Regions of the Accepted Tolerance for the Recognition of Binary Imprints in Robotics

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Abstract: - The tactile matrix sensors often generate the binary imprints of the explored shapes in robotics. For the shape recognition, their vectors of the characteristic features define both the prototypes and the unknown shapes. The similarity between two shapes is estimated with different distances. The paper recommends the reference distances and defines the regions of the accepted tolerance, associated with a prototype. The shapes of these regions in the (p_1, p_2) parameter plane and (p_1, p_2, p_3) parameter space are presented and analyzed for various distances.

Key-Words: - recognition, binary imprint, parameter, prototype, region of the accepted tolerance, distance.

1. Introduction

In the shape recognition process, many prototypes (known shapes) are first touched and the robot memorizes the values of characteristic parameters; the same parameters describe each unknown shape that must be recognized. The similarity between two shapes can be estimated with different distances, but the choice of the proper one is a very difficult problem. This paper proposes the identification of an unknown shape with a prototype if that shape is inside the region of the accepted tolerance of the prototype. Different reference distances are defined and the resulted regions of the accepted tolerance, associated with a prototype, are presented. By comparing these regions, we get the proper distances for the identification of similar or very different shapes.

2. Problem Formulation

The procedure for the recognition of the shapes using their binary imprints [11], presented in [8,9], consists of three steps: choosing the descriptors, prototype learning and classification, unknown shape identification. This procedure assures a certain recognition (when x is identified with only one prototype), an ambiguous recognition (when x presents the same similarity with minimum two prototypes), or a rejection of the unknown shape (when x cannot be identified with a prototype). Some observations about this recognition method should be mentioned.

a) For a quick recognition of each unknown shape, it is touched only once, regardless of its location in the sensory plane [2,4,6]. In these conditions, only characteristic features (parameters) invariant or quasi-invariant to rotation and translation of the 2D-shape in the plane of the tactile matrix sensor must describe the binary imprint of the unknown shape.

b) In the learning step of the recognition process, many 2D-shapes called *prototypes* are touched; v different binary imprints of each shape are analyzed and described by f parameters invariant or quasi-invariant to rotations and translations of the shape in the sensory plane [7,8,10]. The characteristic feature vectors of those v different imprints of the i prototype are $m_i^k = \left[m_{i,1}^k, m_{i,2}^k, ..., m_{i,f}^k\right]^T$, $k = \overline{1,v}$. Based on these vectors, the i prototype can be defined by its characteristic feature vector, $m_i = \left[m_{i,1}, m_{i,2}, ..., m_{i,f}\right]^T$, where

$$m_{i,j} = 0{,}5\left(\min_{k=\overline{1,v}} m_{i,j}^k + \max_{k=\overline{1,v}} m_{i,j}^k\right). \qquad (1)$$

The recognition procedure imposes distinct value domains for at least one of the characteristic parameters that describe any two prototypes.

The vector of the minimum values of the parameters, associated with the i prototype, is $e_i = [e_{i,1}, e_{i,2}, ..., e_{i,f}]^T$ with

$$e_{i,j} = \min_{k=\overline{1,v}} m_{i,j}^k . \qquad (2)$$

c) The same f parameters describe the unknown shape x. Its characteristic feature vector is $x = [x_1, x_2, ..., x_f]^T$ and results by processing only one binary imprint of the shape x. The similarity between x and m_i is established based on the distance $d(x, m_i)$ between x and m_i. The unknown shape is identified with m_i if

$$d(x, m_i) \le d_{ref}(m_i), \qquad (3)$$

where $d_{ref}(m_i)$ is a reference distance.

This paper proposes some reference distances, presents the obtained regions of the accepted tolerance and analyzes the shapes of these regions.

3. Assignation of the reference distance

The distances recommended in [1,3,12] to estimate the similarity between an unknown shape $x = [x_1, x_2, ..., x_f]^T$ and the prototype $m_i = [m_{i,1}, m_{i,2}, ..., m_{i,f}]^T$ are the following: $d_1(x, m_i)$ - Hamming distance, $d_2(x, m_i)$ -Euclidean distance, $d_\infty(x, m_i)$ - maximum distance, $d_\infty^w(x, m_i)$ - maximum weighted distance.

