The effects of spatial correlation and the LOS component on the capacity of broadband MIMO channels

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Abstract: - The effects of spatial correlation and LOS (Line of Sight) component on capacity of broadband MIMO channels are investigated. For this purpose a flexible, parameterised model for the MIMO channel is proposed. The capacity gain that can be achieved with the use of CSI (Channel State Information) on the transmitter side is examined for a variety of different channel parameters. Channel knowledge at the transmitter is difficult to attain and will normally require some kind of feedback from the receiver. Complexity and bandwidth are the costs of having CSI at the transmitter. The channel conditions under which capacity gain could compensate for these additional costs are identified.
Relative phases of the LOS components of the respective SISO channels turn out to have an enormous impact on both capactiy and capacity gain. This fact allows to extract some conclusions about the importance of antenna placement for fixed receiver and transmitter and the significance of CSI at the transmitter when mobility is considered.

Key-Words: - OFDM, MIMO, channel capacity, spatial correlation, CSI, beamforming.

1. Introduction

Over the last years a lot of papers on the topic of smart antennas have been published. The main motivation for the research on that field is the increase in capacity that can be achieved with the use of several antennas at one or both ends of a wireless channel [1].
Currently, there are some ongoing activities to identify technologies to be employed in future fourth generation wireless systems [2]. MIMO approaches are a main topic in these activities. Due to its simplicity and good properties to combat multipath propagation is OFDM a major candidate for 4th generation systems. For this reason the focus of our present work is set on OFDM in a MIMO context. This is new with respect to previous work on the issue of capacity of MIMO channels [1] [3] [4], where flat fading channels are assumed.
Systems that employ several antennas at either one or both sides of a radio channel can, generally speaking, be viewed as MIMO approaches, the SISO, SIMO and MISO cases being understood as a particular case of them. Furthermore MIMO systems can be classified in two sets: systems that employ total or partial channel knowledge on the transmitter (e.g. beamforming, eigenbeamforming ...) and systems that don’t require any channel knowledge on the transmitter side and uniformly distribute the energy in all directions (e.g. BLAST, space-time coding ...).
The use of CSI at the transmitter requires under FDD schemes the existence of a backward channel so that the receiver can send channel estimates to the transmitter. As a consequence the bandwidth available for communication purposes decreases. The decrease in bandwidth becomes considerable when the radio channel varies very quickly and frequent updating of the CSI is required. This is the case for high mobility scenarios with rich scattering. In [3] it is shown that CSI at the transmitter does not yield any significant advantage with reference to the channel capacity when the MIMO channel is strongly uncorrelated, e.g. in a rich scattering environment [5]. Therefore, in this case, CSI at the transmitter side would provide little or no benefit while the cost in terms of bandwidth would be high. In this paper channel conditions are identified in which the presence of CSI at the transmitter could result advantageous.
Fourth generation terminals should be able to work in a variety of scenarios and environments which might largely differ from each other. Besides, bandwidth is scarce and must be utilised in an efficient way. As a result there is a need for adaptive or robust MIMO approaches that can cope with the most diverse channel conditions and make an efficient use of the bandwidth. By analysing the relationship between channel conditions, channel capacity and availability of CSI at the transmitter a base for further research toward the conception of unitary overall working MIMO approaches is provided.
Different from [3], here not only is the effect of the absolute value of the LOS components on channel capacity analysed but also the effect of the relative phases of LOS components. The spatial correlation coefficients of NLOS components and delay spread are considered as well.
The paper is structured as follows: in section 2 expressions for capacity of broadband channels with and without CSI at the transmitter are derived, in section 3 the channel model employed for the calculation of channel capacity under different conditions is explained, in section 4 the results of our study are pre-

sented and commented. Finally, conclusions are drawn in section 5.

