Multi-user Detection for Wideband CDMA based on the Conjugate Gradient Method

KIYOMI IGARASHI†, YOSHIKAZU ARAI†, TOMOYA SAKAI†,
ATSUSHI FUKASAWA†, and YUMI TAKIZAWA††

† Graduate School of Science and Technology
Chiba University
1-33 Yayoi-cho, Inage-ku, Chiba 263-8522
JAPAN

†† Institute of Statistical Mathematics
Ministry of Education & Science
4-6-7 Minami azabu, Minato-ku, Tokyo 106-8569
JAPAN

Abstract: - Multiple users communicate with individually assigned codes on a single radio carrier frequency in a DS-CDMA system. Mutual interference among codes limits the capacity of the system. Conventional detectors of receivers are for single-user, which are designed neglecting the mutual interference. Multi-user detection is expected to enhance the capacity.

In this paper, a new scheme is proposed corresponding to solve a huge matrix equation under the multiple access interference (MAI) environments. This scheme is related to the mathematics; the Conjugate Gradient Method (CGM). The modeling of multi-user detection is shown, then the algorithm is given based on the mathematics; the Conjugate Gradient Method (CGM). The characteristics of the proposed scheme are evaluated with the comparison of the conventional single user detector. The proposed scheme is found to have reduced complexity comparing conventional technologies based on the serial and the parallel configurations.

Key-Words: - multiple access interference (MAI), multi-user detection, the conjugate gradient method (CGM)

1 Introduction

A Code Division Multiple Access (CDMA) system has been in practice for recent mobile radio communications, intelligent traffic systems (ITS), and satellite communications, etc.

The Wideband CDMA (W-CDMA) system has been proposed by the authors. This system is the PCD Radio Air Interface Standard TIA/EIA IS-665[1]. This features; (1)wideband spreading fills a given full radio frequency band, (2)coherent transmission using a pilot signal of a common PN code within a radio air interface system, (3)Walsh code set is assigned for control channels composing radio air interface system.

Unique code is assigned to individual users to transmit data. Spectrum of sending data is widely spread by a spread code. Detection of data is done by detecting the unique code assigned to a special user. Mutual interference among codes limits the capacity of the system. Current user detection is designed as a single detector because multi-user detection has never been given practically. Then multiple access interference (MAI) from users except the target user has been neglected.

Multi-user detection is expected to enhance the capacity of the CDMA system. The previous configurations ever proposed have been designed under the condition that transmission data and radio channel characteristics are simultaneously unknown. Convergence is not certified for solution, because that belongs to the blind deconvolution problem. To avoid this difficulty, the W-CDMA has been featured to utilize pilot signal, and this is the specific feature of the W-CDMA.

The authors have been studying fundamental multi-user detection schemes including serial and parallel configurations for simplified and reduced size with sufficient characteristics under the actual environments.

This paper proposes a novel scheme based on the mathematics; the Conjugate Gradient Method (CGM).

The algorithm and the characteristics of the proposed scheme is written in this paper with the comparison of the conventional single user detector. The proposed scheme is found reduced complexity comparing conventional configurations.

2 Conventional Detection Method

Considering single path condition, the received signal is represented as follows.

$$r(t) = \sum_{i=1}^{K} d_i(t) PN_i(t) \gamma_i(t) + n(t) \quad (1)$$

where, K is the number of channels, $d_i(t)$, $PN_i t)$, $\gamma_i(t)$ are data, code, and amplitude of the i-th user respectively. The detected signal $y_i(t)$ at the i-th user is written as follows.

$$y_i = \frac{1}{T_b} \int^{T_b} r(t) PN_i(t) dt \quad (2)$$

$$= \frac{1}{T_b} \int^{T_b} \left(\sum_{j=1}^{K} \gamma_j(t) PN_j(t) d_j(t) + n(t) \right) PN_i(t) dt \quad (3),$$

$$= \gamma_i d_i + \sum_{\substack{j=1 \\ j \neq i}}^{K} \rho_{j,i} \gamma_j d_j + \frac{1}{T_b} \int^{T_b} n(t) PN_i(t) dt \quad (4)$$

where,

$$\rho_{i,j} = \frac{1}{T_b} \int^{T_b} PN_i(t) PN_j(t) dt = \begin{cases} 1 & i = j \\ \rho_{i,j} & i \neq j \end{cases} \quad (5)$$

