect the DM's goal utility assessment, where k and k_i are constants which are determined by questions posed to the DM.

Step 6 A new surrogate objective function is defined:

$$S_1(\mathbf{x}) = \sum_{i=1}^{p} w_i G_i(\mathbf{x}) \quad (7)$$

where

$$w_i = 1 + \frac{r}{G_i(\mathbf{x}_1)} \left. \frac{\partial u(\mathbf{G})}{\partial G_i} \right|_{G_i} \quad (8)$$

Step 7 An alternative solution is generated maximising the surrogate solution S_1 finding a solution called $\mathbf{x}_2$ used to generate $\mathbf{G}_2$ and $\mathbf{U}_2$:

$$\mathbf{G}_2 = \begin{bmatrix} G_1(\mathbf{x}_2) \\ G_2(\mathbf{x}_2) \\ \vdots \\ G_p(\mathbf{x}_2) \end{bmatrix} \quad \mathbf{U}_2 = \begin{bmatrix} z_1(\mathbf{x}_2) \\ z_2(\mathbf{x}_2) \\ \vdots \\ z_p(\mathbf{x}_2) \end{bmatrix} \quad (9)$$

Step 8 A vector $\mathbf{V}_1$ that expresses the tradeoff between goal value and its probability of achievement is generated:

$$\mathbf{V}_1 = \begin{bmatrix} (G_1(\mathbf{x}_2), & 1 - a_1) \\ (G_2(\mathbf{x}_2), & 1 - a_2) \\ & \vdots \\ (G_p(\mathbf{x}_2), & 1 - a_p) \end{bmatrix} \quad (10)$$

where $1 - \alpha_i$ is such that,

$$prob[z_i(\mathbf{x}) \geq z_i(\mathbf{x}_2)] \geq 1 - a_i \quad (11)$$

Step9 The DM has to answer the following question: "Are all the $z_i(\mathbf{x}_2)$ values satisfactory?" [3] if the answer is affirmative the vector $\mathbf{U}_2$ is a solution if not go to step 10.

Step 10 The $z_k(\mathbf{x})$ with the least satisfactory pair of $(G_k(\mathbf{x}_2), 1\text{-}\alpha_k)$ is selected and the DM specifies a new probability for that pair.

Step 11 The solution space is redefine creating a new x-space.

Step 12 A new surrogate objective function is generated and a sequential search for a satisfactory solution is performed going back to step 7 or step 6 as many times as necessary.

4 Case Study

The Black Mesa Region problem was presented by Goicoechea et al. [6].

In Northern Arizona on the Navajo Nation lands, there is an area of 5,700 hectares that will be strip mined for coal in a 30 years period. The area has been used as rangeland and this activity has caused heavy overgrazing. This resulted in a development of a programme for designing and implementation of multiple land uses. This development programme can then be given to a management agency.

Five objectives are considered [6]: 1. Livestock production, 2. Augmentation of water runoff, 3. Farming of selected crops, 4. Control of sedimentation rates, and 5. Fish pond harvesting.

It is desired to maximise objectives 1,2,3 and 5 while objective 4 has to be minimised. The decision variables considered are twelve and are expressed in hectares of mined land:

1. No reclamation program current management practises (x_1)
2. Contour furrowing livestock production good range conditions (x_2)
3. Contour furrowing livestock production poor range conditions (x_3)
4. Runoff augmentation compacted earth treatment (x_4)
5. Runoff augmentation compacting and salt treatment (x_5)
6. Runoff augmentation plastic cover and gravel (x_6)
7. Wheat production (x_7)
8. Corn production (x_8)
9. Alfalfa production (x_9)
10. Barley production (x_{10})
11. Sorghum production (x_{11})
12. Fish production pond base (x_{12})

The objectives are defined as follows:

Objective 1 Livestock production

$$f_1(\mathbf{x}) = \sum_{i=1}^{12} l_i x_i \quad \text{animal units} \quad (9)$$

where l_i is the number of livestock heads in animal units month per hectare of land (AUM/ha), and i is the number of decision variable applied.

