Blind Channel Estimation in Presence of Carrier Offsets for DS/CDMA

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Abstract: - Most blind channel estimation methods for Direct-Sequence Code-Division Multiple-Access (DS/CDMA) systems does not take into account the carrier offset resulting from imperfect carrier recovery. The method presented here estimates jointly the carrier offset and propagation channel thus allowing for the carrier offset compensation. Our technique combines the subspace decomposition with a Jacobi-like Constant Modulus Algorithm (CMA) and exploits the fact that CM criterion is invariant to the presence of residual carriers. Channel identifiability is proved for the case when different users have different carrier offsets. Numerical studies illustrate the efficiency of the proposed algorithm.

Key-words: - multiuser detection, system identification, blind channel estimation, carrier offset, CDMA, constant-modulus algorithm

1 Introduction

This paper deals with the problem of joint channel and carrier offset estimation for DS-CDMA systems. It is well known that conventional (RAKE) receiver does not account for the multiple-access interference (MAI) thus leading to severe performance degradation for moderate and high system loads. As opposed to RAKE, multiuser detectors exploit structural properties of the CDMA propagation channels to effectively combat the MAI at the expense of somewhat increased computational complexity.

Generally, in a CDMA system multipath propagation takes place so the received observation contains the sum of delayed and attenuated replicas of the transmitted signal. Multiuser detectors rely on the knowledge of multipath delays and fadings for the extraction of the desired signal thus making the issue of channel estimation of major practical importance.

The channel estimation method considered in this work is blind, that is, it does not require training sequences. Our method also estimates and compensates the carrier offset which results from the frequency shift between the transmitter and receiver oscillators. It should be noted that most blind channel estimation methods for CDMA systems (see, for example, [3] and references therein) do not account for the carrier offset. As a result, in presence of frequency shifts the detector output contains a sequence of rapidly rotating symbol estimates. Recently, several joint channel and carrier offset estimation techniques were proposed. Maximum-likelihood (ML) methods [2] are theoretically optimal, although the ML solution usually cannot be obtained in a closed-form and one has to use computationally demanding Viterbi or Expectation-Maximization algorithms. ESPRIT-based approach [8] is suboptimal but much less complex.

Here, another suboptimal method is presented which is based on subspace decomposition combined with a Constant Modulus Algorithm (CMA). The subspace decomposition yields the zero-forcing equalizer for the transmitted data up to a unitary factor. The unitary factor is estimated by minimization of a CM criterion through a Jacobi-like algorithm [1], in what it differs from other CMA techniques [6, 9]. The essential property of our method is that the value of the CM criterion is not affected by residual carriers. Once the equalizer is estimated, offsets are extracted from the equalized data by exploiting the finite alphabet (FA) property of the transmitted symbols. In numerical studies, our technique is compared to the method of [5] which estimates the carrier offset as a root of a certain matrix polynomial.

Notations: Throughout the paper, ^{T}, *, ^{H} and † are used to denote transpose, conjugate, conjugate transpose and Moore-Penrose pseudoinverse operations, respectively. Matrix and vector quantities are typed in boldface. $\Re\{x\}$ and $\Im\{x\}$ denote real and imaginary parts of x, respectively.

2 Data model

In a DS/CDMA system with K users, the received signal after matched filtering and chip-rate sampling can be written as

$$x(n) = \sum_{k=1}^{K} \sqrt{\epsilon}_k \sum_i w_k(i) h_k(n-i)\, e^{j\phi_k n} + v(n),$$

where, for user k, ϵ_k denotes the transmitted energy, $w_k(i)$ is the chip sequence, $h_k(i)$ is the channel impulse response and ϕ_k stands for the carrier offset. $v(n)$ denotes the white Gaussian receiver noise. In the sequel, impulse responses $h_k(i)$ are supposed to have finite support limited to L non-zero samples. The chip sequence $w_k(i)$ in DS-CDMA can be expressed as