The detected signal is obtained as ;

$$y_i = \gamma_i d_i + MAI_i + z_i \quad (6).$$

Where, MAI_i is the component of Multi-Access Interference (MAI). The matrix representation of this equation is;

$$\boldsymbol{y} = \boldsymbol{R} \gamma \boldsymbol{d} + \boldsymbol{z} \quad (7)$$

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_i \\ \vdots \\ y_K \end{pmatrix}, \boldsymbol{d} = \begin{pmatrix} d_1 \\ \vdots \\ d_i \\ \vdots \\ d_K \end{pmatrix}, \boldsymbol{\gamma} = \begin{pmatrix} \gamma_1 \\ \vdots \\ \gamma_i \\ \vdots \\ \gamma_K \end{pmatrix}, \boldsymbol{z} = \begin{pmatrix} z_1 \\ \vdots \\ z_i \\ \vdots \\ z_K \end{pmatrix},$$

$$\boldsymbol{R} = \begin{bmatrix} 1 & & & \rho_{i,j} \\ & 1 & & \\ & & \ddots & \\ \rho_{j,i} & & & 1 \end{bmatrix} \quad (8),$$

where, $\boldsymbol{R}$ is the correlation matrix of $K \times K$.

3 Multi-User Detection Method

The capacity of DS-CDMA system depends on the characteristics of cross correlation of spectral spreading code. The values of the cross correlations, which is not zero, bring mutual interference. The multi-user detection should consider this mutual interference with other users' codes. This interference canceling problem is shown as the simultaneous linear equations(7). The inverse matrix of $\boldsymbol{R}$ can cancel interference components of the system. But it is tough problem to calculate all components of the correlation and to obtain its inverse matrix.

New multi-user detection method based on the Conjugate Gradient (CG) method is proposed. The CGM is known for an iterative solution method of simultaneous linear equations. The proposed method achieves to cancel interference from other users' channels without calculation of the autocorrelation and its inverse matrixes.

3.1 Conjugate Gradient Method

3.1.1 Theoretical

The CG method is an iterative solution method for simultaneous linear equations which has a positive definite matrix $\boldsymbol{A}$ as follows.

$$\boldsymbol{Ax} = \boldsymbol{b} \tag{9}$$

The solution by the CG method converges with finite adaptation steps. In the case of simultaneous equations with n unknown, the number of adaptation steps is less than n. The CG method is equivalent to that the solutions minimize the following equation.

$$f(\boldsymbol{x}) = \frac{1}{2}(\boldsymbol{x}, \boldsymbol{Ax}) - (\boldsymbol{x}, \boldsymbol{b}) \tag{10}$$

The approximate solution $\boldsymbol{x}^{(k+1)}$ at the (k+1)-th adaptation step is obtained from $\boldsymbol{x}^{(k)}$ at the k-th adaptation as follows.

$$\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} + \alpha^{(k)} \boldsymbol{p}^{(k)} \qquad (11),$$

where, $\alpha^{(k)}$ is adaptation coefficient, $\boldsymbol{p}^{(k)}$ is adaptation vector.

The adaptation coefficient $\alpha^{(k)}$ should satisfy the following.

$$\frac{df}{d\alpha^{(k)}} = 0 \tag{12}$$

Then,

$$\alpha^{(k)} = \frac{(\boldsymbol{p}^{(k)}, \boldsymbol{r}^{(k)})}{(\boldsymbol{p}^{(k)}, \boldsymbol{Ap}^{(k)})} \qquad (13),$$

$$\boldsymbol{r}^{(k)} = \boldsymbol{b} - \boldsymbol{Ax}^{(k)} \tag{14}$$

where, $\boldsymbol{r}^{(k)}$ is residual vector.