2. Capacity of Broadband Channels

2.1. SISO Channels

To begin with, the expression for the channel capacity of a broadband channel with one antenna at each side is derived. It will be assumed that OFDM is the modulation scheme employed and that the length of the cyclic prefix is enough to avoid ISI (intersymbol interference). The relationship between the transmitted and received signal can be expressed in the time domain as follows:

$$\mathbf{y} = \mathbf{h} \otimes \mathbf{x} + \mathbf{n} \tag{1}$$

The channel matrix **h** will be a square circulant matrix of dimension N, being N the number of OFDM subcarriers. The received signal **y**, the transmitted signal **x** and the Gaussian noise **n** are column vectors of dimension N [6]. If we transform expression 1 to the frequency domain by means of a DFT we obtain

$$\mathbf{Y} = \mathbf{H} \otimes \mathbf{X} + \mathbf{N} \tag{2}$$

In this expression **Y**, **X** and **N** are the DFT transforms of **y**, **x** and **n** in the previous expression. As a result of this transformation the channel **H** becomes diagonal [6].

The capacity of channel **H** is defined as the greatest mutual information of **X** and **Y** that can be reached over the channel

$$C(\mathbf{H}) = \max_{\mathbf{X}} \quad I(\mathbf{X}, \mathbf{Y}) \tag{3}$$

Given expression 2 it can be easily shown that the mutual information of **X** and **Y** will be the difference of the entropies of **Y** and **N**. In [7] it is shown that circularly symmetric complex Gaussian vectors are entropy maximizers. Therefore the vector **Y** must be circularly symmetric complex Gaussian in order to obtain the maximum of mutual information [8]. The noise **N** is also a circularly symmetric complex Gaussian vector. In this case the following relationship holds for the capacity of the channel

$$C(\mathbf{H}) = \log_2\left(\det\left(\mathbf{I_N} + \frac{1}{\sigma^2}\mathbf{H}\mathbf{Q}\mathbf{H}^{\dagger}\right)\right), \tag{4}$$

where **Q** is the covariance matrix of vector **X** and σ^2 is the noise variance.

Since the radio channel can be seen as a stationary stochastic process over a long period of time, we will remove the capacity dependence on the channel from expression 4 by applying the expectation operator. As a result we obtain the expression for the so called ergodic capacity

$$C = \varepsilon_{\mathbf{H}}\left\{\log_2\left(\det\left(\mathbf{I_N} + \frac{1}{\sigma^2}\mathbf{H}\mathbf{Q}\mathbf{H}^{\dagger}\right)\right)\right\} \tag{5}$$

Taking into account the Hadamard's inequality for positive definite matrices [8], we conclude that matrix **Q** which maximizes the determinant in expression 5 must be diagonal and, given that **H** is also diagonal, we obtain

$$C = \varepsilon_{\mathbf{H}}\left\{\sum_{k=0}^{N-1}\log_2\left(1 + \frac{1}{\sigma^2}\lambda_k P_k\right)\right\} \tag{6}$$

In this expression P_k is the kth element in the diagonal of **Q**, which represents the power transmitted on subcarrier k, and $\lambda_k = H_k \cdot H_k^*$ is the instantaneous power of the stochastic process which models the channel at subcarrier k.

Assuming that P_t is the maximum mean power allowed there are two cases that will be considered.

In the first case there is no CSI available at the transmitter and as a consequence the same power P_t will be transmitted on each subcarrier. In this case the capacity is given by

$$C = \varepsilon_{\mathbf{H}}\left\{\sum_{k=0}^{N-1}\log_2(1 + \rho\lambda_k)\right\}. \tag{7}$$

where $\rho = P_t/\sigma^2$ is the signal-to-noise ratio.

Now, if channel model is Ricean, λ_k is then distributed as a non-central chi-square distribution [9] and capacity can be numerically calculated by means of the following equation

$$C = N(K+1)e^{-K}\int_0^{\infty}\log_2(1 + \rho\lambda_k) \, e^{-(K+1)\lambda_k} I_0\left(2\sqrt{K(K+1)\lambda_k}\right)d\lambda_k \tag{8}$$

where K is a factor that expresses the relation between the LOS and NLOS power components of the channel (section 3) and I_0 is the modified Bessel function of the first kind of the 0th order. Since the distribution of λ_k will not depend on the subcarrier position k, the capacity of the whole OFDM symbol is found to be simply N times the capacity of any particular subcarrier.

In the second case it is assumed that CSI is available at the transmitter, i.e. the values of λ_k at the time of transmission are known. In this case, expression 6 can be optimised over the choice of P_k for each k subject to the following mean power constraint

$$\frac{1}{N}\sum_{k=0}^{N-1}P_k \le P_t \tag{9}$$

The solution to this optimisation problem is the well known water-filling algorithm [8].

