Modelling of Rate-Independent Hysteresis with Feed-Forward Neural Networks

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Abstract: - A rate-independent hysteresis model is presented, based on the nonlinear function approximation capabilities of feed-forward neural networks. The general approach to hysteresis modelling consists of two steps. Firstly, the memory state of the hysteretic system is determined, based on the input and output history. Secondly, the memoryless relation between magnetic state and system output is approximated by a feed-forward neural network. The model obeys the wiping-out and the congruency properties of hysteresis. The proposed technique is verified by experiments. Compared to the classical Preisach hysteresis model, the new model requires less memory and less training data. The accuracy is good and can be adapted to the requirements of the application, as the congruency property of the model can be relaxed.

Key-Words: **-** Hysteresis modelling, Rate-independent hysteresis, Dynamic system, Neural networks

1 Introduction

Hysteresis phenomena are encountered in many areas of engineering science. Examples include magnetic hysteresis, mechanical hysteresis, etc. Hysteresis is a memory effect, in the sense that the output $y(t)$ at time t of a transducer exhibiting hysteresis lags behind its input $u(t)$ at time t. The output $y(t)$ is thus dependent on the current input $u(t)$ and the previous input history $u(t_{prev})$ for all $t_{prev} < t$.

The accurate modelling of hysteresis phenomena is important for the design of devices exhibiting hysteresis. Many hysteresis models exist, but they all have their limitations concerning the types of hysteresis phenomena they can handle [1]. Major drawbacks are the computation speed, the large amount of identification data required and the memory needed for the storage of the model parameters. Previous studies have indicated the potential of neural network techniques to provide an alternative to classical models [2,3]. This paper proposes a new general approach to model hysteresis with feed-forward neural networks and compares this new technique with widely used classical approaches. Since the authors work in the area of magnetic hysteresis, the presented techniques are illustrated with results for soft ferromagnetic SiFe laminated steel, used in electrical motors and transformers. The application area includes the CAD-design of such devices.

The paper is organized as follows. Section 2 describes the major properties of hysteresis and focuses on rate-independent hysteresis. The widely used classical Preisach model of hysteresis and its limitations are outlined in section 3. The newly proposed neural network hysteresis model is described in section 4 and experimentally verified in section 5. Section 6 summarizes the main results.

2 Hysteresis: description and properties

The hysteresis transducer, introduced in section 1, can be considered as a nonlinear dynamic system with input u_k, state x_k and output y_k, all at time point k, in discrete representation. Besides nonlinearity, a hysteresis transducer often exhibits saturation. Only scalar hysteresis systems are considered, thus u, x, and y are scalars. The mathematical description of such a system is:

$$x_k = g(x_{k-1}, u_{k-1}) \tag{1a}$$
$$y_k = h(x_k, u_k), \tag{1b}$$

where g and h are nonlinear functions. The input-state equation (1a) is a dynamic relation, thus including memory, since the calculation of the current system state x_k requires the knowledge of the state x_{k-1} at the previous time step. The state-output equation (1b), on the other side, is a static, memoryless relation, as the complete input history is included in the current state x_k through (1a). In the context of hysteretic systems, x_k is the memory state of the system at the time point k, in the sense of the values of a set of system parameters that contains sufficient information to determine the output y_k, when the input u_k is known [4].

The above discussion could suggest that a good hysteresis model should take into account the complete input history starting from $k = 0$. However, in the case of hysteresis, many experiments have shown that only small parts of the input history are relevant for the current memory state x_k [1]. The determination of the relevant part of the input history is detailed below.

2.1 Rate-independent hysteresis

An important class of hysteresis phenomena is the case of rate-independent hysteresis. This type of hysteresis, also called quasi-static hysteresis, occurs when the rate of change of the input is so low that it can be neglected and thus does not influence the output.