$$w_k(Nl+m) = b_k(l)c_k(m), \quad m = 0, 1, \ldots N-1.$$

Here, $b_k(l)$ stands for the sequence of i.i.d. information bearing symbols, N defines the spreading factor and $\{c_k(m)\}$ is the spreading sequence. We also assume that 1) $K \leq N$ 2) the system is quasisynchronized up to L chips, i.e., the unknown timing ambiguity can be incorporated into the impulse response $h_k(i)$; 2) $L \ll N$. In this case, the intersymbol interference (ISI) can be neglected and one can write

$$\begin{aligned} \mathbf{x}(l) &= \sum_{k=1}^{K} \sqrt{\epsilon}_k b_k(l) e^{j\phi_k N l} \mathbf{Z}_k \mathbf{C}_k \mathbf{h}_k + \mathbf{n}(l) \\ &= \mathbf{GEF}(l)\mathbf{b}(l) + \mathbf{n}(l), \qquad (1) \end{aligned}$$

where

$$\begin{aligned} \mathbf{x}(l) &\stackrel{\text{def}}{=} [x(Nl+N-1) \ \ldots \ x(Nl)]^T \\ \mathbf{Z}_k &\stackrel{\text{def}}{=} \text{diag}\left(1 \ \ e^{j\phi_k L} \ \ \ldots \ \ e^{j\phi_k(N-1)}\right) \end{aligned}$$

$$\mathbf{C}_k \stackrel{\text{def}}{=} \begin{pmatrix} c_k(N-1) & & \mathbf{0} \\ c_k(N-2) & \ddots & \\ \vdots & \ddots & c_k(N-1) \\ \vdots & & \vdots \\ c_k(0) & \ldots & c_k(L-1) \end{pmatrix}$$

$$\begin{aligned} \mathbf{h}_k &\stackrel{\text{def}}{=} [h_k(L-1) \ h_k(L-2) \ \ldots \ h_k(0)]^T \\ \mathbf{n}(l) &\stackrel{\text{def}}{=} [v(Nl+N-1) \ \ldots \ v(Nl)]^T \\ \mathbf{G} &\stackrel{\text{def}}{=} [\mathbf{Z}_1\mathbf{C}_1\mathbf{h}_1, \ \mathbf{Z}_2\mathbf{C}_2\mathbf{h}_2, \ \ldots, \mathbf{Z}_K\mathbf{C}_K\mathbf{h}_K] \\ \mathbf{E} &\stackrel{\text{def}}{=} \text{diag}(\sqrt{\epsilon_1} \ \sqrt{\epsilon_2} \ \ldots \ \sqrt{\epsilon_K}) \\ \mathbf{F}(l) &\stackrel{\text{def}}{=} \text{diag}\left(e^{j\phi_1 Nl} \ e^{j\phi_2 Nl} \ \ldots \ e^{j\phi_K Nl}\right) \\ \mathbf{b}(l) &\stackrel{\text{def}}{=} [b_1(l) \ b_2(l) \ \ldots \ b_K(l)]^T. \end{aligned}$$

Finally, the block of T observations of $\mathbf{x}(l)$ can be expressed as

$$\mathbf{X} = [\mathbf{x}(0), \ \mathbf{x}(1), \ \ldots, \mathbf{x}(T-1)] = \mathbf{AS} + \mathbf{N},$$

where $\mathbf{A} = \mathbf{GE}$ denotes 'overall' channel matrix, $\mathbf{S} = [\mathbf{F}(0)\mathbf{b}(0), \ \mathbf{F}(1)\mathbf{b}(1), \ \ldots, \mathbf{F}(T-1)\mathbf{b}(T-1)]$ and $\mathbf{N} = [\mathbf{v}(0), \ \mathbf{v}(1), \ \ldots, \mathbf{v}(T-1)]$.