2.2. MIMO Channels

Equation 2 can be used to describe a MIMO system employing OFDM. In this case **H** becomes a block diagonal matrix with N blocks. Each block refers to a particular subcarrier, the number of columns being the number of transmit antennas and the number of rows the number of receive antennas [6]. M_t antennas at the transmitter and M_r antennas at the receiver are assumed. Then **X** has NM_t elements while **Y** and **N** have NM_r elements. Following the same reasoning as above for the capacity of the MIMO channel the following expression results

$$C = \varepsilon_{\mathbf{H}}\left\{\log_2\left(\det\left(\mathbf{I}_{NM_r} + \frac{1}{\sigma^2}\mathbf{HQH}^{\dagger}\right)\right)\right\} \quad (10)$$

As stated above, capacity will be maximised when the product $\mathbf{HQH}^{\dagger}$ is diagonal. Having a block diagonal covariance matrix $\mathbf{Q}$ with blocks of dimension $M_t \times M_t$ suffices to assure that diagonalization can be reached. Moreover, it can be easily shown that the possible entries of $\mathbf{Q}$ outside the blocks, in the best case, will not contribute to the value of the determinant. As a consequence, in the following, we will consider a block diagonal $\mathbf{Q}$ without loss of generality. This is equivalent to choosing the information on different subcarriers to be independent, i.e. not correlated. If there is no correlation between subcarriers the transmitted information in the sense of entropy is maximised, which intuitively and taken into account the frequency orthogonality of the channel matrix agrees with the objective of maximising the mutual information. In this case it follows from 10

$$C= \varepsilon_{\mathbf{H}}\left\{\sum_{k=0}^{N-1}\log_2\left(\det\left(\mathbf{I}_{M_r} + \frac{1}{\sigma^2}\mathbf{H}_k\mathbf{Q}_k\mathbf{H}_k^{\dagger}\right)\right)\right\} \quad (11)$$

Again P_t is assumed to be the maximum mean power allowed.

If there is no CSI available on the transmitter side a diagonal $\mathbf{Q}_k$ is taken for every k, where all entries are equal to P_t / M_t in order to comply with the maximum mean power constraint. This covariance matrix achieves capacity when we don't have any information about the channel, the entries of $\mathbf{H}_k$ are independent and identically distributed and the channels are Rayleigh modelled [7]. In this case capacity is given by

$$C = \varepsilon_{\mathbf{H}}\left\{\sum_{k=0}^{N-1}\log_2\left(\det\left(\mathbf{I}_{M_r} + \frac{\rho}{M_t}\mathbf{H}_k\mathbf{H}_k^{\dagger}\right)\right)\right\} \quad (12)$$

where once again $\rho = P_t / \sigma^2$ is the signal-to-noise ratio. Since the statistics for the matrix $\mathbf{H}_k$ will not depend on the subcarrier position k, it can be inferred from expression 12 that the capacity of a OFDM symbol in absence of CSI at the transmitter will be N times the capacity of any particular subcarrier.

If now the identity $\det(I + AB) = \det(I + BA)$ is applied to equation 11 and the singular value descomposition $\mathbf{H}_k^{\dagger}\mathbf{H}_k = \mathbf{U}_k\Lambda_k\mathbf{U}_k^{\dagger}$ is performed, it follows

$$C = \quad (13)$$

$$\varepsilon_{\mathbf{H}}\left\{\sum_{k=0}^{N-1}\log_2\left(\det\left(\mathbf{I}_{M_t} + \frac{1}{\sigma^2}\mathbf{U}_k^{\dagger}\mathbf{Q}_k\mathbf{U}_k\Lambda_k\right)\right)\right\}$$

When CSI is available at the transmitter the expression inside the determinant can be diagonalised by choosing $\mathbf{Q}_k = \mathbf{U}_k\mathbf{D}_k\mathbf{U}_k^{\dagger}$, where $\mathbf{D}_k$ is diagonal. The result is

$$C = \varepsilon_{\mathbf{H}}\left\{\sum_{k=0}^{N-1}\log_2\left(\det\left(\mathbf{I}_{M_t} + \frac{1}{\sigma^2}\mathbf{D}_k\Lambda_k\right)\right)\right\} \quad (14)$$