Objective 2 Water runoff

$$f_2(\mathbf{x}) = \sum_{i=1}^{12} r_i x_i \qquad \text{m}^3 \tag{10}$$

where r_i is the water runoff yield in cubic meters per hectare (m^3/ha), and i is the number of decision variable applied.

Objective 3 Selected crops

$$f_3(\mathbf{x}) = \sum_{i=1}^{12} c_i x_i \qquad \text{kg} \tag{11}$$

where c_i is the crop yield in kilograms per hectare (kg/ha), and i is the number of decision variable applied.

Objective 4 Sediment

$$f_4(\mathbf{x}) = \sum_{i=1}^{12} s_i x_i \qquad \text{m}^3 \tag{12}$$

where s_i is the sediment yield in cubic meters per hectare (m^3/ha), and i is the number of decision variable applied.

Objective 5 Fish yield

$$f_5(\mathbf{x}) = \sum_{i=1}^{12} p_i x_i \qquad \text{kg} \tag{13}$$

where p_i is the fish yield in kilograms per hectare (kg/ha), and i is the number of decision variable applied.

There are three constraints to be considered:

Constraint 1 Land

$$\sum_{i=1}^{12} x_i = b_i \tag{14}$$

where b_i is the area to be strip-mined in a 2-year sub-period . If the total area to be strip-mined in a 30-year period is 5,700 ha, then

$b_i = \frac{5700}{15} = 380$ hectares every two years.

Constraint 2 Capital

$$\sum_{i=1}^{12} q_i x_i = b_q \tag{15}$$

where q_i is the cost of implementing the ith decision variable, and b_q is $200,000. This is an estimated value and was modified from the original problem ($35,000) Goicoechea et al.(1979).

Constraint 3 Water

$$\sum_{i=1}^{12} w_i x_i = b_w \tag{16}$$

where w_i is the water consumption of the ith decision variable, and b_w is the water available for a 2-year subperiod through runoff practices and rainfall. Therefore, this value is a random variable since rainfall is unpredictable.

The values of parameters l_i, r_i, c_i, s_i, p_i, q_i,and w_i can be found in Goicoechea et al. [6] and Goicoechea et al. [3] note that the values are given as the expected value with its standard deviation.

5 Initial results

As mentioned above the case study is solved using both a GA and RISKOptimiser. In both methods two models, each with variations, are constructed, one that does not take risk into account and the other that does. This is done so that the outputs could be compared to see if the decision made based on the risk-free model would be the same as that made with the one that accounted for risk.

The objective functions are calculated by using equations 9 to 13. Since the objective is to maximise the five objective functions, keeping in mind that objective four is to be minimised, therefore it is written as $-f_4$, this is written as $z(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x}), -f_4(\mathbf{x}), f_5(\mathbf{x})]$. This basically means finding the maximum of the sum of the five objectives. It is important to consider that the units of the objectives are of different dimension, and therefore, they cannot be added directly. They have to be normalised to make them dimensionless quantities. Each objective normalised will be called a goal. These goals are found by using equation 4 subject to the land, capital and water constraints. Therefore the main output of the model is the value of the sum of the goals. To obtain z_{imin} and $z_i(\mathbf{x}^*)$ a minimisation and maximisation of each individual objective function is run using both a GA and @RISK 4.0.

As mentioned above the water constraint is a random variable since rainfall is unpredictable. To generate values for the water available for each 2-year subperiod it is necessary to run a Monte Carlo simulation. This simulation is done for fifteen iterations and then these values are used as the values for b_w for the fifteen 2-year periods. The values obtained are: 2,702,473; 1,721,549; 1,047,110; 338,251; 475,079; 803,335; 2,512,024; 845,213; 1,944,114; 705,202; 1,425,694; 409,607; 2,275,253; 1,576,443; 1,200,811 each value corresponds to a 2-year subperiod.