For a rate-independent hysteresis system, the memory state x_k and the output y_k depend only on the extreme (minimum and maximum) values of the input u_k, reached in the past, but not on the intermediate values. The relevant part of the input history is thus the sequence of extreme input values [1]. Fig.1 illustrates this property: Fig.1a shows two different inputs that exhibit an identical sequence of extreme input values. The resulting input–output diagrams in Fig.1b are identical as well. This property has been confirmed by experiments with hysteresis systems with a very low input rate of change, for example in the case of magnetization of ferromagnetic materials at very low frequency.

2.2 Wiping-out property

Further reduction of the relevant part of the input history is based on the wiping-out or deletion property of hysteresis, illustrated in Fig.2. Experiments have shown that extreme input values are deleted from the relevant input history when surpassed by larger input values [1]. In Fig.2, when the input changes from point 1 to point 5, cycling between the extreme values at point 2 and 4, and 3 and 5, a major hysteresis loop is formed. During the input change from point 6 to point 8, a minor loop is traced. When the minor loop closes at point 9 and the input rises further to point 10, the effect on the output is exactly the same as when the input rises monotonically from point 5 to point 10. When the input exceeds the last maximum at point 6, the maximum at point 6 and the minimum at point 7,

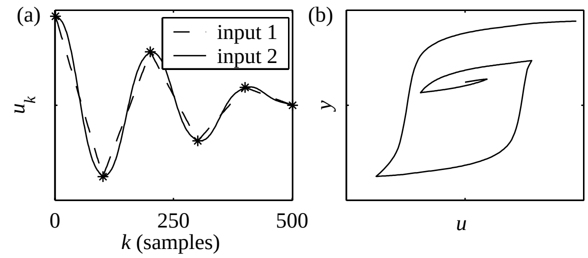

Fig.1: Rate-independent hysteresis: (a) different input sequences with identical extreme values; (b) resulting identical input-output trajectories

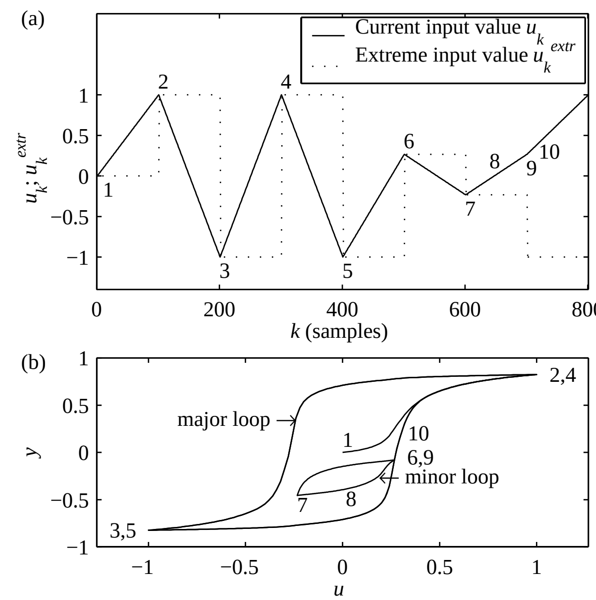

Fig.2: Wiping-out (deletion) property of rate-independent hysteresis: (a) input sequence and corresponding extreme values; (b) input-output trajectory

which determine the minor loop, are evaded from memory. The further evolution of the output is as if the minor loop never existed.

In conclusion, the relevant part of the input history at each time point k consists of exactly one stored extreme input value u_k^{extr}, i.e. the last extreme value, kept in memory. This value can be easily determined from the (known) input history, as illustrated in Fig.2 [4].

