Boundary between CCM and DCM in DC/DC PWM Converters

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Abstract: - It is presented a modality to compute the boundary between the discontinuous (DCM) and continuous (CCM) operating modes in the DC/DC fourth-order PWM converters with parasitic resistances. To reach this goal, an averaged model of the converter has been used. The boundary between DCM and CCM, considered to be the critical load resistance R_{crit}, or, equivalently, the critical value of the inductor conduction parameter k_{ctrit}, is found as the positive root of a second-degree equation with unknown R_{crit} or k_{ctrit}. The algorithm carried out and implemented with MATLAB environment allows studying the effect of the parasitic resistances and the coupling of inductors over the boundary R_{crit} or k_{tcrit}, the steady-state properties and the external characteristics of the converter. The simulation results show that the parasitic resistances and the inductor coupling change the boundary between the discontinuous and continuous conduction modes of a fourth-order PWM converter.

Key-Words: - DC/DC fourth-order PWM converters, boundary between CCM and DCM

1 Introduction

In the elementary PWM converters (buck, boost, buck-boost), which are second-order converters, the discontinuous conduction mode (DCM) appears at the moment when the diode current becomes zero. This will occur at a specific operating point (duty ratio D and load resistance R). The current through the inductor becomes discontinuous and would stay discontinuous for the third switched interval D_3T_s. Here, the boundary between CCM and DCM is defined by the critical value of the inductor conduction parameter (k_{crit}) or by the critical value of the load resistance (R_{crit}). Considering the converter to be ideal, this boundary has been calculated as the positive root of a second-degree equation [1]. Including the parasitic resistances of the circuit, the degree of the equation with the unknown R_{crit} or k_{crit} remains unchanged [2].

A fourth-order converter consists of two inductors, two capacitors, and a single switch realized by transistor and diode combination, such as the non isolated Cuk, Sepic and Zeta converters and the ripple-free input-current PWM boost converter. In these converters, the two inductors conduct some constant current I, even for the third switched interval D_3T_s. In this third switched interval D_3T_s, the sum i_1+i_2 of two inductors currents that traverses the diode is zero [3] – [5]. Using the individual inductor voltage and current waveforms and the volt-second-balance law on the inductors, for the assumed 100% efficiency, the boundary between CCM and DCM has been easily found as the positive root of a second-degree equation, for a non-isolated Cuk converter with DCM and a ripple-free input-current boost converter with DCM [3], [4].

2 Boundary between CCM and DCM

It is known that the boundary between CCM and DCM is established by the specific value of the parameter D_2, namely $D_2=1-D$, regardless of the order (two or four) of PWM switching converters. So, a PWM converter operates with CCM when $D_2>1-D$ and with DCM when $D_2<1-D$ [1], [3].

For the elementary or second-order PWM converters (buck, boost, buck-boost) no parasitic included, the boundary between CCM and DCM is given either as the critical value of the inductor conduction parameter k, written k_{crit}, or as the critical value of the load resistance R, written R_{crit}. It is natural to express the boundary in terms of the load resistance R, rather than the dimensionless parameter k. The parameter k is defined as

$$k = \frac{2Lf_s}{R} = \frac{R_{nom}}{R} \quad (1)$$

where R_{nom} is a design parameter called nominal load resistance of the converter and f_s is the constant switching frequency. Considering (1) for k_{crit}, the load boundary R_{crit} results as

$$R_{crit} = \frac{R_{nom}}{k_{crit}}. \quad (2)$$

Similarly, the boundary between CCM and DCM for the fourth-order converters (non-isolated Cuk, Sepic, Zeta and ripple-free input-current boost converters etc.) is given either as the critical value of conduction parameter k_{tcrit} ($k_t = 2L_e f_s/R$) through an equivalent inductor L_e ($L_e = L_1//L_2$) or as the critical value of the load resistance R_{crit} [3]-[5]. Like the second-order converters with ideal circuit components, these critical values have been found as the positive root of a second-degree equation with unknown k_{tcrit} or R_{crit}.

The expressions of k_{tcrit} as functions on duty ratio and dc voltage conversion ratio are shown in Table 1, for a non-isolated Cuk PWM converter and a ripple-free input-current boost PWM converter [3], [4]. The dc voltage conversion ratio of the converter with DCM is written M_D.

Table 1: k_{tcrit} for a non-isolated Cuk PWM converter and a ripple-free input-current boost converter (no parasitic case)

Converter	$k_{tcrit} = f(D)$	$k_{tcrit} = f(M_D)$
Cuk	$(1-D)^2$	$\frac{1}{(1+M_D)^2}$
Ripple-free Input-current Boost	$D(1-D)^2$	$\frac{M_D - 1}{M_D^3}$

The functions $R_{crit} = f(D)$ and $R_{crit} = f(M_D)$ are given in Table 2, for the same ideal converters [3, 4].