The characteristic features vector and the vector of the minimum values of the parameters, associated with the i prototype, are represented in the parameter space by two points: $M_i(m_{i,1}, m_{i,2}, ..., m_{i,f})$ and $E_i(e_{i,1}, e_{i,2}, ..., e_{i,f})$, respectively. Around each M_i point there is a *region of the accepted tolerance* [5,12],

$$R_t^i(d) = \{x | d(x, m_i) \le d_{ref}(m_i)\}, \qquad (4)$$

where $d(x, m_i)$ is one of the above specified distances and $d_{ref}(m_i)$ is the associated reference distance. The unknown shape x is identified with m_i if $x \in R_t^i(d)$.

The reference distances must be defined for d_1, d_2, d_∞ and d_∞^w.

- If $d(x, m_i)$ is a classical distance,

$$d(x, m_i) = d_q(x, m_i), \ q = 1,2,\infty, \qquad (5)$$

the reference distance

$$d_{q,ref}(m_i) = d_q(e_i, m_i), \ q = 1,2,\infty \qquad (6)$$

is proposed in [9] for the identification of x with m_i. So, when a classical distance $d_q(x, m_i)$ is used, the region of the accepted tolerance for the i prototype is

$$R_t^i(d_q) = \{x | d_q(x, m_i) \le d_q(e_i, m_i)\}. \qquad (7)$$

- The maximum weighted distance [9,12] between x and m_i is

$$d_\infty^w(x, m_i) = \max_{k=\overline{1,f}} w_{i,j} |x_k - m_{i,k}|, \qquad (8)$$

where the coefficients for weighting are

$$w_{i,j} = \frac{\min_{j=\overline{1,f}} e_{i,j}}{e_{i,j}} = \frac{e_{i,min}}{e_{i,j}}, \ j = \overline{1,f}. \qquad (9)$$

The proposed reference distance,

$$d_{\infty,ref}^w(m_i) = d_\infty^w(e_i, m_i) = e_{i,min}, \qquad (10)$$

determines the following region of the accepted tolerance:

$$R_t^i(d_\infty^w) = \{x | d_\infty^w(x, m_i) \le e_{i,min}\}. \qquad (11)$$

4. Regions of the Accepted Tolerance Obtained Using Various Distances

In the f-dimensional parameter space, the regions of the accepted tolerance have different shapes for the same m_i prototype, depending on the used distance and associated reference distance. Let us consider two situations:

- Each shape is described by two parameters (p_1 and p_2), when the regions $R_t^i(d_1)$, $R_t^i(d_2)$, $R_t^i(d_\infty)$ and $R_t^i(d_\infty^w)$ are 2D-shapes in the (p_1, p_2) plane;

- Each shape is described by three parameters (p_1, p_2 and p_3), when the same regions are 3D-shapes in the (p_1, p_2, p_3) space.

4.1. Regions of the Accepted Tolerance in the (p₁,p₂) Parameter Plane

The value domains of the p_1, p_2 parameters generate the shaded rectangles in fig. 1, 2, 3 and 5, for the i prototype.

The classical distances [1,3,12] and the associated reference distances, defined by the relations (6) and (10), determine the regions of the accepted tolerance of the i prototype, represented in fig. 1, 2 and 3; the M_i point is the center of each region.

- For the Hamming distance, the reference distance is

$$d_{1,ref}(m_i) = d_1(e_i, m_i) = \sum_{k=1}^{f} \left| e_{i,k} - m_{i,k} \right|, \qquad (12)$$

and $R_t^i(d_1)$ is the surface of a square with $\sqrt{2} d_{1,ref}(m_i)$ the side and the diagonals parallel to Op_1 and Op_2 (fig. 1).

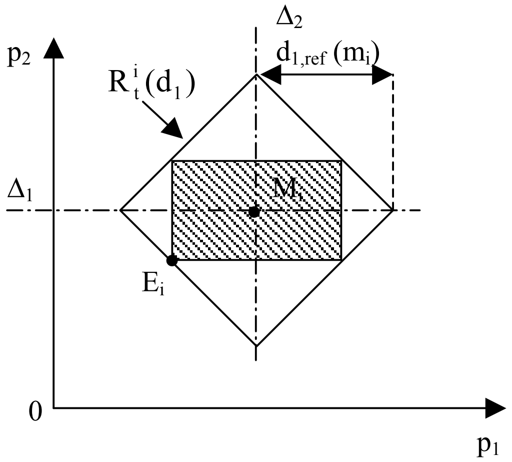

Fig. 1

- When the Euclidean distance is used, the reference distance is

$$d_{2,ref}(m_i) = d_2(e_i, m_i) = \sqrt{\sum_{k=1}^{f} \left(e_{i,k} - m_{i,k} \right)^2}, \qquad (13)$$

and $R_t^i(d_2)$ is the surface of a circle with $d_{2,ref}(m_i)$ the radius (fig. 2).

