NONLINEAR WAVELET PACKET DENOISING OF IMPULSIVE VIBRATION SIGNALS

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Abstract: In this paper an application is presented of the wavelet packet method for denoising of impulsive vibration signals. Vibration response of machines often includes signals with periodic excitation of resonances. The aim is to extract information regarding the physical mechanism which generates the impulsive characteristics of the signals. The signals are transformed using the wavelet packet method, and the resulting coefficients are nonlinearly modified. The reconstructed time waveform of the signal using the modified coefficients may reveal the impulsive characteristics, resulting in a safer identification of the source of the impacts. The approach and its parameters are evaluated on industrial signals resulting from defective bearings.

Key-Words: - wavelet packet, vibration , impulsive signal,denoising

1 Introduction

The Fourier analysis is widely used for the detection of periodicities of vibration signals. It is based on the assumptions of stationary signals, however, many signals contain numerous transient characteristics. For example, the vibration response of rotating machines often includes impact generated transient signals, typical cases being the vibration response of machines subjected to defects or wear of certain machine parts. In those cases the the impact periodicity is usually the interesting information which characterizes the impact source, while the overall frequency content of the signals is not significant.

A common approach to processing of vibration signals is to measure the vibration level and generate the Fast Fourier transform of the measured signals. Often, frequency spectrum peaks characterizing the impact source are not easily observed, or may have multiple interpretations.

In order to overcome this problem a number of time domain and frequency domain methods have been proposed [1-4]. The aim is to develop signal processing methods that are able to extract patterns that relate to the source of the impact mechanism. Joint time and frequency domain methods, such as the Short Time Fourier Transform, the Wigner-Ville Distribution and the Wavelet Transform, have been widely used in many signal processing areas.

Wavelets have been established as a widespread tool, due to their flexibility and to their efficient computational implementation [5-8]. They have been introduced in vibrations [9] and there are specific case studies for bearing fault detection and for other machine components [10]. In many cases the application of wavelets has been combined and enriched by using additional features, such as Gaussian/exponential-enveloped functions [11], and de-noising methods [12].

The Wavelet Packet Transform is a generalization of the wavelet transform and has been used in signal processing for denoising or compression of signals [13-14]. Applications in machining process have also been proposed [15]. In this paper a wavelet packet transform is used as a tool for the denoising of vibration signals with impulsive characteristics. The aim is to extract the impulsive information and reject the unwanted information due to other factors. The unwanted information is assumed to exist as a part of each signal component. Thus, a nonlinear modification of the all the wavelet packet coefficients is applied and the signal is reconstructed. In chapter 2, a brief review of the basics of the wavelet transforms is presented. In section 3, the wavelet based denoising and the parameters of the implementation are discussed. In section 4 the proposed approach is evaluated on industrial measurements for two types of bearing faults.

2 Wavelet Transforms

2.1 The Continuous Wavelet Transform

The continuous wavelet transform (CWT) of a finite energy signal *x(t)* with the analyzing wavelet *ø(t)* is the convolution of *x(t)* with a translated and scaled wavelet :

$$W(a,b) \;=\int_{-\infty}^{+\infty} x(t)y_{a,b}(t)dt \qquad (1)$$

The wavelet coefficient *W(á,b)* measures the similarity between the signal *x(t)* and the analyzing wavelet *ø(t)* at different scales as defined by the parameter *a*, and different time positions as defined by the parameter *b*.

$$y_{a,b}(t)=|a|^{-\frac{1}{2}}\, y\left(\frac{t-b}{a}\right) \qquad (2)$$

Small values of *a* give a contracted version of the basic wavelet and allows the analysis of high frequency components, while large values strech the basic wavelet and allows analysis of low frequency components of the signal. The factor $á^{-1/2}$ is used for energy preservation.

Equations (1) and (2) indicate that the wavelet analysis is a time-frequency analysis, or, more properly termed, a time-scale analysis. The wavelet transform can be also considered as a special filtering operation. At successively larger scales the frequency resolution improves and the time resolution decreases.