Table 2: R_{crit} for a non-isolated Cuk PWM converter and a ripple-free input-current boost converter (no parasitic case)

Converter	$R_{crit} = f(D)$	$R_{crit} = f(M_D)$
Cuk	$\frac{R_{nom}}{(1-D)^2}$	$(1+M_D)^2 R_{nom}$
Ripple-free Input-current Boost	$\frac{R_{nom}}{D(1-D)^2}$	$\frac{M_D^3}{M_D - 1} R_{nom}$

3 Equation of the Boundary between CCM and DCM for Nonideal Converters

Despite its complexity, the state-space averaging (SSA) method is the most popular approach used for the modelling of the dc-dc switching converters. This method allows including all the parasitic resistances of the circuit even in the initial stage and supplies both the static and dynamic models of the converter. The following parasitic resistances of the converter (written p_i with $i = \overline{1,6}$) have been considered here: the loss resistance of the inductors (r_1 and r_2), the equivalent-series resistance of the capacitors (r_3 and r_4), the conducting-state resistance of the transistor (r_5) and the diode (r_6).

Using the SSA method, a fourth-degree equation in unknown D_2 has been carried out for the fourth-order converters with parasitic included [8]:

$$m_4 D_2^4 + m_3 D_2^3 + m_2 D_2^2 + m_1 D_2 + m_0 = 0. \quad (3)$$

In order to obtain the equation (3), the matrices A and B that describe the averaged steady-state model of the fourth-order PWM converter with DCM have been set in the form:

$$A = A_e + D_2 A_{23}$$
$$B = B_e + D_2 B_{23}$$

where

$$A_e = DA_1 + D'A_3 = [a_e(m,n)]$$
$$B_e = DB_1 + D'B_3 = [b_e(m)]$$
$$A_{23} = A_2 - A_3 = [a_{23}(m,n)]$$
$$B_{23} = B_2 - B_3 = [b_{23}(m)]$$
$$D' = 1 - D$$

with $m = n = \overline{1,4}$.

Generally, the coefficients m_i with $i = \overline{1,4}$, from the equation (3), are functions on the load resistance R. At the boundary between the two operating modes, D_2=1-D and R becomes R_{crit}. In order to distinguish this last element, the components of the matrices A_e and A_{23}, which are functions on R, have been split in two terms: one term is independent of R and another that is function on R. For example, it writes:

$$a_e(m,n) = a_{e0}(m,n) + r_1(m,n) f_1(R)$$

$$a_{23}(m,n) = a_{230}(m,n) + r_2(m,n) f_2(R).$$

Replacing D_2 with 1-D and R with R_{crit} into the equation (3), a second-degree equation in unknown R_{crit} results for the fourth-order PWM converter with parasitic included, like as for the second-order PWM converters [2]:

$$n_2 R_{crit}^2 + n_1 R_{crit} + n_0 = 0. \tag{4}$$

The coefficients n_2, n_1 and n_0 from the equation (4) are functions of the duty ratio D, all the parameters and parasitic resistances of the circuit, the switching frequency and the coupling coefficient k_c of the inductors: $R_{crit}=f\,(D, L_1, L_2, C_1, C_2, p_i, f_s, k_c)$. The general form of the coefficients from the equation (4), as well as those from the equation (3), allows studying four PWM converter configurations, namely: the non isolated Cuk, Sepic and Zeta converters, and the ripple-free input current boost converter.

A second-degree equation in unknown k_{tcrit} is obtained if the equation (2) is introduced into (4):

$$n_0 k_{tcrit}^2 + n_1 R_{nom} k_{tcrit} + n_2 R_{nom}^2 = 0. \tag{5}$$

Like as R_{crit}, the coefficients of this last equation depend on the duty ratio D, all the parameters and parasitic resistances of the circuit, the switching frequency and the coupling coefficient k_c of the inductors: $k_{tcrit}=f\,(D, L_1, L_2, C_1, C_2, p_i, f_s, k_c)$.

The same second-degree equation with the unknown R_{crit} or k_{tcrit} can be obtained by equaling the expressions of M_C and M_D, deduced from the averaged steady-state models corresponding to DCM and CCM, and replacing D_2 with 1-D in M_D.

