Calibration Method for Light Sectioning Measurement Systems

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Abstract: - This paper presents a new approach in calibrating a light sectioning measurement system. The developed method permits the calibration of such a system without previous knowledge of the exact positions of camera and laser. The basic concepts of this new approach and its advantages compared to common methods are discussed. Further, a possible implementation in an industrial process is given.

Key-Words: - Light Sectioning, Camera Calibration, Grassmann Coordinates, Grassmannian Reduction, Least-Square Fitting

1 Introduction

In many industrial applications quality control concerning geometric measurement is employed using optical measurement systems. In that context, common tasks are fulfilled by means of light sectioning. The most popular variant of this image processing method is based on trigonometric calculation. In the present paper, the main disadvantages of this variant are explained and an alternative is proposed.

1.1 Structure of a Light-Sectioning System

As can be seen in Figure 1, a typical measurement system using light-sectioning is established by the object to be measured, a laser projecting the plane of light onto the object and a suitably positioned camera observing the intersection of the plane of light and the object.

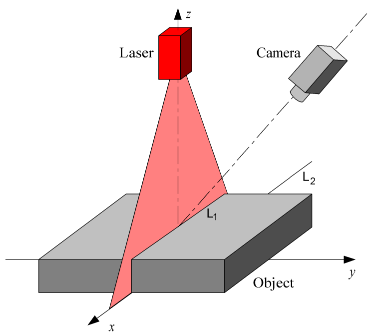

Figure 1: Structure of a typical light sectioning system

1.2 Common Light-Sectioning Methods

A well known and commonly used method of light sectioning (see [1]) uses trigonometric relations to calculate the desired geometric quantity. This paper presents a new solution based on projective geometry. This approach has practical and numerical advantages and uses a simple target with known geometry and registration marks (see Section 3.3). That implies that neither laser nor camera position has to be known a priori.

Similar to the camera calibration technique which is proposed in [2], the presented concept is based on a single calibration image. However, this new method reduces the number of coordinate systems from five to three. Additionaly, the orientation of the plane of light can be obtained without the knowledge of the focal length. Further the more robust singular value decomposition is used rather than QR decomposition.

Images currupted by distortion can be corrected applying algorithms proposed in [3], which are based on works presented in [4].

2 Problem Formulation

Determination of sufficiently exact positions (relative or absolute) of camera and laser of a light sectioning measurement system for calibration purpose is a critical task. The preliminary idea of this new calibration approach is to avoid the need for any previous knowledge of these positions. The procedure should be based on image processing algorithms and geometric dependencies.

Therefore the design of a suitable calibration object is essential.

Further, it is desirable that internal parameters of the camera (focal length, resolution, geometric size of the camera chip) are not necessarily needed for the calibration procedure.

3 Calibration of the Light Sectioning Measurement System

The light-sectioning measurement system can be calibrated using a geometrical approach without knowing the position and orientation of the camera and the laser. The developed calibration method is performed in three steps:

- Segmentation of the calibration image by means of image processing.
- Determination of position and orientation of a coordinate frame related to the plane of light.
- Calculation of the homogeneous projection matrix $\mathsf{H}_{\mathsf{Laserpl-Cam}}$ between camera- and laser-plane.

3.1 Principle of calibration

- The homogeneous transformation of a plane onto the camera image is fully determined by four known corresponding points.
- Given two known planes on the calibration target and the image of the points of intersection with the plane of light, the orientation of the laser plane can be obtained.
- Therefore the homogeneous transformation and its inverse relating the plane of light to the image plane of the camera is completely known.

3.2 Description of Central Concepts

Basic components of the presented calibration procedure are the concept of homogenous coordinates and linear coordinate transformation using homogenous coordinates (see [5]), which are not discussed in this document.