Now, capacity can be maximised over the choice of the entries in $\mathbf{D}_k$ for every k constrained to the maximum mean power P_t

$$\frac{1}{N}\sum_{k=0}^{N-1}\mathrm{tr}(\mathbf{Q}_k) = \frac{1}{N}\sum_{k=0}^{N-1}\mathrm{tr}(\mathbf{D}_k) \le P_t \quad (15)$$

The optimal solution to this optimisation problem is given by the water-filling algorithm [8]. In this case the capacity of the OFDM symbol is greater than the simple addition of capacities of N MIMO flat-fading channels (subcarriers).

3. Channel model

The channel model employed in the present work is based on models from [10]. We take model B, which is Rayleigh and whose RMS spread delay amounts to 100 ns. The vector of delay taps and power taps of that model is given in Table 1.

Table1: delay taps (ns) and power taps (dB)

ns	0	10	20	30	50	80
dB	-2.6	-3.0	-3.5	-3.9	0.0	-1.3
ns	110	140	180	230	280	330
dB	-2.6	-3.9	-3.4	-5.6	-7.7	-9.9
ns	380	430	490	560	640	730
dB	-12.1	-14.3	-15.4	-18.4	-20.7	-24.6

Next, we compute an equivalent power vector for an homogeneous delay vector characterised by a sampling rate of 50 ns, which is the sampling rate for HiperLAN/2. This computation is done by clustering power values from the original vector to the next sample of the new vector. In this way a channel impulse response consisting of 16 taps placed at 50ns intervals is obtained. The vector of tap variances of the normalised channel impulse response remains σ_h^2={-5.85, -6.12, -6.5, -11.5, -11, -13.2, -15.3, -17.5, -19.7, -21.9, -23, -26, -inf, -28.3, -inf, -32.2}dB. This model is Rayleigh and therefore an instance of the channel can be generated by taking independent Gaussian random values for the real and imaginary part of each tap with half of the respective tap variance. The taps will be uncorrelated with each other according to a WSSUS model.

The LOS component is modelled by a mean value m in the first tap. As the channel impulse response is normalised, the following identity must hold

$$|m|^2 + \sum_{i=0}^{L-1}\sigma_i^2 = 1 \quad (16)$$

Here is L the length of the channel impulse response and σ_i^2 are the tap variances. When m = 0, the channel is Rayleigh and the tap variances are given by vector σ_h^2. The relation between the LOS component in the first tap and the rest of the channel power (NLOS) is expressed by a factor K defined as follows

$$K = \frac{|m|^2}{\sum_{i=0}^{L-1} \sigma_i^2} = \frac{|m|^2}{1-|m|^2} \quad (17)$$

In order to satisfy identity 16 when $K \neq 0$ the vector of tap variances σ_h^2 is divided by 1+K.

A MIMO channel can be characterised as a set of $M_t \times M_r$ SISO channels that in general will be correlated to some extent (figure 1).

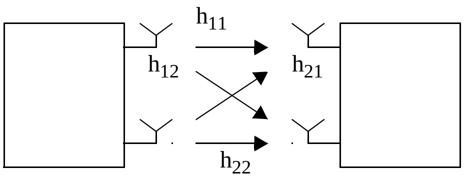

Fig.1 MIMO channel

For the shake of simplicity, in the following 2 antennas at both ends will be assumed. In this case, the generation of an occurrence of MIMO channel will require the generation of 4 instances of SISO channels in the way exposed above.

In general, there will be some spatial correlation, i.e. the 4 stochastic processes that describe the behaviour of the respective SISO channels at a particular point in the delay domain (tap) will show correlation.