3 Identifiability

In blind system identification, we are interested in estimating the propagation channel based only on the observations $\mathbf{x}(l)$ and on known (for example, statistical) properties of channel, channel input and noise. It should be noted that exact values of transmitted symbols are unavailable. However, one may suppose that transmitted symbols belong to a finite alphabet [7]. On the other hand, some of the channel parameters may be known *a priori*, for example, spreading codes $c_k(m)$. The basic question is whether this information is sufficient to identify the unknown channel parameters, say, the coefficients $h_k(n)$ and carrier offsets ϕ_k. The following theorem explores the identifiability of the overall channel and transmitted symbols in the absence of noise.

Theorem **1** *Let*

$$\mathbf{X} = \mathbf{AS} = \bar{\mathbf{A}}\bar{\mathbf{S}}, \qquad (2)$$

where $\mathbf{A}$ *(*$\bar{\mathbf{A}}$*) are arbitrary* $N \times K$ *matrices of full column rank,* $\mathbf{S}$ *(*$\bar{\mathbf{S}}$*) are* $K \times T$ *matrices of full row rank with the elements* $s_{kl} = b_k(l)e^{jNl\phi_k}$, $\bar{s}_{kl} = \bar{b}_k(l)e^{jNl\bar{\phi}_k}$.

If

$$\phi_1 \neq \phi_2 \neq \ldots \neq \phi_K \ (\textit{mod}\ 2\pi/N) \qquad (3)$$

$$E\left[b_k(l)b_{k'}^*(l)\right] = 0, \quad k \neq k' \qquad (4)$$

then

$$\bar{\mathbf{A}} = \mathbf{A}\mathbf{T}^{-1},\ \bar{\mathbf{S}} = \mathbf{T}\mathbf{S}, \tag{5}$$

where $\mathbf{T}$ contains exactly one non-zero element in each row and column.

Proof: See appendix A.

The above theorem indicates that there exist an ordering, amplitude and phase ambiguity in determining user symbols (or columns of $\mathbf{A}$). This ambiguities result from the fact that, under general assumptions of the above theorem, the matrices $\bar{\mathbf{A}}$ and $\bar{\mathbf{S}}$ as given by (5) are admissible solutions to (2).

We would like to note that identifiability conditions can be relaxed (for example, users may have identical carrier offsets). These cases are not elaborated here due to space limitation.

4 Channel estimation algorithm

In this section, an algorithm is developed which estimates the unknown channels $\mathbf{h}_k$ and offsets ϕ_k of all system users assuming known spreading codes (matrices $\mathbf{C}_k$) and constant modulus property of the transmitted symbols ($|b_k(l)| = 1 \ \ \forall k, l$). The procedure is carried out in the following four steps:

1. Estimation of the overall channel matrix $\mathbf{A}$ up to a unitary factor $\mathbf{Q}$;

2. Estimation of $\mathbf{Q}$ using a Jacobi-like constant-modulus algorithm;

3. Equalization of the received data and estimation of offsets ϕ_k;

4. Estimation of propagation channels $\mathbf{h}_k$ using the estimates $\hat{\mathbf{A}}$, $\hat{\phi}_k$ and the *a priori* knowledge of $\mathbf{C}_k$.

In what follows, each of the above steps is explained in detail.

4.1 Estimation of A up to a unitary factor

Consider the covariance matrix of the observation $\mathbf{x}(l)$:

$$\mathbf{R} \stackrel{\text{def}}{=} E\left[\mathbf{x}(l)\mathbf{x}(l)^H\right] = \mathbf{A}\mathbf{A}^H + \sigma^2\mathbf{I}. \tag{6}$$