The adaptation vector $\boldsymbol{p}^{(k)}$ should satisfy the following.

$$(\boldsymbol{s}^{(k)}, \boldsymbol{Ap}^{(k)}) = 0 \qquad (15),$$

where, $\boldsymbol{s}^{(k)}$ is error vector which is defined as follows.

$$\boldsymbol{s}^{(k)} = \boldsymbol{A}^{-1}\boldsymbol{b} - \boldsymbol{x}^{(k)} \tag{16}$$

Equation (15) means that the adaptation vector $\boldsymbol{p}^{(k)}$ has been given to satisfy the vectors $\boldsymbol{Ap}^{(k)}$ and $\boldsymbol{s}^{(k+1)}$ are orthogonal each other. The vector $\boldsymbol{p}(i)$ and $\boldsymbol{p}(j)$ are conjugate each other when they satisfy the following orthogonal relation.

$$(\boldsymbol{p}^{(i)}, \boldsymbol{Ap}^{(j)}) = 0 \qquad (15),$$

From this relation, new vector $\boldsymbol{p}^{(k+1)}$, which is conjugate to the former vector $\boldsymbol{p}^{(k)}$, gives optimal value. The adaptive solution converges to strict solution with n time steps.

3.1.2 Calculation procedure

Simultaneous linear equation with n unknown is;

$$\boldsymbol{Ax} = \boldsymbol{b} \qquad (16).$$

where,

$$\boldsymbol{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}, \boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \tag{17}$$

The (k+1)-th adaptive solution $\boldsymbol{x}^{(k+1)}$ is;

$$\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} + \alpha^{(k)} \boldsymbol{p}^{(k)} \qquad (18),$$

where,

$$\boldsymbol{x}^{(k)} = \begin{pmatrix} x_1^{(k)} \\ x_2^{(k)} \\ \vdots \\ x_n^{(k)} \end{pmatrix}, \ \boldsymbol{p}^{(k)} = \begin{pmatrix} p_1^{(k)} \\ p_2^{(k)} \\ \vdots \\ p_n^{(k)} \end{pmatrix}, \ \boldsymbol{r}^{(k)} = \begin{pmatrix} r_1^{(k)} \\ r_2^{(k)} \\ \vdots \\ r_n^{(k)} \end{pmatrix} \tag{19}$$

(1) The approximate solution at the 0th adaptation step as an initial value is;

$\boldsymbol{x}^{(0)}$

(20).

(2) The residual for the 0th adaptation is;

$$\boldsymbol{r}^{(0)} = \boldsymbol{b} - \boldsymbol{A}\boldsymbol{x}^{(0)} \quad (21).$$

(3) The adaptation vector at the 0th adaptation is;

$$\boldsymbol{p}^{(0)} = \boldsymbol{r}^{(0)} \quad (22).$$

$$k = 0 \quad (23).$$

(4) The k-th adaptation coefficient $\alpha^{(k)}$ is;

$$\alpha^{(k)} = \frac{(\boldsymbol{r}^{(k)}, \boldsymbol{p}^{(k)})}{(\boldsymbol{p}^{(k)}, \boldsymbol{A}\boldsymbol{p}^{(k)})} \quad (24).$$

(5) The approximate solution at the (k+1)-th adaptation is;

$$\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} + \alpha^{(k)}\boldsymbol{p}^{(k)} \quad (25).$$

(6) The residual at the (k+1)-th adaptation is;

$$\boldsymbol{r}^{(k)} = \boldsymbol{r}^{(k)} - \alpha^{(k)}\boldsymbol{A}\boldsymbol{p}^{(k)} \quad (26).$$

(7) The k-th adaptation coefficient $\beta^{(k)}$ for $\boldsymbol{p}$ is;

$$\beta^{(k)} = \frac{(\boldsymbol{r}^{(k+1)}, \boldsymbol{r}^{(k+1)})}{(\boldsymbol{r}^{(k)}, \boldsymbol{r}^{(k)})} \quad (27).$$

(8) The adaptation vector at the (k+1)-th adaptation is;

$$\boldsymbol{p}^{(k+1)} = \boldsymbol{r}^{(k+1)} + \beta^{(k)}\boldsymbol{p}^{(k)} \quad (28).$$

(9) Convergence criterion is tested.
If it is not enough, then

$$k = k + 1 \quad (29),$$

Go to (4).