2.2 The Discrete Wavelet Transform

The discrete wavelet transform is performed by choosing fixed values

$$a = a_0^m \qquad b = nb_0a_0^m \qquad (3)$$

where *m,n* are integers. The discrete wavelet analysis can be implemented:

$$W(m,n) = a_0^{\frac{-m}{2}} \int_{-\infty}^{+\infty} x(t)y\,(a_0^{-m}t - nb_0)dt \qquad (4)$$

An orthonormal basis can be constructed for $a_0 = 2$ and $b_0 = 1$

$$y_{m,n} = 2^{-m/2} y\,(2^{-m}t - nb_0)dt \qquad (5)$$

A fast algorithm can be implemented by using the scaling filter which is a lowpass filter *L* related to the scaling function *ö*(*t*), and the wavelet filter, which is a highpass filter *H*, related to the wavelet function *ø*(*t*).

The computation of these filters and their properties have been widely analyzed in [5, 6].

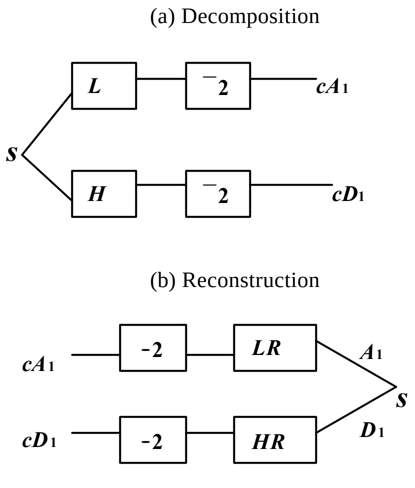

Figure 1. Basic steps of discrete wavelet transform (a) Decomposition, (b) Reconstruction

The fast wavelet algorithm can be implemented in two opposite directions, decomposition and reconstruction. In the decomposition step in Fig. 1(a), the discrete signal *s* is convolved with a low-pass filter *L* and a high-pass filter *H*, resulting in two vectors cA_1 and cD_1.

The elements of the vector cA_1 are called approximation coefficients and the elements of the vector cD_1 are called detail coefficients. The symbol 2 denotes downsampling i.e. omitting the odd indexed elements of the filtered signal, so the number of the coefficients produced by the basic step is approximately the same as the number of elements of the discrete signal *s*. In the reconstruction step in Fig 1(b) a pair of filters *LR* and *HR* are convolved with the vectors cA_1 and cD_1 respectively. Two signals are produced resulting in a reconstruction signal A_1 called Approximation, and a reconstruction signal D_1 called Detail. The symbol 2 denotes upsampling e.g. inserting zeros between the elements of the vectors cA_1 and cD_1. An important property of this step is

$$s = A_1 + D_1 \qquad (6)$$

The procedure of the basic step is repeated on the approximation vector cA_1 and successively on every new approximation vector cA_j. This idea is presented by means of a wavelet tree with *J* levels, where *J* is the number of iterations of the basic step.

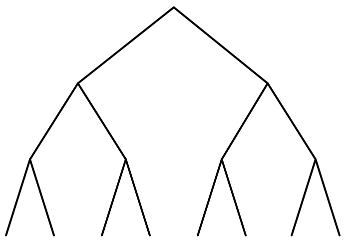

Figure 2. An example of three level wavelet packet decomposition tree

2.3 The Wavelet Packet Transform (WPT)

The wavelet packet transform is a generalization of the wavelet transform. Let us define two functions $W_0(t)=\varphi(t)$, $W_1(t)=\psi(t)$ where $\varphi(t)$ and $\psi(t)$ are the scaling and wavelet functions respectively. Then in an orthogonal case we can write functions $W_m(t)$, $m=0,1,2,\ldots$, as

$$\begin{aligned} W_{2m}(t) &= 2\sum_{n=0}^{2N-1} h(n)W_m(2t-n) \\ W_{2m+1}(t) &= 2\sum_{n=0}^{2N-1} g(n)W_m(2t-n) \\ W_{j,m,n}(t) &= 2^{-j/2}W_m(2^{-j}t-n) \end{aligned} \tag{7}$$

where j is a scale parameter and n is a time localization parameter. The analyzing functions $W_{j,m,n}$ are called wavelet packet atoms.