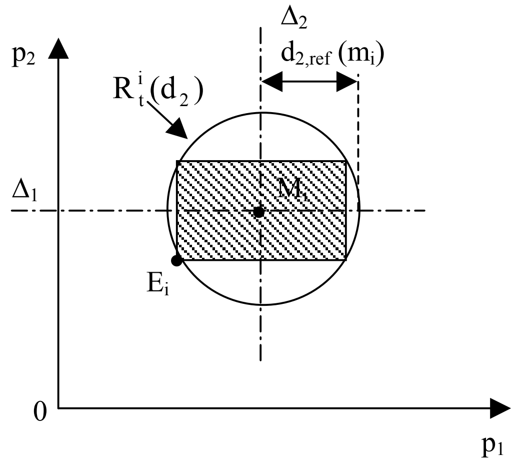

Fig. 2

- The reference distance for the maximum distance is

$$d_{\infty,ref}(m_i) = d_\infty(e_i, m_i) = \max_{k=\overline{1,f}} \left| e_{i,k} - m_{i,k} \right|, \qquad (14)$$

and $R_t^i(d_\infty)$ is the surface of a square with $2d_{\infty,ref}(m_i)$ the side and the sides parallel to Op_1 and Op_2 (fig. 3).

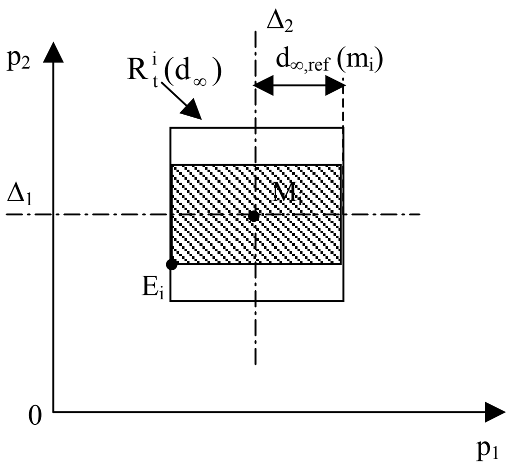

Fig. 3

- For the maximum weighted distance, the reference distance is $d_{\infty,ref}^{w}(m_i) = e_{i,min}$; $R_t^i(d_\infty^w)$ is the surface of a rectangle whose sides represent the value domains of the p_1, p_2 parameters (the shaded surfaces in fig. 1, 2, 3 and 5).

4.2. Regions of the Accepted Tolerance in the (p_1,p_2,p_3) Parameter Space

In the (p_1, p_2, p_3) parameter space, the M_i point is the center of all regions of the accepted tolerance, $R_t^i(d)$, $d = d_1, d_2, d_\infty, d_\infty^w$. These regions are presented below.

- $R_t^i(d_1)$ is the combined interiors of two opposing pyramids of the exact same size, with a common square base; the side of the base is $\sqrt{2} d_{1,ref}(m_i)$ and the height is $d_{1,ref}(m_i)$;
- $R_t^i(d_2)$ is the interior of a sphere, with $d_{2,ref}(m_i)$ the radius;
- $R_t^i(d_\infty)$ is the interior of a cube with $d_{\infty,ref}(m_i)$ the side;
- $R_t^i(d_\infty^w)$ is the interior of a parallelepiped whose sides are the value domains of the p_1, p_2 and p_3 parameters.

4.3. Discussion

Let us consider P_i^f a polytope centered in M_i and whose sides are the value domains of the $p_1, p_2, ..., p_f$ parameters. This polytope is precisely $R_t^i(d_\infty^w)$.