Let $h_{ij}(n)$ be the random variable that describes the behaviour of the NLOS-component of tap n for the SISO channel defined by transmit antenna j and receive antenna i. The degree of spatial correlation between two SISO channels at a particular tap n will be given by a correlation coefficient defined as follows:

$$\rho_{iji'j'}(n) = \frac{\varepsilon\{h_{ij}(n)h_{i'j'}(n)^*\}}{\sqrt{\varepsilon\{|h_{ij}(n)|^2\}\varepsilon\{|h_{i'j'}(n)|^2\}}} \quad (18)$$

The following assumptions are made: first, the variance of the channel impulse response at tap n will be the same for every SISO channel

$$\sigma_n^2 = \varepsilon\{|h_{ij}(n)|^2\} \qquad \forall i, j, \quad (19)$$

second, the correlation coefficient is the same for each tap n

$$\rho_{iji'j'} = \frac{\varepsilon\{h_{ij}(n)h_{i'j'}(n)^*\}}{\sigma_n^2} \qquad \forall n \quad (20)$$

The first assumption seems reasonable as signals suffering similar delays propagate over a similar distance and therefore suffer similar attenuation.

The second might not be strictly true in a real channel but at the time this paper is written no model was found that exactly describes the relationship between delay and spatial correlation.

In [11] it is shown that the correlation coefficient can be expressed as the product of a correlation coefficient that accounts for the correlation seen from the receiver side and another that accounts for the correlation seen from the transmitter side.Thus

$$\rho_{iji'j'} = \rho_{ii'}\rho_{jj'}. \quad (21)$$

With two antennas at both channel ends we have $\rho_{11} = \rho_{22} = 1$, $\rho_{tx} = \rho_{12} = \rho_{21}^*$ and $\rho_{rx} = \rho_{12} = \rho_{21}^*$, ρ_{tx} and ρ_{rx} being defined as follows

$$\rho_{tx} = \frac{\varepsilon\{h_{x1}(n)h_{x2}(n)^*\}}{\sigma_n^2} \quad (22)$$

$$\rho_{rx} = \frac{\varepsilon\{h_{1x}(n)h_{2x}(n)^*\}}{\sigma_n^2} \quad (23)$$

In these expressions the subindex x can be substituted by either 1 or 2 without affecting the value of the correlation coefficients [11].

To generate occurrences of the MIMO channel we proceed on a tap basis. Firstly, independent complex Gaussian random values for tap n are generated with the corresponding variance σ_n^2. As a result we obtain matrix **H'**

$$\mathbf{H}'(n) = \begin{bmatrix} h_{11}'(n) & h_{12}'(n) \\ h_{21}'(n) & h_{22}'(n) \end{bmatrix} \quad (24)$$

Now, given a correlation coefficient for the transmit side ρ_{tx} and a correlation coefficient for the receive side ρ_{rx}, spatial correlation can be introduced by pre-multiplying and post-multiplying with two transformation matrices $\mathbf{K}_R$ and $\mathbf{K}_T$ [5],

$$\mathbf{H}(n) = \mathbf{K}_R \cdot \mathbf{H}'(n) \cdot \mathbf{K}_T. \quad (25)$$

These transformation matrices satisfy $\mathbf{\Phi}_T = \mathbf{K}_T^\dagger \cdot \mathbf{K}_T$ and $\mathbf{\Phi}_R = \mathbf{K}_R \cdot \mathbf{K}_R^\dagger$, where

$$\mathbf{\Phi}_T = \begin{bmatrix} 1 & \rho_{tx} \\ \rho_{tx}^* & 1 \end{bmatrix}, \quad (26)$$

and

$$\mathbf{\Phi}_R = \begin{bmatrix} 1 & \rho_{rx} \\ \rho_{rx}^* & 1 \end{bmatrix}. \quad (27)$$

Repeating this process for the 16 taps, four vectors result with each 16 entries corresponding to the four spatially correlated SISO channels that form the MIMO channel.

In the next section we investigate the effect that the channel parameters considered in our model have on the channel capacity both when CSI is available at the transmitter and in absence of CSI at the transmitter. The channel parameters which will be considered are the spatial correlation on the receiver side ρ_{rx}, the spatial correlation on the transmitter side ρ_{tx}, the factor K, which expresses the relation between the powers of the LOS component and the NLOS components of the channel, and the relative phases of the LOS components m_{ij} $(i, j \in \{1, 2\})$ of the respective SISO channels.