Combining all results, the hysteretic system can be presented as:

$$y_k = f_1\left(u_k, u_k^{extr}, y_k^{extr}\right), \tag{2}$$

with f_1 a nonlinear function of three variables and y_k^{extr} the output value corresponding to u_k^{extr}. Comparing with (1), it is clear that the memory state

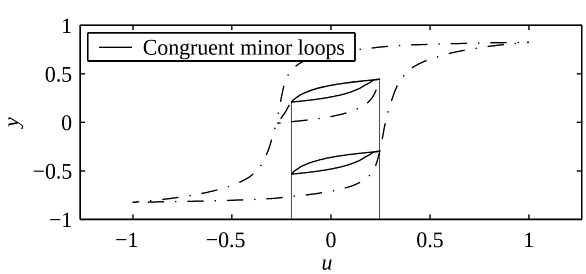

Fig.3: Congruency property of rate-independent hysteresis

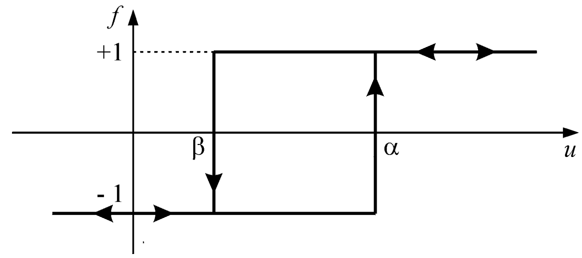

Fig.4: Elementary Preisach hysteresis operator

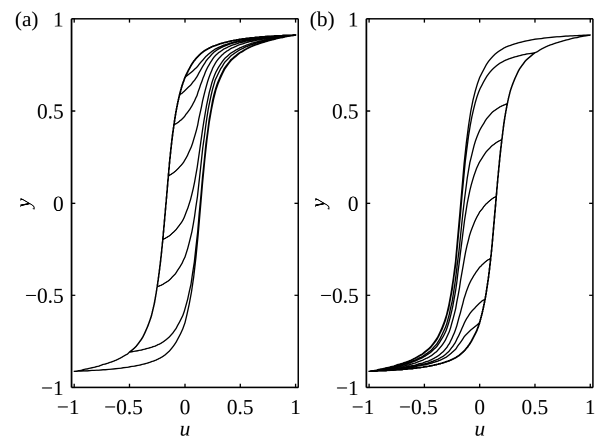

Fig.5: First order reversal curves from various starting points up to saturation: (a) ascending curves; (b) descending curves

of the system can be represented by the two parameters u_k^{extr} and y_k^{extr}, known at each time point k. The task of the hysteresis model is to determine the function f_1 and identify its parameters based on measurements.

2.3 Congruency property

Further simplification of (2) can be achieved taking into account the congruency property of minor loops, which is approximately valid for many soft magnetic materials. The congruency property states that minor loops, formed when the input is cycled between two identical extreme values, are congruent, independent of the mean output level at which they are situated, see Fig.3 [1]. The hysteretic system can thus be presented as:

$$y_k = y_k^{extr} + f_2\left(u_k, u_k^{extr}\right), \tag{3}$$

with f_2 a nonlinear function of two variables. The function f_2 is easier to identify than f_1 and would thus yield a simpler hysteresis model. However, using (3) instead of (2) would result in reduced accuracy for materials that exhibit minor loops significantly deviating from congruency.

In the following, the possibilities for the identification of the functions f_1 and f_2 are discussed.

3 Preisach hysteresis model

In order to allow a comparison between the classical modelling techniques and the newly proposed neural network method, the widely used Preisach hysteresis model is described below.

The Preisach model is a phenomenological hysteresis model used mainly in magnetism, aiming at approximating the macroscopic hysteresis behaviour of ferromagnetic materials. The model is constructed as a superposition of elementary rectangular Preisach hysteresis operators $\gamma_{\alpha\beta}$ with up-switching field α and down-switching field β (Fig.4) [1]. Each operator yields +1 or –1 depending on its memory state (determined from the input history) and the current input. A weight function $\mu(\alpha,\beta)$, called the Preisach distribution function, is used for the superposition of all Preisach operators with different values of α and β. It can be proven [1] that the resulting classical Preisach model describes both major and minor loops and obeys the wiping-out and congruency properties. Besides, the model can also be presented as:

$$\Delta y = y_k - y_k^{extr} = Ev\left(u_k^{extr}, u_k\right), \tag{4}$$

with $Ev\left(u_k^{extr}, u_k\right)$ the Everett function [5]. The Preisach distribution function is the second derivative of the Everett function. Comparing (3) and (4), it is clear that the function f_2 can be directly derived from the Everett function Ev. The classical Preisach model is thus a model of the type of Eq. (3).