The dc characteristics of the converter as the average values of the currents that traverse the inductors, the voltage drops across the capacitors and the dc voltage conversion ratio ($M=V_O/V_I$) of converter are obtained through the SSA method, for both DCM and CCM. The two dc voltage conversion ratio M_C for CCM and M_D for DCM have the same value ($M_C= M_D$) on the boundary between CCM and DCM. Consequently, plotting the external characteristics of the converter for both continuous and discontinuous operating modes, that is $M_C=f\,(D, R)$ and $M_D=f\,(D, R)$, the boundary R_{crit} between CCM and DCM could be found as the intersection of these two external characteristic families of converter: $R_{crit} = R|_{M_c = M_D}$. The value k_{tcrit} can be calculated with the formula (2). But this way of determination of the boundary between DCM and CCM is not recommended because this is a graphical method that cannot prove a satisfactory accuracy.

4 Simulation results

MATLAB environment offers a very simple implementation of this algorithm carried out for establishing the boundary between DCM and CCM in the fourth-order PWM converter.

The analytical way for establishing R_{crit} or k_{tcrit} in the fourth-order PWM converters with parasitic resistances has been studied on a ripple-free input-current boost converter with the specifications: L_1=5.1 μH, L_2=0.7 μH, C_1=18 μF, C_2=1000 μF and f_s=300 kHz [4]. The following parasitics have been considered here: $r_1=r_2=r_4=r_5=r_6=0.1\ \Omega$ and $r_3=0.01\ \Omega$.

For this converter, the effect of the inductor coupling and the parasitic resistances of the circuit over R_{crit} can be seen in Figures 1 and 2.

In Fig. 1, the function $R_{crit}=f\,(D, k_c)$ is plotted for several values of D and k_c, for an ideal converter (p_i=0). As it can be seen from this figure, the increasing of the value of k_c causes the decreasing of R_{crit} for all the values of D.

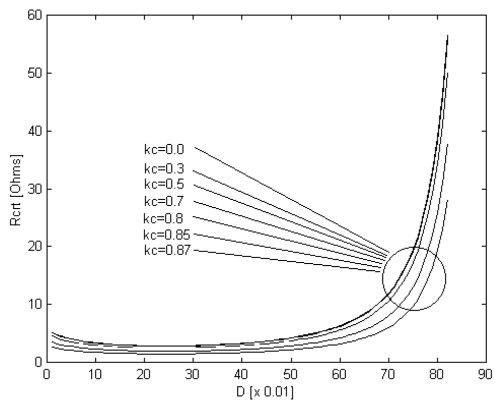

Fig. 1: Plots of R_{crit} as function on the duty ratio and the coupling coefficient of inductors

The function $R_{crit}=f\,(D, k_c, p_i)$ is plotted in Fig. 2 for the same values of D and k_c as in Fig. 1.

For the converter with parasitics, the effect of the value of k_c over R_{crit} is changed in function on the value of the duty ratio. So, the increasing of the value of the coupling coefficient causes the increasing of R_{crit} at small values of D and the decreasing of R_{crit} at big values of D. As k_c and D tend to 1, R_{crit} tends to zero.

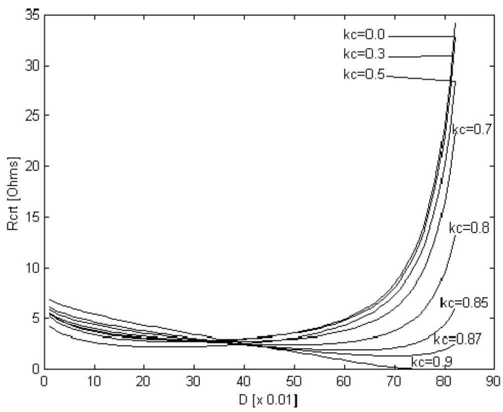

Fig. 2: Plots of R_{crit} as function on the duty ratio, the coupling coefficient of inductors and the parasitic resistances of converter

The effect of the coupling coefficient k_c of the inductors and the parasitic resistances of the circuit over k_{tcrit} is shown in Fig. 3 and 4. Here, the function k_{tcrit}=f (D, k_c) is plotted for the same values of D and k_c as for R_{crit}, and for the two cases: no parasitic included (Fig. 3) and parasitic included (Fig. 4).

As it can be seen from Fig. 3, k_{tcrit} is a continuous function on the duty ratio D for all the values of the coupling coefficient k_c from k_c=0 (separate inductors) to k_c=0.87. This function has a maximum at D=0.25 that decreases with the increasing of k_c in the ideal converter case.

The parasitic resistances of the circuit cause to move and to decrease the maximum of the function k_{tcrit} with the increasing of k_c from zero to 0.8 (Fig. 4). This maximum increases again if k_c surpasses 0.8. So, $k_{tcrit.max}$=0.17 at D=0.25 for k_c=0 (separate inductors), $k_{tcrit.max}$=0.107 at D=0.5 for k_c=0.8 and $k_{tcrit.max}$=0.155 at D=0.7 for k_c=0.87.