In practice a fast algorithm is applied by using the basic step of Fig.1. The difference is that both details and approximations are split into finer components, resulting in a wavelet packet tree. In Fig. 2 an example of a wavelet packet decomposition tree of three levels is presented.

3. Denoising

3.1 Wavelet Based Denoising

The wavelet decomposition allows searching an optimal decomposition among L trees if a signal has been decomposed at L levels. Wavelet decomposition involves the selection of an optimal decomposition tree among 2^L. Several criteria have been proposed for the optimization of the decomposition as the Shannon entropy.

An application of the wavelet analysis is to remove undesired components from the signal through a denoising approach. The linear denoising approach assumes that the udesired components (noise) are located in certain scales and the signal is reconstructed without those components.

The nonlinear denoising assumes that the noise components exist in each coefficient vector and involves a thresholding approach in order to remove those components.

$$y = \begin{cases} \mathrm{sgn}(y)\,|y|-t, & |y|>t_{hr} \\ 0, & |y|<t_{hr} \end{cases} \tag{8}$$

or in a more generalized form [12]

$$y = \begin{cases} \mathrm{sgn}(y)(|y|-qt_{hr}), & |y|>t_{hr} \\ 0, & |y|<t_{hr} \end{cases} \tag{9}$$

where $0<q<1$ and when $q=0$ hard thresholding is applied, when $q=1$ soft thresholding is applied.

There are several criteria for the selection of threshold [13]:

Steins unbiased risk estimate (SURE) is an adapted threshold selection rule.

$$t_{hr} = \sqrt{2\log_e(n\log_2(n))} \tag{10}$$

where n is the number of samples of the decomposition level.

Fixed threshold approach FIXTHRESH

$$t_{hr} = \sqrt{2\log(n)} \tag{11}$$

calculates the threshold with respect to the length of the signal.

The HEURISTIC SURE approach a variant of the SURE approach

The MINIMAXI procedure

$$t_{hr} = 0.3936 + 0.1829\log(n) \tag{12}$$

These models assume noise distributed with zero mean and variance of 1 and have to be rescaled when dealing with unscaled noise.

A method proposed in [12] is based on the continuous wavelet transform using the Morlet wavelet. A simple inverse transform which requires only one integration is used [7]

$$x(t) = C_{1\psi}^{-1}\int W(a,b)\frac{da}{a^{3/2}} \tag{13}$$

and in a discrete form

$$x(k) = \frac{1}{C_{1\psi}}\sum_{A} W(a,k)a^{-3/2} \tag{14}$$

if A is the domain of a.

3.2 Parameter selection

The purpose of this application is to isolate the impulsive components of the signal and reject the rest of the signal. These characteristics are assumed to exist in the largest coefficient values of each coefficient vector.

The signal is decomposed using the wavelet packet transform. It is decomposed at a specific depth L, for example L=3 results in 2^3 coefficient vectors. This decomposition has the advantage that the wavelet coefficients of different coefficient vectors, which belong to the same level L, are equivalent in terms of signal energy. By selecting decomposition which produces coefficient vectors which belong to the same level, the same threshold value for all coefficient vectors can be selected. The threshold is selected as a portion of the maximum of the absolute value of the set of all the wavelet packet coefficients

$$t_{hr} = \frac{\max(|c_{of}|)}{k_{th}} \qquad (15)$$

Hard thresholding is applied because it was found to be more effective for the purpose of isolation of impact generated peaks of signals.

A variation of this method is also proposed.

According to this variation the coefficients y are modified according to the formula:

$$y_{mod} = |y|^{k_m} \operatorname{sign}(y) \qquad (16)$$

where k_m integer. This nonlinear approach modifies the coefficients in a way that coefficients with larger absolute values contribute more in the reconstruction of the final signal. The RMS level of the signal is modified because of Eq. 16.This is a qualitative process aiming at isolating of impulsive characteristics.

4 Results and evaluation

The method is tested on impulsive vibration signals resulting from defective rolling element bearings.

Defects or wear cause impacts at frequencies governed by the operating speed of the unit and the geometry of the bearings, which in turn excite various machine natural frequencies.

For example the characteristic defect frequency f_o of a bearing with an outer race fault is

$$f_o = 0.5z(1 - \frac{d}{E}\cos a)f \qquad (17)$$

where d is the roller diameter, E is the pitch diameter , z is the number of rolling elements, a is the angle of contact.