The identification of the classical Preisach model is often carried out by the measurement of the so-called first order reversal curves up to saturation (Fig.5). Assuming that the congruency property holds, these curves yield the value of $Ev(u_k^{extr}, u_k)$ for all possible combinations of u_k^{extr} and u_k.

Attempts have been made to relax the congruency property of the classical Preisach model, since many hysteresis systems deviate significantly from this property. However, no model exists today that approximates the general relation (2). The noncongruent Preisach models all assume a predetermined dependence of the width of the minor loops on the mean output level [6] and are thus valid for certain types of hysteretic materials only.

Concluding, the practical use of the Preisach model requires a large amount of memory to store the Everett function, consisting of the model parameters. Besides, an accurate identification of these parameters requires a dense set of reversal curves, hence a large amount of measurement data. The deviation of real materials from the congruency property limits the accuracy of the Preisach model.

4 Hysteresis modelling with feed-forward neural networks

The general description of hysteresis, developed in section 2, Eqs. (2) and (3), reduces the problem of hysteresis modelling to the identification of functions like f_1 and f_2. A feed-forward neural network (FFNN) from the multilayer perceptron type (MLP) can be used to accomplish this task, as it is proven that such a network, with at least one hidden layer, can approximate any smooth nonlinear function of an arbitrary number of variables with arbitrary accuracy [7]. In the context of description of hysteresis as a dynamic system in (1a) and (1b), the FFNN, static in nature, can be used to describe the memoryless state-output relation (1b) [4]. The state x_k of the system should then be determined beforehand by some other algorithm. Eqs. (2) and (3) show that this is possible in the case of rate-independent hysteresis.

An arbitrary accurate hysteresis model, strictly obeying the wiping-out and congruency properties, can thus be constructed based on (3), whereby the nonlinear function f_2 is determined by a feed-forward neural network (network 1) with two inputs: u_k and u_k^{extr} (Fig.6). The network input u_k^{extr} can be derived at each time point as outlined in section 2. A possible network training set consists of a selection of ascending and descending first order reversal curves up to saturation (Fig.5). Indeed, such a training set contains sufficient information about the sought function f_2. Any other training set containing similar information can be used as well.

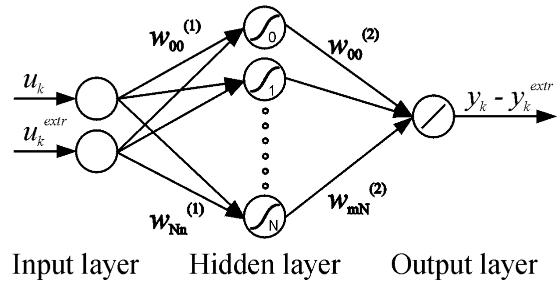

Fig.6: Congruent loops neural network model (network 1)

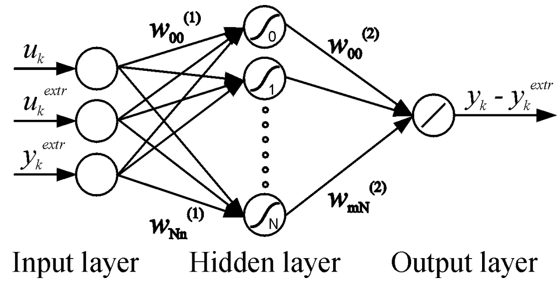

Fig.7: Noncongruent loops neural network model (network 2)