5.1 Problem solution using PROTRADE

To solve the problem using PROTRADE a GA is implemented; the initial population is expressed in real-valued vectors generating randomly x_i (decision variables) with an initial population of 80.

In this work, the algorithm is run for 200 cycles performing the crossover and mutation operator 80 times for each cycle. The selection method used is tournament selection; initially the tournament size was two (binary tournament [7]) but it was found that with a tournament size of three the results where better. The crossover used is “arithmetic crossover” [8]. The mutation operator selected is that proposed by Michalewicz [8] where the new child is a random value generated from a domain, in this case [0,380].

The crossover and mutation probabilities are 0.5 and 0.15 respectively, finding the results shown in Table 1. These results are found without considering risk. The water constraint used for the GA is 2,702,473 m^3, which is that, corresponded with the maximum value that was obtained when the Monte Carlo simulation was performed.

Practices	**No risk** (in Hectares)	**Risk** (in Hectares)
x_1	194	100
x_2	3	37
x_3	0	16
x_4	4	14
x_5	3	19
x_6	51	45
x_7	2	0
x_8	2	9
x_9	89	70
x_{10}	10	0
x_{11}	3	65
x_{12}	18	5

Table 1 Results of the land allocation using the GA and PROTRADE

It is important to mention that in this experiment just the first four steps of PROTRADE method are applied. This means that for the moment a multiplicative form will not represent the DM’s preferences and they will be only considered once the results are found.

In the no risk model the parameter values for equation 9 to 13 are the expected values defined by Goicoechea et al. [3].

To introduce uncertainty on the decision variables it is necessary to define a normal distribution as follows:

$$\text{NORMDIST} = \frac{1}{s\sqrt{2p}} e^{-\frac{1}{2}\left(\frac{x-m}{s}\right)^2} \qquad (17)$$

This distribution will be used to generate random values for the decision variables’ parameters. Table 1 shows the results of the GA considering risk.

The maximum values for the objective function represented as the addition of the five objectives are 1.3442 (no risk) and 0.639 (risk).

In conclusion the results found considering uncertainty (risk) in the decision variables are significantly different to those found in the no risk model.

5.2 Solution using @RISK and RISKOptimiser

The risk-free model was constructed as follows:

Since @RISK works in a spreadsheet format four tables were constructed. One containing the decision variables, which were represented by probability distributions; one with the constraints; one including the minimum and maximum values of the optimisation of the individual objective functions under the constraints; and the other containing the objective functions and the goals. The decision variables were assigned probability distributions because they represent the area allocated to the different practices, which can vary from zero to three hundred and eighty hectares (0, 380 ha). These distributions were all Uniform Distributions (written as RiskUniform (min, max)), but with different ranges.

Before a simulation can be started, constructing a correlation matrix identifies all dependent and independent variables. The correlation coefficients are obtained by performing a sensitivity analysis.

The difference occurs when risk is introduced to the model. The mean value of 0.3553 increased by approximately ten percent (10%) to 0.4574.

The next step is to find an optimal solution for the problem using RISKOptimiser. This model is almost identical to that of @RISK. The difference is in the settings and options used. Once the model has been constructed, it is defined in the RISKOptimiser to search for a maximum. Values of 0.5 and 0.15 are used for the crossover and mutation rates respectively. These values are the same than those used in the GA in order to be able to compare. The constraints applied are the land, capital and water constraints; the values of the goals had to be between 0 and 1 inclusive. Recall that the water available is a random variable, that is b_w is a random variable.

The population size is set to 80. This is the same than that used in the GA. An optimisation is performed which uses a stopping condition of a thousand (1000) iterations. An optimisation stopping condition of five hundred (500) simulations is used.

The first set of optimisations is performed with the water constraint equal to 2,702,473 m^3. Therefore, when risk is considered the decision made is different.

The maximum values for the objective function represented as the addition of the five objectives are 1.329 (no risk) and 0.639 (risk).