Assuming that $\mathbf{A}$ is of full column rank (what generally holds in practice), the eigendecomposition of $\mathbf{R}$ can be written as

$$\mathbf{R} = \mathbf{U}_s\mathbf{\Lambda}_s^2\mathbf{U}_s^H + \sigma^2\mathbf{I}, \tag{7}$$

where $\mathbf{U}_s$ is the matrix of K signal subspace eigenvectors and $\mathbf{\Lambda}_s^2 = \text{diag}(\lambda_1^2, \lambda_2^2, \ldots, \lambda_K^2)$ is the diagonal matrix of corresponding eigenvalues. The remaining $N-K$ eigenvectors (which are orthogonal to signal subspace) span the noise subspace . Comparing (6) and (7) yields

$$\mathbf{A}\mathbf{A}^H = \mathbf{U}_s\mathbf{\Lambda}_s^2\mathbf{U}_s^H$$

or

$$\mathbf{A} = \mathbf{A}_0\mathbf{Q}^H,$$

where $\mathbf{A}_0 \stackrel{\text{def}}{=} \mathbf{U}_s\mathbf{\Lambda}_s$ and $\mathbf{Q}$ stands for some unitary $K \times K$ matrix. Therefore, $\mathbf{A}_0$ estimates $\mathbf{A}$ up to a unitary matrix $\mathbf{Q}$ which remains to be determined. Clearly, in practice one deals with the estimate of $\mathbf{R}$, for example,

$$\hat{\mathbf{R}} = \mathbf{X}\mathbf{X}^H/T.$$

The signal subspace vectors and eigenvalues can then be computed through a Singular Value Decomposition (SVD) of $\hat{\mathbf{R}}$.

4.2 Estimation of the unitary factor Q

The output of the equalizer $\mathbf{A}_0^\dagger$ applied to the noise-free observation $\mathbf{X} = \mathbf{A}\mathbf{S}$ has the form

$$\mathbf{Z} = \mathbf{A}_0^\dagger\mathbf{X} = \mathbf{Q}^H\mathbf{S}.$$

Let us introduce the following CM criterion:

$$C(\mathbf{V}) = \sum_{k=1}^{K}\sum_{l=0}^{T-1}\left(|(\mathbf{V}\mathbf{Z})_{k,l}|^2 - 1\right)^2,$$

where $(\mathbf{V}\mathbf{Z})_{k,l}$ stands for the $\{k,l\}$th element of $\mathbf{V}\mathbf{Z}$. As $\mathbf{Q}\mathbf{Z} = \mathbf{S}$ has the constant modulus property, we will have $C(\mathbf{Q}) = 0$. Therefore, $\mathbf{Q}$ can be estimated by minimizing $C(\mathbf{V})$. To account for the noise, one may write

$$\hat{\mathbf{Q}} = \min_{\mathbf{V}} C(\mathbf{V}), \tag{8}$$

where the minimization is carried over all $K \times K$ unitary matrices $\mathbf{V}$. We propose iterative procedure of solving (8) by decomposing the whole minimization problem into a sequence of Givens rotations. Specifically, the ith estimate of $\mathbf{Q}$ is obtained as

$$\hat{\mathbf{Q}}_i = \prod_{1 \leq p < q \leq K} \mathbf{\Theta}_{p,q}(\alpha, \theta)\, \hat{\mathbf{Q}}_{i-1},$$

where $\hat{\mathbf{Q}}_{i-1}$ is the estimate obtained at $(i-1)$th iteration and $\mathbf{\Theta}_{p,q}(\alpha,\theta)$ is the elementary Givens rotation

$$\mathbf{\Theta}_{p,q}(\theta,\alpha) = \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & & \vdots & & \vdots \\ 0 & \cdots & c & \cdots & s & \cdots & 0 \\ \vdots & & \vdots & \ddots & \vdots & & \vdots \\ 0 & \cdots & -s^* & \cdots & c & \cdots & 0 \\ \vdots & & \vdots & & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix},$$

where $c \overset{\text{def}}{=} cos\,\theta$, $s \overset{\text{def}}{=} e^{j\alpha} sin\,\theta$ are located in rows (and columns) p and q. The parameters (θ,α) are chosen to minimize the criterion $C(\hat{\mathbf{Q}}_i)$ at each iteration. In the appendix, it is shown that this minimization amounts to computing the least eigenvector of a 3×3 matrix. The proposed optimization algorithm is summarized below (using informal notation):