4. Capacity calculation

To obtain the results presented in this section capacity results were averaged over 1000 occurrences of the intended channel. The channel is first generated in the time domain (16 taps) and later transformed into the frequency domain using 64 subcarriers as specified in Hiperlan/2.
First, the capacity gain, which is reached by employing CSI at the transmitter, was calculated for several values of K for a SISO channel. The result is represented in figure 2.

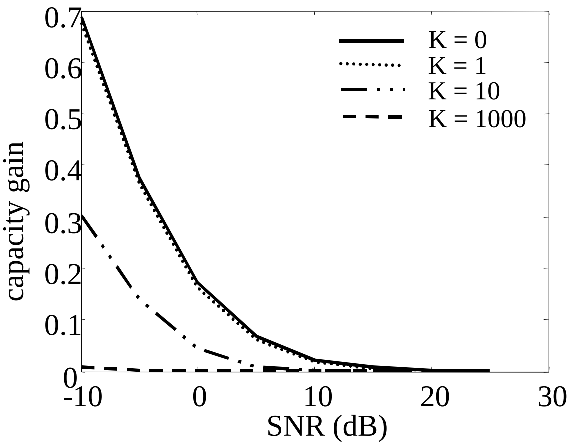

Fig.2 Capacity gain:. $(C_{CSI} - C_{noCSI}) / C_{noCSI}$.

It can be seen that the gain we obtain with CSI at the transmitter increases as K decreases. When the LOS component predominates, e.g. K=1000, the channel becomes flat-fading and the water-filling algorithm delivers a uniform distribution of power in the frequency domain as the optimum. This is the distribution we use in absence of CSI (7). On the other hand, when the NLOS component grows the channel becomes increasingly frequency selective and therefore the optimum water filling solution increasingly differs from the uniform distribution. As a consequence more capacity gain can be attained.
It can be also observed that capacity gain decreases very fast with increasing SNR. This behaviour can be understood by taking a close look at the way the water-filling algorithm works. Taking it to the limit, if we have almost infinite SNR the difference between the uniform distribution and the optimal water-filling distribution will be negligible. On the other hand if we have a very small SNR the way we distribute power over the subcarriers will have a decisive impact on the achievable capacity.
As a conclusion for the SISO channel, CSI at the transmitter offers significant capacity gain only for low values of SNR and in environments where NLOS propagation predominates. However CSI will be specially difficult to achieve for low SNR values and therefore the cost to obtain it in terms of bandwidth and computational complexity might be very high.
For the MIMO case two antennas at both transmitter and receiver side are assumed.
We first look at the effect of the correlation coefficients on the capacity gain that can be obtained through availability of CSI at the transmitter (figure 3). While capacity decreases with growing spatial correlation, the capacity gain that can be reached with the help of CSI at the transmitter increases. Now, the capacity gain can be significant even for relatively high values of SNR. For both graphics represented in figure 3 the absolute values of the correlation coefficients were chosen to be the same at both transmit and receive sides. It should be mentioned that no significant capacity variation was noted when varying the phase of the correlation coefficients. In figure 4 curves are shown for the case of spatial correlation at only one side. Capacity proved to be independent of the side in which correlation applies.

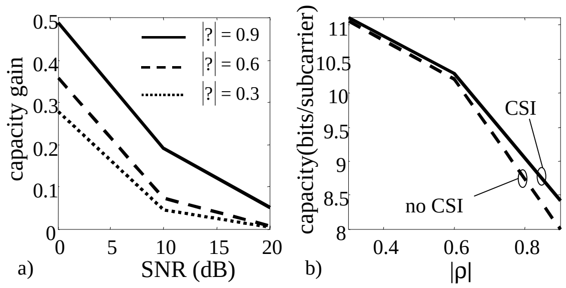

Fig.3 a) capacity gain curves for three absolute values of the correlation coefficient at both receive and transmit sides. b) capacity curves for 20 dB SNR with and without CSI at the transmitter.