Therefore the decisions that were made from the model are as follows:

Practices	No risk (in Hectares)	Risk (in Hectares)
x_1	165	99
x_2	91	32
x_3	0	16
x_4	7	16
x_5	0	19
x_6	32	57
x_7	2	1
x_8	0	9
x_9	61	62
x_{10}	5	0
x_{11}	0	63
x_{12}	18	7

Table 2 Results of the land allocation using RISKOptimiser

6 Analysing the results

It can be seen that in the GA when risk was not considered (Table 1), x_3 is the only practice that had no land allocated to it for the first 2-year period. While in the @RISKOptimiser model when risk was not considered (Table 2), x_3, x_5, x_8 and x_{11} practices had no land allocated to them for the first 2-year period. The land allocated to x_5, x_8 and x_{11} practices in the GA is 3,2 and 3 ha respectively. While x_1 (no reclamation program) had the highest allocation in both models 194 ha in the GA and 165 ha in the RISKOptimiser. With this comparisons it is possible to conclude firstly, that in terms of numerical results the GA had a better performance when risk was not included. Because the maximum value found in the GA is 1.3442 and in the RISKOptimiser model is 1.329. Secondly, in terms of time performance RISKOptimiser is faster than the GA. Finally, the GA allows the programmer to have a better control of the model and to know exactly what occurs in the algorithm.

When risk was considered, however, there was a drastic change in some of the allocations. What remained unchanged was the management practice, which was given the highest amount of land. This remained to be x_1, even though this quantity decreased by 40% (from 165 to 99) in the RISKOPtimiser model and by 51% (from 194 to 100) in the GA. The drastic changes occurred when all of the practices, which were not allocated any land, before, were now allocated some land, and in some cases, it was quite a big change. For example, x_{11} was now allocated 63 ha in the RISKOptimiser model and 65 in the GA.

7 Conclusion

It is therefore shown that the consideration of risk is very important to decision-making. The decisions made based on models that did not include risk, were different to the decisions made when it was considered.

In future work some research can be directed into further investigation into RISKOptimiser and the Macro command. May decision-makers preferences be able to be analysed with RISKOptimiser through the use of a macro. Fuzzy logic rules can also be investigated to see if they can be used to implement the decision-maker's preferences either on their own or as the basis of a macro.

8 Acknowledgement

The first author acknowledges support from Consejo Nacional de Ciencia y Tecnología (México) and Instituto Tecnológico y de Estudios Superiores de Monterrey - Campus Estado de México.

References:

[1] Palisade Europe, 2000a, *Announcing @RISK 4.0!*, 7 No. 1, 3-8 and 21-22.

[2] J.P. Ignizio, *Linear Programming in single- and multiple-objective systems*, Englewood Cliffs, London: Prentice-Hall, 1982.

[3] A. Goicoechea, D. R. Hansen, and L. Duckstein, *Multiobjective Decision Analysis with Engineering and Business Applications*, New York, Chichester: John Wiley and Sons, 1982.

[4] C-L. Hwang and A. S. Md. Masud, *Multiple Objective Decision Making- Methods and Applications*, Lecture Notes in Economics and Mathematical Systems, Berlin, New York : Springer-Verlag, 1979.

[5] Palisade, 2000b, "@RISK 4.0", 2001, http://www.palisade-europe.com/html/risk.html, (1st July, 2001).

[6] A. Goicoechea, L. Duckstein, and M. Fogel, Multiple Objectives Under Uncertainty: An Illustrative Application of Protrade, *Water ResourcesResearch*, Vol. 15, No. 2, April 1979, pp. 203-210.

[7] T. Bäck, D. Fogel, and Z. Michalewicz, Preface, *Evolutionary Computation 1*, T. Bäck, D.B Fogel, T. Michalewicz (Eds.), Chapter 24, Bristol and Philadelphia, UK: Institute of Physics Publishing, 2000.

[8] Z. Michalewicz, *Genetic Algorithms + Data Structures = Evolution Programs*, Berlin; New York: Springer-Verlag, 1994.