All these results differ from that obtained for an ideal converter. Using the formula given by the Table 1 for a ripple-free input-current boost converter, it finds $k_{tcrit.max}$=0.148 at D=0.33. Hence, if k_t is greater than 0.148, then the converter operates in the continuous conduction mode for all D. The simulation results disable this general conclusion.

After establishing R_{crit}, the steady-state model of converter with coupled or separate inductors and all the parasitic included and its external characteristics $M = f(R / R_{nom})$ can be computed and plotted for both DCM ($R>R_{crit}$) and CCM ($R<R_{crit}$) [6].

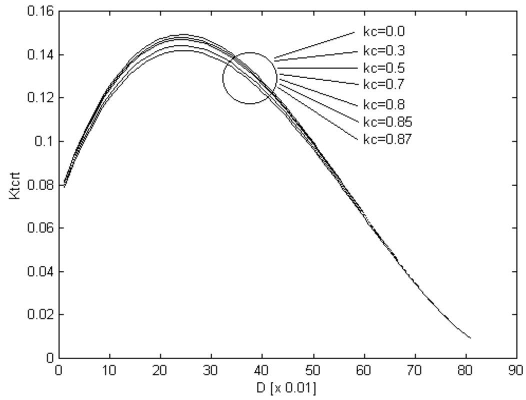

Fig. 3: Plots of k_{tcrit} as function on the duty ratio and the coupling coefficient of inductors

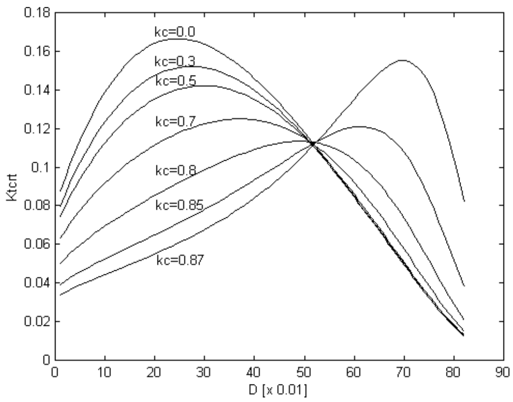

Fig. 4: Plots of k_{tcrit} as function on the duty ratio, the coupling coefficient of inductors and the parasitic resistances of converter

5 Conclusion

The order of the equation of the boundary between DCM and CCM, with unknown R_{crit} or k_{tcrit}, is an invariant of the PWM converters. It is two, regardless of the order of converter (two or four), coupled or separate inductors and the including or non-including of the parasitics.

Both the parasitic resistances and the inductor coupling change the boundary between the discontinuous and continuous conduction modes of a fourth-order PWM converter.

The algorithm carried out for establishing the boundary between DCM and CCM in a fourth-order PWM converter and implemented with MATLAB environment allows studying the effect of the parasitic resistances and the coupling of inductors over the boundary R_{crit} or k_{tcrit}, the steady-state properties and the external characteristics of converter.

References

[1] S. Cuk and M. D. Middlebrook, A General Unified Approach to Modelling Switching DC-to-DC Converters in Discontinuous Conduction Mode, *PESC'77 Record*, 1977, pp.36-57

[2] E. Niculescu and A. Cirstea, Computation of Averaged Characteristic Coefficients for Elementary PWM Converters, *COM.P.EL.'98 Proceedings, IEEE Catalog 98TH8358*, 1998, pp. 193-200

[3] S. Cuk, Discontinuous Inductor Current Mode in the optimum topology switching converter, *PESC'78 Record*, 1978, pp. 105-123

[4] J. Wang, W.G. Dunford and K. Mauch, Analysis of a Ripple-Free Input-Current Boost Converter with Discontinuous Conduction Characteristics, *IEEE Transactions on Power Electronics,* Vol. 12, No. 4, 1997, pp. 684-694

[5] R. W. Erickson, *Fundamentals of Power Electronics*, Chapman & Hall (ed.), International Thomson Publishing, 1997

[6] E. Niculescu and E. P. Iancu, Modeling and Analysis of the DC/DC Fourth-Order PWM Converters, *COM.P.EL.'2000 Proceedings, IEEE Catalog 00TH8535,* 2000, pp. 83-88

[7] E. Niculescu and E. P. Iancu, State Sensitivity of the Fourth-Order PWM Converters, *CIFA'2000 Proceedings,* 2000, pp. 865-870

[8] E. Niculescu and E. P. Iancu, Decay Interval of the Inductor Currents in PWM Converters, *Annals of the University of Craiova***,** Year 24, No. 24, 2000, pp. 151-160