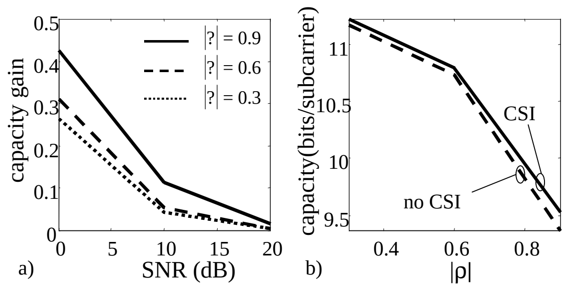

Fig.4 a) capacity gain curves for three absolute values of the correlation coefficient at only one side of the channel. b) capacity curves for 20 dB SNR with and without CSI at the transmitter.

So far it was assumed that there is no LOS component in the MIMO channel. To generate a LOS component of factor K a matrix of mean values **M** is introduced as the first tap of the MIMO channel and the NLOS components of the four SISO channels are scaled according to equations 16 and 17

$$\mathbf{M} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} . \qquad (28)$$

The absolute value of all four entries of **M** is equal and determined by factor K. However, the phases might differ depending on the relative position of the antennas. This phase diversity of the LOS component turns out to have a dramatic effect on both capacity and capacity gain.

To obtain values for capacity and capacity gain with LOS component we average not only over the occurrences of NLOS components but also over the phases of the entries of **M**, which are assumed to be uniformly distributed between 0 and 2π and independent of each other. This would correspond to an scenario in which communication takes place between two MIMO terminals for a long period of time, where at least one of the terminals is mobile. Using a mobile terminal the position of the antennas would randomly change and therefore the phases of the LOS components would also vary in a random way during the communication process. The average is done over 100000 ocurrences.

The variation of capacity and capacity gain with factor K is shown in figure 5. Capacity gain grows for small values of SNR as the LOS component (factor K) decreases. For high values of SNR capacity gain becomes insignificant showing little variation with K. Capacity grows with increasing factor K. For values of K over 10 capacity remains almost constant.

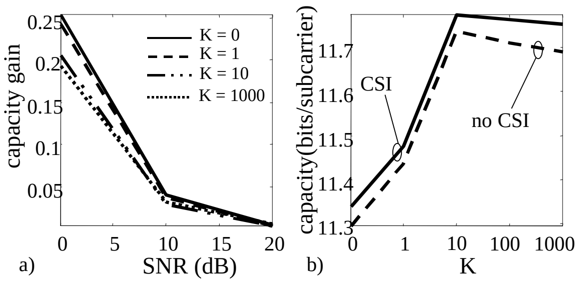

Fig.5 a) capacity gain curves for four values of factor K. No spatial correlation is assumed. b) capacity curves for 20 dB SNR with and without CSI at the transmitter.

Figure 5 b) is apparently in contradiction with results presented in [3]. There, capacity is shown to decrease with increasing K. This difference is due to the fact that in [3] all LOS components of the MIMO channel are assumed to have the same phase. In the most of the cases this assumption will not be valid. Antenna separation is expected to be at least half of the wavelength in order to avoid a high spatial correlation, which, as shown above, reduces capacity. As a result the phase difference between the multiple LOS components, which will be caused by the length difference of the multiple SISO channels, could be anyone between 0 and 2π.

The three pairs of curves represented in figure 6 correspond to three different choices of the phase in the entries of matrix **M**.

The pair of curves depicted in figure 5 b) corresponds to the *average case* in figure 6. As explained above, in this case phases are chosen randomly and averaged over a number of occurrences.

The *best case* results when the singular value descomposition of **M** gives a diagonal matrix with two identical singular values. This property holds when M has the following form

$$\mathbf{M} = \sqrt{\frac{K}{K+1}} \cdot \begin{bmatrix} 1 & e^{j\phi} \\ -e^{-j\phi} & 1 \end{bmatrix} \qquad (29)$$

The *worst case* is the one considered in [3]. Here the singular value descomposition of matrix **M** only gives a singular value different from zero. This property holds when the phases in the entries of **M** are all equal, e.g.

$$\mathbf{M} = \sqrt{\frac{K}{K+1}} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}. \qquad (30)$$