3.2.1 Homogeneous Mapping between Planes

Starting from the homogeneous definition of a point $\boldsymbol{p_0}$ on a plane

$$\boldsymbol{p_0} = \begin{bmatrix} x_0 \\ y_0 \\ 1 \end{bmatrix}, \tag{1}$$

the homogeneous projection H of $\boldsymbol{p_0}$ to a point $\boldsymbol{p_r}$ on another plane can be formulated as

$$\boldsymbol{p}_r = \mathsf{H}\boldsymbol{p}_0, \tag{2}$$

where

$$\boldsymbol{p_r} = \begin{bmatrix} x_r \\ y_r \\ w_r \end{bmatrix}. \tag{3}$$

Obviously, the homogeneous projection H is defined by a 3x3-matrix of the form

$$\mathsf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}. \tag{4}$$

Assuming that one of the nine parameters in H can be interpreted as a scaling, the remaining eight entries of the matrix can be determined with the help of four points given in both planes. The two affine coordinates (x_r^a and y_r^a) of the projected point $\boldsymbol{p_r}$ are calculated using the homogeneous coordinate w_r (see [5]).

$$\begin{aligned} x_r^a &= \frac{x_r}{w_r} = \frac{h_{11}x_0+h_{12}y_0+h_{13}}{h_{31}x_0+h_{32}y_0+h_{33}} \\ y_r^a &= \frac{y_r}{w_r} = \frac{h_{21}x_0+h_{22}y_0+h_{23}}{h_{31}x_0+h_{32}y_0+h_{33}} \end{aligned} \tag{5}$$

Rewriting and expanding the above equations gives:

$$\begin{aligned} -h_{11}x_0 - h_{12}y_0 - h_{13} + h_{21}0 + h_{22}0 + h_{23}0 + \\ + x_r^a\,(h_{31}x_0 + h_{32}y_0 + h_{33}) = 0 \end{aligned}$$

$$\begin{aligned} h_{11}0 + h_{12}0 + h_{13}0 - h_{21}x_0 - h_{22}y_0 - h_{23} + \\ + y_r^a\,(h_{31}x_0 + h_{32}y_0 + h_{33}) = 0 \end{aligned}$$

Writing these equations for n corresponding pairs of points ($n \geq 4$) and formulating as matrices, we can write:

$$\mathsf{G}\boldsymbol{h} = 0, \tag{6}$$

where

$$\mathsf{G} = \begin{bmatrix} -x_0(1) & -y_0(1) & -1 & 0 & 0 & 0 & x_r^a(1)x_0(1) & x_r^a(1)y_0(1) & x_r^a(1) \\ 0 & 0 & 0 & -x_0(1) & -y_0(1) & -1 & y_r^a(1)x_0(1) & y_r^a(1)y_0(1) & y_r^a(1) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -x_0(n) & -y_0(n) & -1 & 0 & 0 & 0 & x_r^a(n)x_0(n) & x_r^a(n)y_0(n) & x_r^a(n) \\ 0 & 0 & 0 & -x_0(n) & -y_0(n) & -1 & y_r^a(n)x_0(n) & y_r^a(n)y_0(n) & y_r^a(n) \end{bmatrix}$$

and

$$\boldsymbol{h} = \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix}.$$

For $n > 4$, Equation 6 is an over-determined linear equation system which can be solved by different methods, such as normal equations method, QR-factorization or singular value decomposition (SVD). Because it is robust and effective, SVD (see [6], [7]) is applied on matrix G to find a non-trivial solution of $\boldsymbol{h}$ in Equation 6. Finally, the components of $\boldsymbol{h}$ are recombined to form the matrix H of the homogeneous projection.

3.2.2 Grassmannian Reduction and Fitting

The calibration process and the actual measurement are based on Grassmann coordinates and Grassmannian reduction (see [8], [9]). These concepts are presented with regard to fitting of lines in the plane. Further, the extendibility of these concepts on arbitrary geometric objects (parallel lines, planes, circles, conic sections, . . .) is shown (see also [10]).