Initialization: $\hat{\mathbf{Q}} = \mathbf{I}$
Iterations: for $i = 1, 2, \ldots, \#$ of iterations (sweeps)
for $1 \le p < q \le K$
$(\theta_{min}, \alpha_{min}) :=$ arg $\min_{\theta,\alpha} C(\mathbf{\Theta}_{p,q}(\alpha,\theta)\,\hat{\mathbf{Q}})$
$\hat{\mathbf{Q}} := \mathbf{\Theta}_{p,q}(\theta_{min},\alpha_{min})\,\hat{\mathbf{Q}}$

Note that like usual CM algorithm [4], the proposed technique can also be applied to non-CM sub-gaussian signals.

4.3 Estimation of offsets and propagation channels

Using the estimate $\hat{\mathbf{Q}}$ obtained from the previous step, one can estimate the overall channel $\mathbf{A}$ as

$$\hat{\mathbf{A}} = \mathbf{A}_0 \hat{\mathbf{Q}}^H. \tag{9}$$

Zero-forcing equalization yields the estimate of $\mathbf{S}$:

$$\hat{\mathbf{S}} = \hat{\mathbf{A}}^\dagger \mathbf{X} = \hat{\mathbf{Q}}\mathbf{Z} + \mathbf{N}',$$

where $\mathbf{N}' \overset{\text{def}}{=} \hat{\mathbf{A}}^\dagger \mathbf{N}$. One may extract the offset information from $\hat{\mathbf{S}}$ by exploiting the finite alphabet (FA) property of the transmitted symbols. More precisely, suppose that $b_k(l)$ takes the values from the alphabet $\Omega = \{\zeta_i,\ i = 1, 2, \ldots, P\}$. Consider the following cost function[1]:

$$f_k(\gamma) \overset{\text{def}}{=} \sum_{l=0}^{T-1} \prod_{i=1}^{P} |\hat{s}_{k,l} e^{-jl\gamma} - \zeta_i|^2. \tag{10}$$

It can be easily seen that when $\hat{\mathbf{S}} = \mathbf{S}$, $f_k(\gamma)$ is zero when $\gamma = N\phi_k$. Other zeroes of f_k may exist. For example, if the constellation Ω is invariant under the rotation by β degrees, then $\gamma_m = N\phi_k \pm m\beta,\ \ m \in \mathcal{Z}$ is also a zero of f_k. Let Γ denote the set of zeros of $f_k(\gamma)$. In order to choose the zero which is the closest to $N\phi_k$, we propose to use the fact that the columns of $\mathbf{A}$ lie in the signal subspace. Indeed, let $\mathbf{a}_k$ and $\hat{\mathbf{a}}_k$ stand for the kth column of $\mathbf{A}$ and its respective estimate obtained from (9). Define

$$\begin{aligned} \mathbf{Z}_k(\gamma) &\overset{\text{def}}{=} \text{diag}\left(1 \quad e^{j\gamma/N} \quad \ldots \quad e^{j\gamma(N-1)/N}\right) \\ \mathbf{h}_k(\gamma) &\overset{\text{def}}{=} \mathbf{C}_k^\dagger \mathbf{Z}_k^{-1}(\gamma)\mathbf{a}_k \\ \hat{\mathbf{h}}_k(\gamma) &\overset{\text{def}}{=} \mathbf{C}_k^\dagger \mathbf{Z}_k^{-1}(\gamma)\hat{\mathbf{a}}_k. \end{aligned}$$

so that $\mathbf{a}_k = \mathbf{Z}_k(N\phi_k)\mathbf{C}_k\mathbf{h}_k(N\phi_k)$. Ideally, if the columns of $\mathbf{U}_n$ form the basis of the noise subspace, we have

$$\|\mathbf{U}_n^H \mathbf{a}_k\|^2 = \|\mathbf{U}_n^H \mathbf{Z}_k(N\phi_k)\mathbf{C}_k\mathbf{h}_k(N\phi_k)\|^2 = 0. \tag{11}$$