A hysteresis model allowing for noncongruent minor loops can be constructed based on (2). However, a more convenient presentation, equivalent to (2), is:

$$y_k = y_k^{extr} + f_3(u_k, u_k^{extr}, y_k^{extr}), \quad (5)$$

with f_3 a nonlinear function of three variables. The function f_3 is determined by a FFNN (network 2) with three inputs: u_k, u_k^{extr} and y_k^{extr}, known at each time point (Fig.7). The network training set consists of much more data than for the model with congruent loops, as sufficient information about the function f_3 should be available for all possible combinations of the three network inputs. The following sets of excitation patterns could be used to provide detailed training data: symmetrical major loops, first order reversals based on the symmetrical major loops, minor loops, etc. The model with network 2 (Fig.7) is a more general representation of hysteresis than the noncongruent Preisach models, mentioned in section 3. Indeed, it can describe arbitrary variations of the width of the minor loops with the output level, thus accurately fitting the experimental results for all types of hysteresis systems.

Note the similarity between (5) and (2). For systems that sufficiently approximate the congruency property, f_3 will be only slightly dependent on y_k^{extr}, thus almost identical to f_2. In this case the simpler model with network 1 (Fig.6) can be used instead. When very high accuracy is required, the model with network 2 (Fig.7) should be employed and much more training data should be provided. In this context, the presented approach allows the easy adaptation of the model complexity to the required accuracy for the application.

5 Experimental verification

The proposed technique is applied to the modelling of quasi-static magnetic hysteresis in a typical SiFe steel lamination. The input u of the hysteresis system is then the magnetic field strength H (A/m) and the output y is the magnetic induction B (T). The network is trained using the Levenberg-Marquardt training algorithm [7]. The purpose is to show that the technique yields accurate results and to compare it with the Preisach approach. It is known that optimization of the network structure and training (number of hidden layers, number of neurons, advanced training techniques such as early stopping, weight decay, etc.) can improve the generalization capability of the network and thus reduce the required amount of training data for an accurate prediction [7]. However, these aspects fall beyond the scope of this paper.

Firstly, the equivalency of the proposed approach with the classical Preisach model is verified. The training and test sets for the congruent loops model with network 1 (Fig.6) were generated by the Preisach model using an experimentally determined Everett function [4]. The network, with 2 hidden layers and 10 neurons in each hidden layer, was trained for 1000 epochs (iterations) with a set of first order reversal curves up to saturation (Fig.5). The results for a test set not used during training (Fig.8a) are presented in Fig.8b. The prediction accuracy of the neural network is excellent and shows the equivalency with the Preisach model. The advantage of the neural network model is that the storage of the network parameters (structure and weights) requires much less memory than the storage of the Everett function. Besides, the number of reversal curves used for the training set presented here is about 20 times less than the number of curves required for accurate identification of the Everett function. Moreover, the prediction of new hysteresis loops after training is very fast.

Secondly, the performance of the congruent loops model with network 1 (Fig.6) is tested for a material that does not obey the congruency property with sufficient accuracy. The first order reversal