Fitting a Line in the Plane: Considering the fact, that a line in the plane is defined by two points $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$, written in homogeneous coordinates:

$$\boldsymbol{p_1} = \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} \quad \textit{and} \quad \boldsymbol{p_2} = \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$$

a third point $\boldsymbol{p}$

$$\boldsymbol{p} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

lies on the line if it is a linear combination of $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$. This can be expressed as:

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0. \qquad (7)$$

Expanding the minors of the first row of this determinant gives

$$x(y_1 - y_2) - y(x_1 - x_2) + (x_1 y_2 - x_2 y_1) = 0, \qquad (8)$$

or

$$xY - yX + N = 0. \qquad (9)$$

The set of the parameters $X : Y : N$ have been proposed by Grassmann (see [8], [9]; also called *Planner Line Coordinates*) rather than variables. These parameters describe a space that is commonly of higher order than the variable space but lead to an equation that is linear in the parameters and can be solved using linear algebra. Formulating this expression as a matrix multiplication

$$\begin{bmatrix} x & y & 1 \end{bmatrix} \begin{bmatrix} Y \\ -X \\ N \end{bmatrix} = 0. \qquad (10)$$

The planner line coordinates X, Y and N represent the line in the plane. Assuming for typical tasks in image processing a set of n points $\boldsymbol{p_i}$, which are corrupted by errors (e.g. measurement noise), an error vector $\boldsymbol{e}$ is introduced on the right side of Equation 10:

$$\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots \\ x_n & y_n & 1 \end{bmatrix} \begin{bmatrix} Y \\ -X \\ N \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix} \qquad (11)$$

or short

$$\mathsf{G}\, \boldsymbol{l} = \boldsymbol{e}. \qquad (12)$$

Ideally, the right side of the above equation becomes zero (i.e. the fitted line contains lies on all the points $\boldsymbol{p_i}$), in which case the matrix G must be singular.

$$\mathsf{G}\, \boldsymbol{l} = 0 \qquad (13)$$

For this reason, singular value decomposition is applied on G to calculate a non-trivial solution for $\boldsymbol{l}$ in Equation 13. There, $\boldsymbol{l}$ is the vector of the line coordinates of the line fitted to the set of points $\boldsymbol{p_i}$. At this point, it has to be mentioned that although this concept seems to lead to more effort in fitting lines, it becomes relevant for problems of higher order.

Fitting a Circle in the Plane: The concept can also be extended to fitting circles, which is required to determine the position of the registration marks.

$$\begin{bmatrix} x_1^2 + y_1^2 & x_1 & y_1 & 1 \\ x_2^2 + y_2^2 & x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ x_n^2 + y_n^2 & x_n & y_n & 1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix}, \qquad (14)$$

where $C_1 : C_2 : C_3 : C_4$ are called *tetracircular Grassmann coordinates*. Fitting of conic sections and quadrics is done in an analogous manner. Further, spheres can be fitted using *pentaspherical coordinates* too (for more information see [11]).

Fitting a Plane in Space: The general equation of a plane

$$E_1 x + E_2 y + E_3 z + E_4 = 0 \qquad (15)$$

can be written as

$$\begin{bmatrix} x & y & z & 1 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix} = 0. \qquad (16)$$

Thus, fitting a plane to a set of n points $\boldsymbol{p_i}$ in space gives:

$$\begin{bmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ x_n & y_n & z_n & 1 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix} \qquad (17)$$

3.3 The Calibration Object

To enable the calibration of the measurement system, a calibration object of specific geometry is necessary (see Figure 2 for a possible realization). With the help of this calibration object, the orientation of the plane of light (projected by the laser) has to be located relative to a fixed spatial coordinate frame. It has to be mentioned, that additional transformations can be avoided by defining the origin of that fixed coordinate frame on the calibration object itself.