Taking into account the fact that only the estimates of $\mathbf{U}_n$ and of $\mathbf{h}_k(\gamma)$ are available, one may choose the zero from Γ as follows

$$\gamma_{opt} = N\hat{\phi}_k = \arg\min_{\gamma\in\Gamma} \|\hat{\mathbf{U}}_n^H \mathbf{Z}_k(\gamma)\mathbf{C}_k\hat{\mathbf{h}}_k(\gamma)\|^2.$$

Finally, $\hat{\mathbf{h}}(\gamma_{opt})$ represents the estimate of the propagation channel.

Remark: It should be noted that for the specific modulation (say, BPSK) simpler techniques other than minimization of (10) can be considered. Actually this issue is under investigation.

5 Numerical studies

In this section, we give some simulation results to illustrate the performance of the proposed technique. A quasisynchronous DS-CDMA system with $K = 4$ users is considered. Each user transmits a sequence of QAM-4 symbols which are then spreaded with a Gold sequence of length $N = 31$. The propagation channel for the kth user ($\mathbf{h}_k$) has 2 dominant paths with independent Rayleigh fadings. In all experiments, the proposed method runs 3 iterations (sweeps) to estimate the

[1] We assume implicitly that the phase and amplitude ambiguities have been already eliminated. In practice, amplitude can be estimated from the covariance matrix and phase ambiguity is removed using differential modulation.

unitary factor $\mathbf{Q}$ as described in section 4.2. The performance of the method presented in [5] (called here Matrix Polynomial, or MP method) is also given for comparison. Both techniques use the grid of 1000 points for offset estimation (MP method estimates offset through the one-dimensional (1-D) spectrum search). The carrier offsets were modeled as random variables uniformly distributed in $[-\pi/(32N); \pi/(32N)]$. However, simulations have shown that due to limited resolution of the MP method it fails to provide reasonable estimation quality for this offset range as the carrier offset accumulated during the chip period is too small. For that reason, offset range for the MP method was chosen to be $[-\pi/32; \pi/32]$. It should be noted that contrary to MP method, the technique proposed here estimates carrier offset accumulated during the symbol period as explained in section 4.3.

In Fig. 1, we plot the channel estimation Mean-Squared Error (MSE) for the first user vs. the Signal-to-Noise Ratio (SNR) (which is the same for all users) for the fixed sample size $T = 256$. The MSE is averaged over 200 Monte-Carlo trials. The MSE of the offset estimation for the first user vs. SNR is shown in Fig. 2. In Fig. 3, the MSE of channel estimation is plotted vs. sample size for the fixed SNR of 8 dB and Figure 4 provides the similar plot but for the MSE of offset estimation.

Note that for moderate and high SNR values our technique outperforms the MP method. Figures 2 and 4 show that, for our technique, starting from a certain SNR value ($\approx$ 8 dB) or a certain sample size (≈ 150 samples) there is no more decrease in MSE of offset estimation. This is explained by the limited accuracy of 1000-point grid search. Further MSE improvement can thus be attained using more grid points.

6 Conclusions

Blind channel estimation for DS-CDMA systems in presence of carrier offsets was studied. The sufficient conditions for channel identifiability were obtained and an algorithm for joint channel and offset estimation was presented. Numerical studies demonstrate the efficiency of the proposed technique.