3.2 Proposed Multi-user Detection Method

3.2.1 CG Multi-user Detection Method

The CG method is applied to multi-user detection. Eq.(7) is solved as Eq.(9) in the CG method. Then,

$$\boldsymbol{b} = \boldsymbol{y} \quad (30),$$

$$\boldsymbol{A} = \boldsymbol{R\gamma} \quad (31),$$

$$\boldsymbol{x}^{(k)} = \boldsymbol{d}^{(k)} \quad (32).$$

Here, the adaptation vector is written $\boldsymbol{q}^{(k)}$ to discriminate the receive signal. This detection operation is applied repeatedly.

The most of processing of this multi-user detection is simple calculation of the inner products and mapping by the matrix $\boldsymbol{R}$. The proposed system is composed of simple configuration and calculation, and its solution convergent rapidly.

The estimated detected data $\boldsymbol{d}^{(k+1)}$ is the following.

$$\boldsymbol{d}^{(k+1)} = \boldsymbol{d}^{(k)} + \alpha^{(k)}\boldsymbol{p}^{(k)} \quad (33)$$

The output from matched filter is used as the initial value $\boldsymbol{d}^{(0)}$ to get rapid convergence.

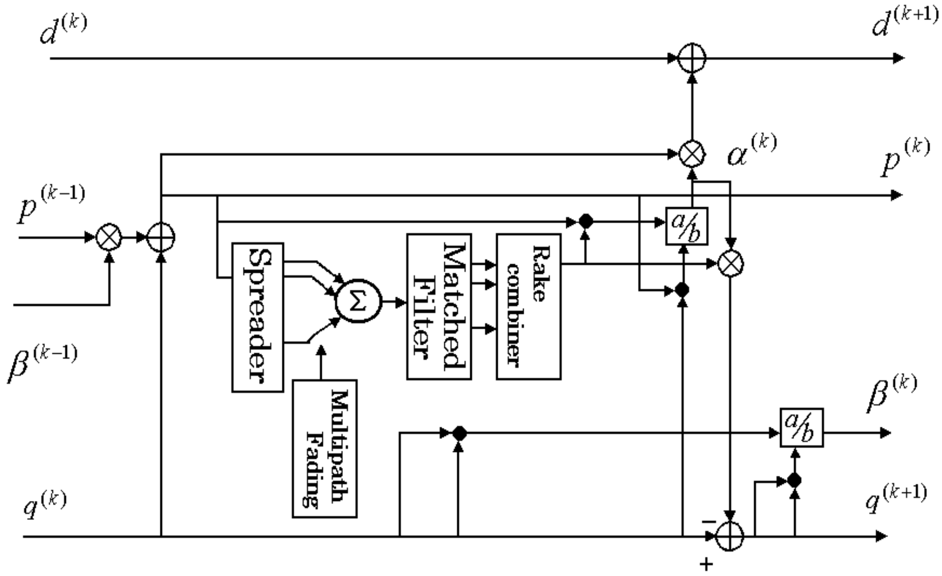

Fig. 1 Configuration of Advanced CG Multi-User Detection

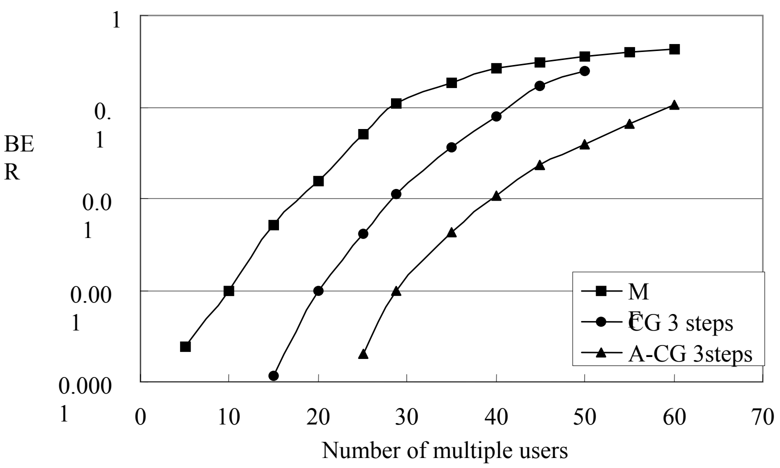

Fig. 2 BER v.s. number of multiple users

3.2.2 Advanced CG Multi-user Detection Method

In general, it is difficult to get strict synchronization of a frame corresponding to one bit of transmission data. The correlation matrix $\boldsymbol{R}$ in Eq.(8) represents components of MAI in the case of established synchronization.

In the case of asynchronous condition, N bits is taken for each user to account interference among users.

The size of the matrix $\boldsymbol{R}$ increases rapidly as $NK \times NK$. The proposed scheme in **3.2.1** is also applicable to this huge matrix. Adaptation steps increases up to NK. Under limited condition of number of adaptation times, the residual error is not reduced sufficiently.

Improvement of configuration is required for achievement of rapid convergence and reduced error.

The advanced CG detection is shown in Fig.1. The operation of vector combining in RAKE receiving has been used in a detection configuration proceeding matched filter.

4 Evaluation

Five component multi-path propagation model specified by the Joint Technical Committee (JTC) of the United States standardization project for 2 GHz Personal Communication Services has been used for evaluation of the proposed method[2].