The general assumption with rolling element bearing faults is that a measured signal contains a low-frequency phenomenon that acts as the modulator to a high-frequency carrier signal. In bearing failure analysis, the low-frequency phenomenon is the impact caused by a defect of a bearing; the high-frequency carrier is a combination of the natural frequencies of the associated rolling element or even of the machine. The goal of denoising is to suppress the oscillation caused by each impact and isolate an impulse sequence.

Two characteristic industrial cases are presented. Both signals were supplied by Alouminum of Greece S.A sampled at 8.33 kHz. The bearing examined in Case A is of type 22228cck/w33 manufactured by SKF. In case B the type of the bearing is1218C3. In both cases the signals were wavelet packet decomposed up to the level L=3, the Daubechies wavelet db4 was used. k_{th}=1.5 and k_m=5 were selected.

In case A, an extended outer race fault (fluting) was created on the outer race by electric arc caused by electric welding in the background of the bearing. In Fig. 3 the measured time waveform is illustrated. The signal is denoised by applying Eq.(15). The resulting waveform is illustrated in Fig. 4. Peaks spaced at the characteristic outer race defect period (approximately 5.2 ms) are observed and the shaft rotation frequency modulation becomes clearer.

In case B the defect was on the outer race, but it has a localized shape. The time domain shape in Fig. 5 does not reveal clear impulsive characteristics. In Fig. 6 the denoised signal applying Eq. (15) is presented. Peaks spaced at the characteristic outer race defect period (approximately 6.4 ms) are observed.

In Fig. 7 the denoised signal applying Eq. (16) is presented. Peaks spaced at the characteristic outer race defect period are also observed.

Spacing of the impacts is approximate due to speed variation and sliding effects.

5 Discussion - Conclusion

Denoising of vibration signals by modifying wavelet packet coefficients was presented.

It offers a better visual inspection of the impulsive content of the time domain signal. The wavelet packet denoising can be helpful when used in combination with the traditional frequency domain methods. It makes diagnosis of faults safer, since the interpretation of peaks in the frequency spectrum may have multiple interpretations. Hard thresholding was applied. Daubechies wavelets were used.

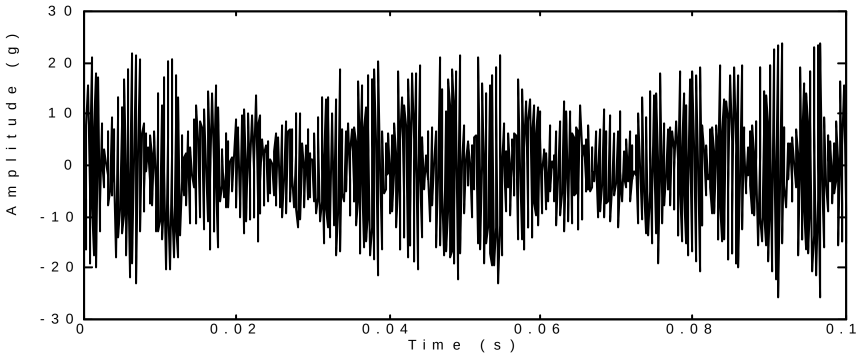

Figure 3. A vibration signal measured on a bearing with an extended outer race fault (case A)

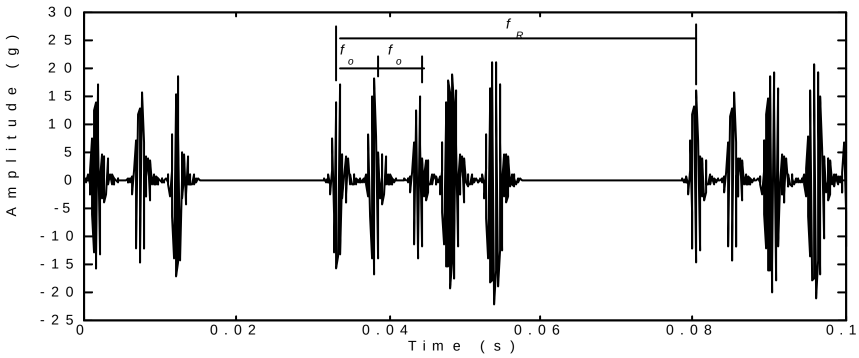

Figure 4. Denoised signal of Fig. 3 by   thresholding WP coefficients.