A Proof of the theorem 1

Proof: First, we solve (2) for $\bar{\mathbf{S}}$ giving

$$\bar{\mathbf{S}} = \bar{\mathbf{A}}^{\dagger}\mathbf{X} = \mathbf{T}\mathbf{S}, \qquad (12)$$

where $\mathbf{T} \stackrel{\text{def}}{=} \bar{\mathbf{A}}^{\dagger}\mathbf{A}$. Denote $\mathbf{t} = [t_1\, t_2 \ldots t_k]$ the mth row of $\mathbf{T}$. Matrix equation (12) contains T linear equations in elements of $\mathbf{t}$ which can be written as

$$\sum_{k=1}^{K} t_k b_k(l)\, e^{jNl\phi_k} = \bar{b}_m(l)\, e^{jNl\bar{\phi}_m}, \;\; l = 0, 1, \ldots, T-1. \qquad (13)$$

As $\bar{\mathbf{S}}$ is of full row rank, there exists $t_q \neq 0$. Multiplying (13) by $b_q^*(l)e^{-jNl\bar{\phi}_q}$ and averaging over $b_k(l)$, $\bar{b}_m(l)$ yields (using (4))

$$t_q\epsilon_q = c_{mq}e^{jNl(\bar{\phi}_m-\phi_q)}, \;\; l = 0, 1, \ldots, T-1 \qquad (14)$$

where $\epsilon_q \stackrel{\text{def}}{=} E[|b_q(l)|^2]$ and $c_{mq} \stackrel{\text{def}}{=} E[\bar{b}_m(l)b_q^*(l)]$. As the left side of (14) is non-zero and does not depend on l, we have

$$\bar{\phi}_m = \phi_q + 2\,n\pi/N, \;\; n \in \mathcal{Z}. \qquad (15)$$

Now let us show that all remaining $t_k,\;\; k \neq q$ are zero. Multiplying (13) by $b_k^*(l)e^{-jNl\phi_q}$ with subsequent averaging over $b_k(l)$, $\bar{b}_m(l)$ using (4) and (15) gives

$$t_k\epsilon_k e^{jNl(\phi_k-\phi_q)} = c_{mk}, \;\; l = 0, 1, \ldots, T-1 \qquad (16)$$

where $c_{mk} \stackrel{\text{def}}{=} E[\bar{b}_m(l)b_k^*(l)]$. As the right side of (16) is constant and $\phi_k \neq \phi_q \pmod{2\pi/N}$ due to (3), we have $t_k = c_{mk} = 0$.

Therefore, $\mathbf{T}$ contains only one non-zero element in each row. As $\bar{\mathbf{S}}$ is of full row rank, each column of $\mathbf{T}$ also contains only one non-zero element.

B Determination of parameters (θ, α)

Denote $\mathbf{M} \stackrel{\text{def}}{=} \hat{\mathbf{Q}}_i\mathbf{Z}$, $\mathbf{M}' \stackrel{\text{def}}{=} \mathbf{\Theta}_{p,q}(\theta,\alpha)\mathbf{M}$. The Givens rotation $\mathbf{\Theta}_{p,q}(\theta,\alpha)$ changes only the rows p and q of $\mathbf{M}$. Therefore,

$$C(\mathbf{M}') = \sum_{l=0}^{T-1}(|\mathbf{M}'_{p,l}|^2-1)^2+(|\mathbf{M}'_{q,l}|^2-1)^2+c_1, \qquad (17)$$

where c_1 denotes the terms which don't depend on θ and α. Substituting $\mathbf{M}'_{p,l} = c\mathbf{M}_{p,l} + s\mathbf{M}_{q,l}$, $\mathbf{M}'_{q,l} = -s^*\mathbf{M}_{p,l} + c\mathbf{M}_{q,l}$ into (17) gives

$$C(\mathbf{M}') = \mathbf{w}^T\mathbf{G}\mathbf{w} + c_2, \qquad (18)$$

where $\mathbf{w} \stackrel{\text{def}}{=} [cos2\theta \;\; sin2\theta cos\alpha \;\; sin2\theta sin\alpha]^T$ and

$$\mathbf{G} \;\; \stackrel{\text{def}}{=} \;\; 2\sum_{l=0}^{T-1}\mathbf{y}_{pq}(l)\mathbf{y}_{pq}(l)^T$$