By studying the regions of the accepted tolerance, obtained in (p_1, p_2) parameter plane and (p_1, p_2, p_3) parameter space, the following conclusions result.

a) $R_t^i(d_1)$, $R_t^i(d_2)$ and $R_t^i(d_\infty)$ have symmetrical shapes in the f-dimensional parameter space. For example

- In the (p_1, p_2) parameter plane, these regions have two symmetry axes that pass through M_i: Δ_1 - parallel to Op_1, and Δ_2 - parallel to Op_2 (figures 1,2,3);

- In the (p_1, p_2, p_3) parameter space, the same regions are 3D-shapes symmetrical in comparison to three planes that pass through M_i and are parallel to the (p_1, p_2), (p_1, p_3) and (p_2, p_3) planes.

But the regions of the accepted tolerance must estimate the polytope of the value domains for the i prototype. If the value domains don't have comparable spreads, the symmetry of the regions is a drawback.

b) All the regions of the accepted tolerance obtained using classical distances are often much larger than $P_i^f \equiv R_t^i(d_\infty^w)$ (fig. 4, where $f = 2$). The minimum difference between the surface of $R_t^i(d)$, $d = d_1, d_2, d_\infty$ and the surface of P_i^f is obtained when the value domains have the same spread for all parameters; in this case, P_i^2 is a square (fig. 5) and P_i^3 is a cube.

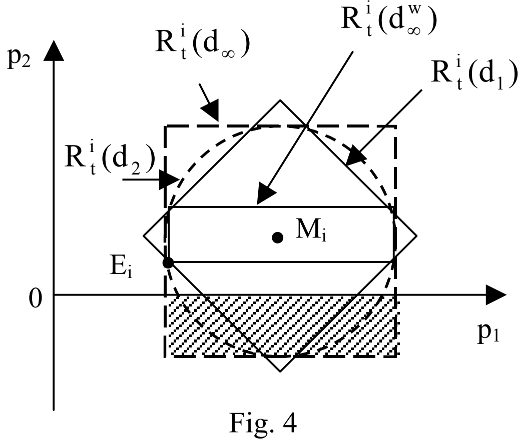

Fig. 4

c) If similar shapes must be recognized, their regions $R_t^i(d_1)$, $R_t^i(d_2)$ and $R_t^i(d_\infty)$ are often not disjoint. So, using classical distances, the recognition can be ambiguous. Because $P_i^f \equiv R_t^i(d_\infty^w)$, d_∞^w is recommended for the recognition of similar shapes.

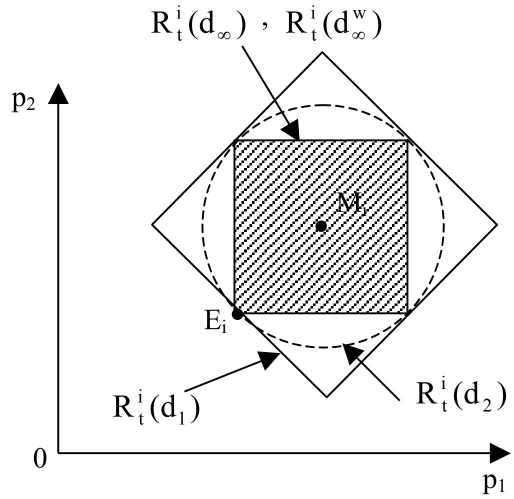

Fig. 5

d) Even if the real values of some parameters (for example p_2) are only positive, the symmetrical regions $R_t^i(d)$, $d = d_1, d_2, d_\infty$ often contain zones where those parameters have negative values (the shaded 2-D surface in fig. 4). These zones must be eliminated from the regions of the accepted tolerance.

5. Conclusion

The region of the accepted tolerance is useful in the recognition of binary imprints, obtained by touch.

An unknown shape is identified with a prototype if that shape is inside the region of the accepted tolerance of the prototype. Each shape is described by f characteristic features and represents a point in the f-dimensional parameter space. Any two shapes must have distinct value domains for at least one of the characteristic parameters.

In order to estimate the similarity between the shapes, four distances can be used: d_1, d_2, d_∞ and d_∞^w. The classical distances (d_1, d_2, d_∞) generate large symmetrical regions of the accepted tolerance; these distances can be used only for very different shapes. The maximum weighted distance (d_∞^w) is recommended for the recognition of similar shapes.

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