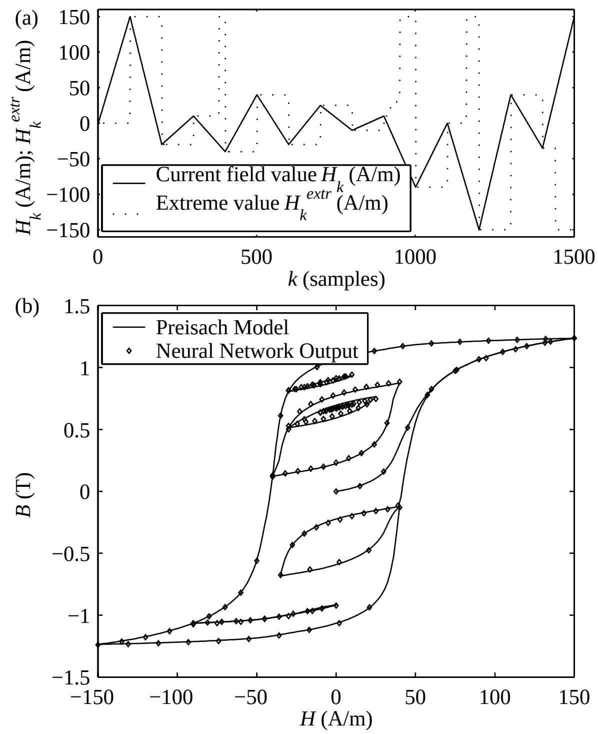

Fig.8: Comparison between congruent loops neural network model (network 1) and classical Preisach model for FeSi steel: (a) input history; (b) input-output trajectory

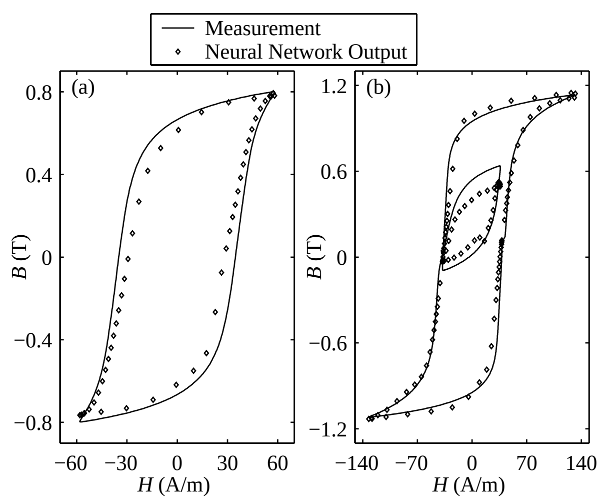

Fig.9: Comparison between congruent loops neural network model (network 1) and measurements for FeSi steel: (a) symmetrical major loop for medium induction level; (b) minor loop situated at low induction level

curves up to saturation were measured and used directly as training data for the network. The network structure is the same as above. This method yields poor results in the absence of congruency, especially when calculating major loops for low and medium induction levels and minor loops situated at low induction levels. To improve the results, symmetrical major loops for different inductions are added to the training set. For a subset of the training set, each input value will thus lead to two different target values, one derived from the first order reversal curves and the other from the symmetrical major loops. After training, the network will yield approximately the mean value of the two target values when an input from this subset is presented. The noncongruency of the material is thus averaged out. Fig.9 shows the results for two measured test sets. The accuracy is good, but less than in the case of Fig.8 due to the deviation of the material from congruency. However, the model still does not require large amounts of storage capacity or measurement data and presents a practical and accurate approach to hysteresis modelling. The accuracy can be further increased using the noncongruent loops model with network 2 (Fig.7), along with a much more extended set of measurement data. As mentioned above, the model complexity can be adapted to the requirements of the application.

6 Conclusions

A new rate-independent hysteresis model was presented, based on the nonlinear function approximation capabilities of feed-forward neural networks. The proposed general approach to hysteresis modelling consists of two steps, performed at each time point. Firstly, the values of the set of parameters, determining the current memory state of the hysteresis system, are calculated, based on the input and output history. In the case of rate independent hysteresis, these parameters are the last relevant extreme input value and the corresponding output value. Secondly, the memoryless relation between magnetic state and system output is approximated by a feed-forward neural network. The model obeys experimentally observed hysteresis properties such as the wiping-out property and the congruency property. The proposed technique is verified by experiments.

Compared to the classical Preisach model approach, the new model requires less memory and less training data. The accuracy is as good as for the congruent loops classical Preisach model. An approach is presented to relax the congruency property of the model and thus yield higher accuracy for hysteresis systems that do not approximate this property sufficiently well. The model yields a practical approach to the handling of hysteresis phenomena, as standard neural network techniques are used.

Further research will focus on the extension of the model to vector hysteresis systems, especially important in magnetics, as well as on the inclusion of dynamic and temperature effects. The generalization capability of the network will be investigated in more details.

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