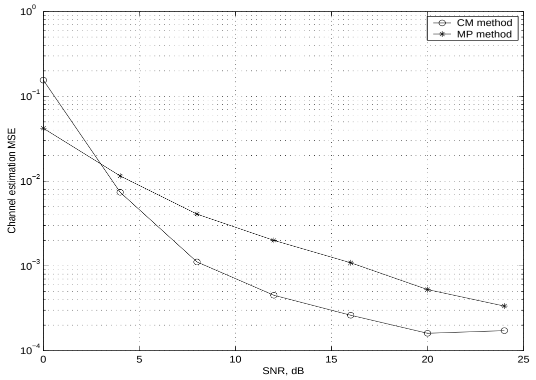

Figure 1: MSE of channel estimation vs. SNR

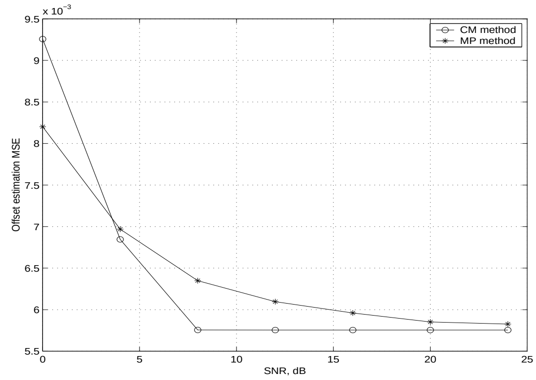

Figure 2: MSE of offset estimation vs. SNR

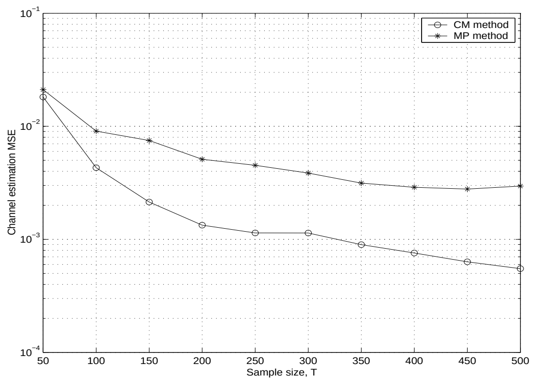

Figure 3: MSE of channel estimation vs. T

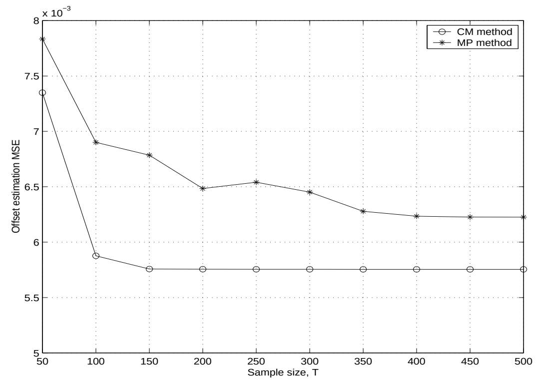

Figure 4: MSE of offset estimation vs. T

$$\mathbf{y}_{pq}(l) \stackrel{\text{def}}{=} [(|\mathbf{M}_{p,l}|^2 - |\mathbf{M}_{q,l}|^2)/2, \Re(\mathbf{M}_{p,l}^* \mathbf{M}_{q,l}), -\Im(\mathbf{M}_{p,l}^* \mathbf{M}_{q,l})]^T.$$

It follows from (18) that $C(\mathbf{M}')$ is minimized by choosing $\mathbf{w}$ as the least eigenvector of $\mathbf{G}$. The parameters c and s of Givens rotation are then easily determined from the elements of $\mathbf{w}$.

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