Since a plane in space is uniquely defined by two non-identical lines, the calibration object establishes two different planes, that are chosen to be parallel for simplicity. The orientation of these two planes has to be known exactly relative to the fixed coordinate frame. Further, the two planes have to be identified during the calibration process. For this reason the two planes are each identified with four calibration marks (see Section 3.1 for further explanations).

The realization of these marks can be achieved by different methods (drilled holes, marks, LED's, optical fibre).

3.4 The Calibration Process

The aim of the calibration process is to obtain the homogeneous transformation T between the laser coordinate frame and the fixed coordinate frame and the homogeneous projection matrix $\mathsf{H}_{\mathsf{Laserpl-Cam}}$ between the laser plane and the camera plane (image). In the measurement process the found laser points in the image are mapped onto the laser plane (therefore $\mathsf{H}_{\mathsf{Laserpl-Cam}}$ is needed). The coordinates of the mapped points are then transformed back into the fixed

coordinate frame (therefore T is needed). The resulting coordinates describe the 3D-structure of the measurement object.
The input data for the calibration is the geometry of the calibration object and a camera image. The camera image has to show the intersection of the plane of light with both planes of the calibration object on the one hand and the eight calibration marks of the calibration object on the other hand. As described at the beginning of this section, the calibration process is performed in three steps.

3.4.1 Segmentation of the Calibration Image

This part of the calibration contains the recognition of both, the calibration marks and the points of the intersection lines of laser plane and calibration object. This is achieved by algorithms typically used in image processing.
The segmentation of the calibration marks is performed using contour extraction. The different contours found with a suitable intensity threshold are then analyzed concerning their similarity to circles.
The intersection of the laser plane and the planes of the calibration object can be seen as bright lines in the image. These lines can roughly be found by detecting the brightest pixel in each column of the image (i.e. maximum of each column of the matrix that represents the image data). As there are not laser points in every column of the image, only those pixels with a intensity value above a certain level (depending on the mean intensity value of the brightest pixels) are considered to be part of an intersection line. To find the exact position of the point in a column that belongs to the laser line, the center of gravity of intensity (see [1]) is calculated.
It is assumed that there are two line segments (one in the left part of the image and one in the right part of the image) that are a projection of the laser plane onto the rear plane of the calibration object and one line segment that is a projection onto the front plane.

3.4.2 Position and Orientation of a Coordinate Frame related to the Plane of Light

According to this assumption each of the obtained points are now mapped onto the appropriate plane of the calibration object using the homogenous projection matrices and transformed into the fixed coordinate frame. Hence the position of all these points in the fixed coordinate frame is known. Because of measurement noise, the points are not exactly in-plane, so a least mean square plane-fit is calculated (see Section 3.2.2). The resulting equation of the plane is:

$$E_1x + E_2y + E_3z + E_4 = 0 \tag{18}$$

The parameters E_1 to E_4 are used to obtain the position and orientation of the laser coordinate frame (i.e both the translational and rotational part of the transformation).
The translational part can be found when the origin of the laser coordinate frame relative to the fixed coordinate frame is known. As defined above the origin of the laser coordinate frame is the intersection point of the y-axis of the fixed coordinate frame and the laser plane. Substituting $x = 0$ and $z = 0$ in Equation 18 results in:

$$E_2y + E_4 = 0 \tag{19}$$

and

$$y = -\frac{E_4}{E_2}. \tag{20}$$

Hence the translational parameters of the transformation are

$$\begin{aligned} \Delta x &= \quad 0, \\ \Delta y &= -\frac{E_4}{E_2}, \\ \Delta z &= \quad 0. \end{aligned} \tag{21}$$