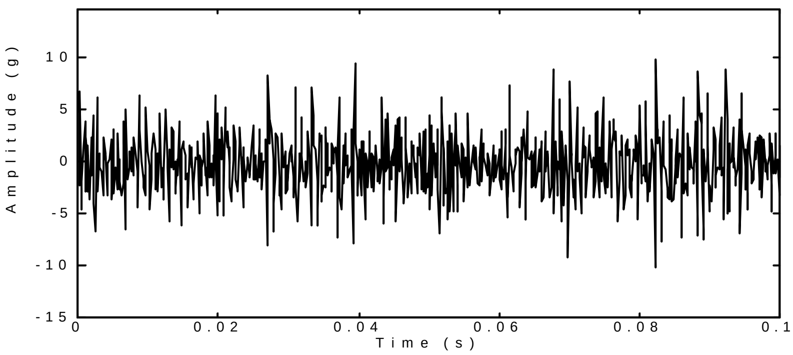

Figure 5. Vibration signal from an outer race localized defect (case B)

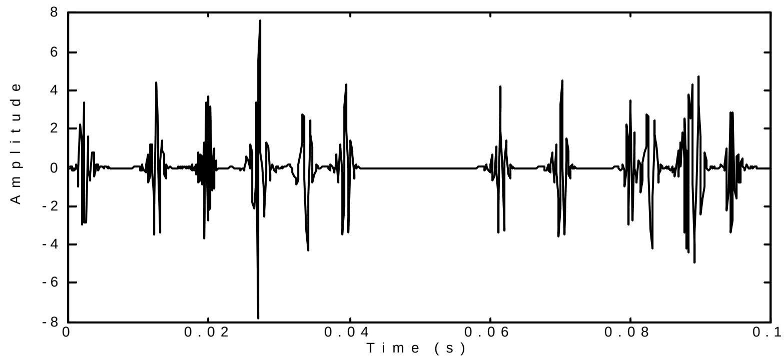

Figure 6. Denoised signal of Fig. 5 by thesholding WP coefficients

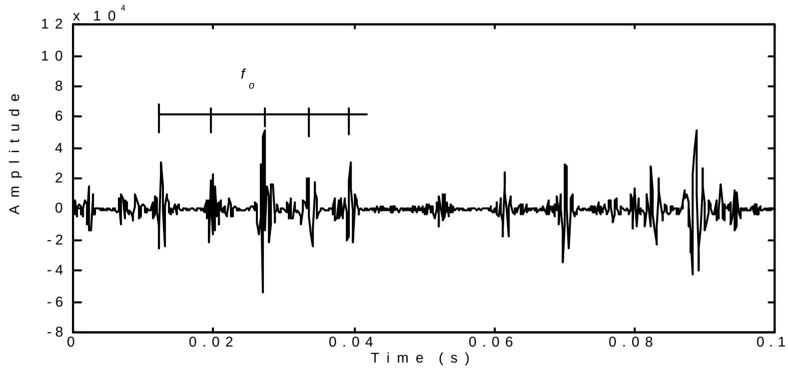

Figure 7. Denoised signal of Fig. 5 by modifying WP coefficients using Eq.(16)

It was observed that the use of lower order wavelets results in more poor representation of the frequency content of each impact response, but results in a more clear detection of the presence of impacts.

The wavelet packet method is simpler than the Morlet denoising method a characteristic of which is redundancy. The Morlet based denoising method seems to be more effective than the wavelet packet denoising method in extracting the frequency content of each impulse response. However, in the tested cases the interesting diagnostic information is the periodicity and the intensity of the impacts rather than the frequency content of each impact response. In several cases, hard thresholding seems to be more effective than soft thresholding in terms of intensity of the impacts.

The selection of thresholding level was selected as a fixed portion of the maximum absolute value of the coefficients making this choice simpler. However, the selection of threshold affects directly the resulting time waveform. The choice of threshold remains an open matter and should be the subject of future work.

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