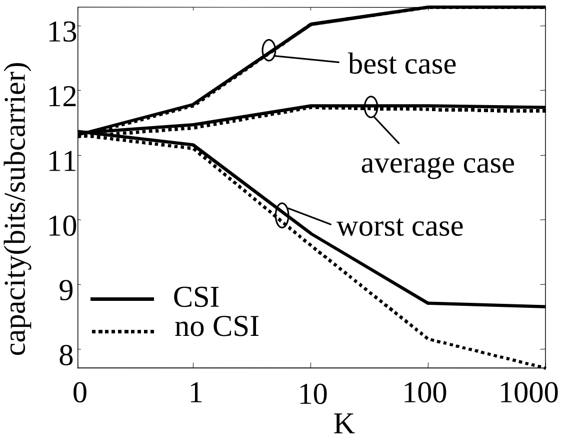

Fig.6 capacity curves for 20 dB SNR and three different choices of the phases of the entries of matrix **M**.

It should be noted that only for the *worst case* the capacity gain that we can reach with CSI at the transmitter is significant. In the other two cases this gain is negligible and capacity remains constant or even increases with growing K. Thus, in the presence of LOS propagation the relative phases of the multiple SISO channels show to have a tremendous impact on capacity. This suggests that when transmitter and receiver are fixed, and LOS propagation exists, the placement of the antenna elements could have a decisive significance regarding the capacity that can be expected and could render CSI at the transmitter completely unnecessary. In the *average case*, which could be thought of as the mobility scenario, CSI at the transmitter provides little gain even when spatial correlation is considered (figure 7).

As a conclusion it could be said that the only situation in which CSI at the transmitter proves really useful is in absence of LOS propagation when the spatial correlation of the channel is high.

At this point it should be recalled, that in this work system aspects such as interference were not taken into account. Beamforming techniques, which make use of CSI at the transmitter, reduce interference in the system and therefore will normally provide an increase in system capacity that was not considered here.

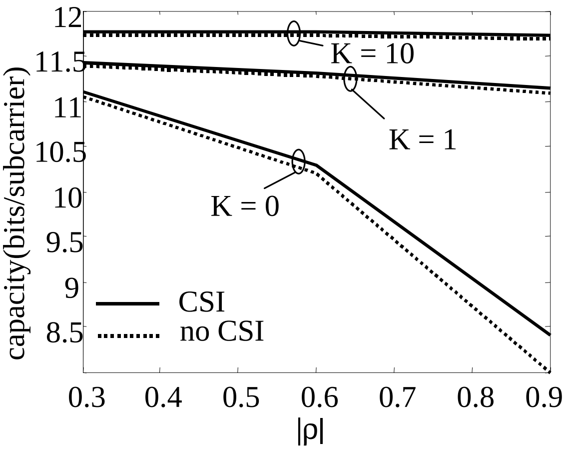

Fig.7 Capacity curves for 20 dB SNR for three different values of K. The curves correspond to the average case in figure 6 and the correlation coefficient applies to both transmit and receive side.

5. Conclusions

A channel model was proposed to study the effects of LOS components and spatial correlation on capacity of a MIMO channel. Utilisation of OFDM was assumed.

For the SISO case it was shown that CSI at the transmitter provides a significant capacity gain only for small values of SNR. However, attaining CSI at the transmitter with small values of SNR might require a high cost in terms of complexity and bandwidth.

For the MIMO channel, capacity decreases with increasing spatial correlation while capacity gain, provided by CSI at the transmitter, increases. This gain shows to be significant (>10%) even for relatively high values of SNR (>10dB). Capacity turned out to be insensitive to the phase of correlation coefficients and to whether correlation applies at the transmitter or receiver side.

In the presence of LOS propagation, capacity and capacity gain largely depend on the relative phases of LOS components corresponding to the particular SISO channels. Three cases were examined: in the *best case* the singular values of the matrix of LOS components are equal, capacity increases with growing factor K and CSI at the transmitter becomes superfluous since capacity gain becomes negligible. In the *worst case* the matrix of LOS components only has a singular value different from zero, capacity decreases dramatically with growing K and CSI at the transmitter provides a considerable capacity gain. In the *average case* capacity remains almost constant and capacity gain is negligible.

In the light of these results the following conclusions could be drawn: for fixed receiver and transmitter and in the presence of LOS propagation the placement of antenna elements will have a decisive effect on capacity. Besides, for mobile transmitter or receiver LOS propagation could render CSI at the transmitter unnecessary even if spatial correlation is high.

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