The results of evaluation for the proposed systems are shown in Fig.2. The horizontal and vertical axes represent number of multiple users and bit error rate. From the left, the first one is the characteristics of matched filter only, the middle is the characteristics of the proposed system with three steps of adaptation without RAKE combining part. The right shows performance of the system composed with three steps adaptation, which fundamental configuration is shown in Fig.1, is shown in Fig.2.

5 Conclusion

The authors have proposed new schemes of the multi-user detection for the Wideband CDMA communication system. The newly proposed scheme has been brought with mathematical viewpoint. The configuration of the proposed method was simplified and the characteristics are equal or higher than the conventional serial and parallel configurations. As a future study, more improvement would be tried with the scheme of error correction.

A new scheme has been intended for multi-user detection in a receiver to enhance capacity of the W-CDMA proposed by the authors, and evaluated by the Information and Communication Institute of the United States of America.

About the scheme and configuration of radio air interface system, W-CDMA IMT-2000 proposed by Japan and Europe are identical except minor difference points including data interface, frame configuration, and pilot signal transmission (continuous or interval). Then the proposed scheme could be applied to two schemes.

References:

[1] A. Fukasawa, T. Sato, Y. Takizawa, T. Kato, M. Kawabe, and R. E. Fisher, “Wideband CDMA System for Personal Radio Communications,” IEEE Communications Magazine, Topics in Personal Communications, Vol.34, No.10, 116-123, Oct. 1996.

[2] “EIA/TIA/IS-665, W-CDMA (Wideband Code Division Multiple Access) Air Interface Compatibility Standard for 1.85-1.99 GHz PCS Applications,” EIA/TIA/95.09.1, Sep. 1995.

[3] S. Moshavi, “Multi-user Detection for DS-CDMA Communications,” IEEE Communications Magazin, pp.124-135, Oct. 1996.

[4] T. Sakai, C. Shimakura, A. Fukasawa, and S. Ohkawa, “Detection of DS-CDMA based on the Conjugate Gradient Method,” The Third Meeting on Universal Personal Communication Theory, IEICE Engineering Sciences Society, Oct. 2-3, 1997.

[5] A. Fukasawa, Y. Takizawa, and M. Hatori, “Basic Understanding on Spread Spectrum Communication and Its Application to Mobile Radio Communications,” Journal of IEICE, Vol.81, No.1, pp.51-59, Jan. 1998.

[6] A. Fukasawa, S. Ohkawa, and Y. Takizawa, “THe Latest Technology of Wideband-Spread CDMA for Mobile Radio Communications,” Journal on C.C.C.C.C., vol. XLIX, Sept-Oct, 1998.