The rotational part of the transformation (i.e. the orientation of the laser coordinate frame) is defined by three angles. The aim is to rotate the x-axis and the y-axis into the laser plane.
The coordinate frame is rotated around the x-axis to turn the y-axis into the laser plane. To obtain the rotation angle the equation of the intersection line between the laser plane and the y-z-plane of the fixed coordinate frame is calculated (by substituting $x = 0$ in Equation 18):

$$E_2y + E_3z + E_4 = 0. \tag{22}$$

The slope of this intersection line corresponds with the desired angle:

$$\phi_x = \arctan\left(-\frac{E_2}{E_3}\right) \tag{23}$$

The next step is to rotate the coordinate frame around the y-axis in order to get the x-axis into laser plane. The current direction of the z-axis is $[0, E_2, E_3]^T$. The normal vector of the laser plane is $[E_1, E_2, E_3]^T$. The angle of rotation around the y-axis corresponds with the angle between these two vectors. It is also possible to rotate the coordinate frame around the z-axis, but this only changes the orientation of the x-axis and y-axis within the laser plane. A specific orientation is not needed, so a rotation around the z-axis is obsolete.
Now the position and orientation of the laser coordinate frame is defined and the matrices of transformation between the fixed coordinate frame and the laser coordinate frame can be calculated.

3.4.3 Calculation of the Matrix $\mathsf{H}_{\mathsf{Laserpl-Cam}}$

This part of the calibration process determines the homogenous projection matrix between the laser plane and the camera plane $\mathsf{H}_{\mathsf{Laserpl-Cam}}$.
To calculate this matrix, at least four points in the laser plane and their corresponding points in the image (camera plane) have to be known. As there are more than four corresponding points known in each plane (the points of the intersection of laser plane and the calibration object), the projection matrix can be found as described in Section 3.2.2 .

4 Implementation

This section deals with the description of the calibration target and the industrial implementation of the

calibration method. Measurement of the width and depth of cracks embedded on the edge of milled steel blocks can be achieved by means of a calibrated measurement head.
Referring to Section 3.3, Figure 2 shows the calibration target we used for the industrial implementation.

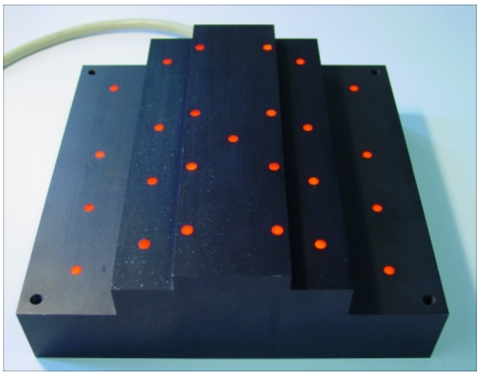

Figure 2: 3-stage-calibration target

The calibration object is a stepped matt-black body with a size of approximately 190x200x70 [mm].
Instead of the two required stages for the calibration procedure the object was extended by an additional stage. Each stage (which is also called *calibration plane*) contains at least four calibration marks. In case of lens-changing and a caused variation of the field of view these modifications lead to a high degree of flexibility.

Figure 3: Steel slab and target in a steel-mill

In Figure 3 the calibration target and the steel slab are shown. In case of a suitable orientated calibration target (referring to the axis of the steel slab) the depth and width of the observed crack can easily be determined by applying the established projection and transform matrices.
The intersection of the laser plane with the calibration target can be seen as line segments. These lines are used for an accurate calibration of the camera. The arrangement of laser and camera is also shown in Figure 3.
After the calibration procedure, the target can be removed and the measuring head can inspect the passing steel slabs. Thereby a certain movement of the slab relative to the measuring head is permitted, whereas every movement of the laser relative to the camera is prohibited. Figure 4 shows form, size and location of an inspected crack.

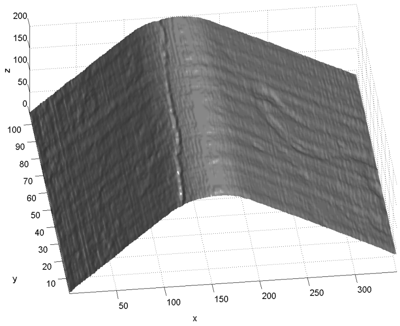

Figure 4: Rendered 3D geometric model of the steel block showing a crack in the edge-region

A calibration picture is shown in Figure 5. The observable line segments and calibration points are now to determine the homogeneous projection matrices H_{rear} and H_{front} (derived in Section 3.1) and the transformation rule T.

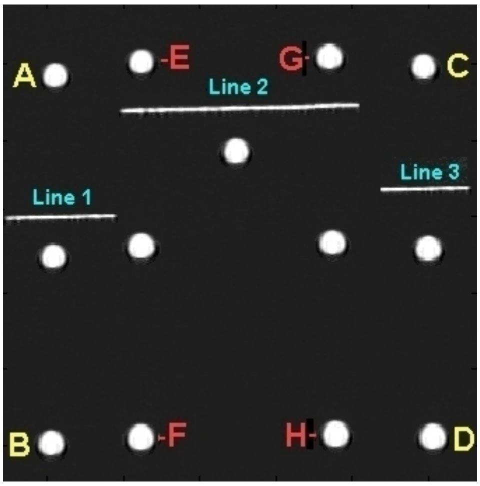

Figure 5: Zoomed Calibration picture

Due to the calibration points A, B, C and D, H_{rear} can be determined for the rear calibration plane. H_{front} is based on the calibration points E, F, G and H. For these two calculations we assume that both, image coordinates [in pixel] and real coordinates of the eight calibration points [in mm] are known. Due to the matrices H_{rear}, H_{front} and the position of the line segments *Line1*, *Line2* and *Line3* it is possible to calculate T.
H_{front}, H_{rear} and T enable the determination of the projection matrix $H_{Laserpl-Cam}$. If a steel slab with a crack is intersected by the laser plane, all measurement points certainly lie in the laser plane. The distance between them is determined in [mm], however, the measurement points should be transformed into the fixed coordinate frame by the help of T to get the radial crack depth.

The fixed coordinate frame ist attached to the calibration target in that way, that x- and y-axes of the coordinate frame span the rear calibration plane. The z-axis is therefore perpendicular to the rear calibration plane and, in the case of a target-orientation according to Figure 3, beyond it radial to the slab axis. Based on z- and x-differences of the measurement points in the fixed coordinate frame (the y-axis is parallel to the slab axis) one can determine the crack-depth.
Figure 6 shows the intersection of a laser plane with a cracked slab and the fitted spline. The data in this figure is not calibrated. The depth of the crack is determined by means of measurement points P_1, P_2, P_3 and the calibration matrices ($\mathsf{H}_{\mathsf{front}}$, $\mathsf{H}_{\mathsf{rear}}$, T, $\mathsf{H}_{\mathsf{Laserpl-Cam}}$).

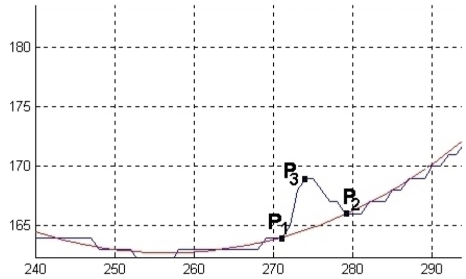

Figure 6: Intersection of laser-plane and slab

The crack in Figure 6 has a depth of 0.71[mm] and a width of 1.49[mm].
The measurement and calibration pictures has been taken with a IVP-SAH5-camera (512x512 pixel) and a 50mm-Cosmicar lens. Due to this combination the perpendicular on the optical axis orientated field of view measure approximately 110x110[mm].

5 Conclusion

In this paper a new calibration method for light sectioning is presented.

- A complete calibration of both camera and laser plane orientation and position is possible from a single image.
- The consistent use of homogeneous coordinates results in a stable system exhibiting no numerical singularities.
- No a priori knowledge or assumption of the laser or camera position and orientation is required. This enables a simple application in industrial environments as shown in Section 4.

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