Analysis of Foundation-Layered Soil Interaction Using Propagating Wave-modes Analysis

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Abstract: - In the engineering analysis of the response of the foundation in layered soil, subjected to direct or seismic excitation, parametric analysis is the most desirable type of analysis because it has potentials for better optimal design. We are presenting a computational approach, which yields dynamic displacements of a foundation and wave motion in layered soil. The computational approach yields wave-modes and their amplitudes as parameters, and the influence of each wave mode on the vibration of the foundation and on the spreading of waves into the surroundings. Computation is accomplished in the frequency domain. It uses the finite element method where the radiating conditions on the fictive boundary are satisfied exactly. We are presenting a brief outline of the key formulas of the computational approach. Numerical results are partially compared to the exact ones, suggesting the efficiency of the approach.

Key-Words: - Layered soil-structure interaction, Propagating Wave-modes, Fictive boundary, FEM

1 Introduction

In the analysis of the soil-structure interaction the soil behaves practically as an infinite space (half-space). Thus, the essential phenomenon is the occurrence of the propagating waves, which propagate in the direction away from the source of excitation. Mathematically these waves satisfy radiating (Sommerfeld) conditions, [1] [2], which imposes the crucial difficulty in the computing procedures. In the case when an non-homogeneity and/or variation of the boundary conditions exist, the solution of the wave equation, which requires the fulfillment of the radiation conditions, is feasible only numerically. Although a variety of numerical and semi-analytical methods are available for the analysis of the soil-structure interaction, none of them is simple and exact at the same time. If we glance over them, we could describe them roughly as follows. Boundary element methods satisfy radiation conditions, but are not simple to apply for complex cases, see for example [3] [4]. By finite-element methods the radiation conditions are satisfied only by using special elements on the fictive boundary, or by evaluating certain computational phases analytically, see for instance [4]-[6]. Operator methods require the implementation of special operators on the fictive boundary, for instance [7]-[12]. We can classify the available methods superficially and briefly as being either a great deal sophisticated and in certain cases exact, or simple and considerably approximate.

To analyze the foundation-layered soil interaction we use the approach that yields exact results when the layers rest over a rigid half-space, [13]. The results are considerably accurate also for the cases where the sub-soil is not rigid, providing that the foundation dimensions are approximately five times smaller than the cumulative depth of the layers. The modeling of the soil and the foundation is performed by FEM, which is comfortable for complex cases. The computation is in the frequency domain and the radiating conditions are exactly satisfied on the fictive boundary. When needed, the transient excitation can easily be analyzed by transforming the results from the frequency to the time domain. The approach yields parametric analysis of interaction, which is particularly advantageous for practical engineering analysis.

2 Problem Formulation and Outline of the Computing Procedure

We are considering a two-dimensional case of antiplane shear wave motion, yet the approach is valid for the case of general wave motion in parallel waveguides. The displacements in a soil with parallel layers, which have no foundation, are governed by the wave equation in the frequency domain, Equation (1), of course with distinct wave numbers k for different layers.

$$\nabla^2 u + k^2 u = 0 \qquad (1)$$

The theoretical solution is a linear combination of wave modes [1], and is presented by Equation (2) when using the co-ordinate system in Fig.1.

$$u(x,y) = \sum_n f_n(y)(A_n e^{-ib_n x} + B_n e^{ib_n x}) \qquad (2)$$

In Equation (2) f_n are eigen-functions, and A_n, B_n are constants which are determined by lateral boundary conditions. They represent wave-modes and amplitudes, respectively. Here, the amplitudes are called also weighting or modal factors. The b_n are distinct wave numbers, which depend on the characteristics of the case under consideration.

Let us consider wave motion in the segment (a cell) between any two cross-sections, for instance cross-sections 1 or 2 in Fig. (1). Then, according to Equation (2), each displacement and stress wave-mode satisfy Equation (3).

$$\begin{Bmatrix} u \\ \tau \end{Bmatrix}_{2,n} = \lambda_n \begin{Bmatrix} u \\ \tau \end{Bmatrix}_{1,n}, n = 1,2,... \qquad (3)$$

Indices 1 and 2 stand for the values on cross-sections 1 and 2, respectively, and n stands for the n-th wave-mode. Regarding Equation 1, the meaning of λ_n is given by Equations (4).

$$\lambda_n = \exp^{ib_n \Delta x} \qquad (4)$$

The constant in the exponent is given by Equation (5).

$$b_k = \frac{\omega}{c_{k,phase}}. \qquad (5)$$

On the other hand, due to the uniqueness of the solution of wave equation, Equation (6) applies, where $\mathbf{T}_{1\text{-}2}$ is the matrix of transfer functions.

$$\begin{Bmatrix} u \\ \tau \end{Bmatrix}_{2,n} = \mathbf{T}_{1-2} \begin{Bmatrix} u \\ \tau \end{Bmatrix}_{1,n}, n = 1,2,... \qquad (6)$$

After equating the right hand sides of Equations (3) and (6), we get the eigenvalue problem, Equation (7).

$$(\mathbf{T} - \lambda \mathbf{I}) \begin{Bmatrix} u \\ \tau \end{Bmatrix} = 0 \qquad (7)$$

By modeling the cell with finite elements, the transfer function becomes the transfer matrix, which is computed from the dynamic stiffness or from the flexibility matrix. Solving Equation (7) by standard routines yields eigenvectors representing displacements and stress wave-modes. Evidently, eigenvalues with negative imaginary part, and real eigenvalues less then a unit, belong to radiating modes.

When the foundation is present, and the excitation as shown in Fig.1, the displacements on any distant cross-section, for instance cross-section 1, must satisfy the radiating conditions. Therefore, wave motion consists of only radiating wave-modes, see Equation (8), where A_n are the amplitudes, and + sign stands for radiating modes.

$$\begin{Bmatrix} \mathbf{u}_1 \\ \boldsymbol{\tau}_1 \end{Bmatrix} = \sum_n \begin{Bmatrix} \mathbf{u} \\ \boldsymbol{\tau} \end{Bmatrix}_n^+ A_n \qquad (8)$$

The excitation displacements $\mathbf{u}_0$ are related to the radiating displacements and stresses on the cross section 1, called fictive boundary, by Equation (9), where $\mathbf{T}_{1\text{-}0}$ is the belonging transfer matrix. .

$$\mathbf{u}_0 = \mathbf{T}_{1-0} \begin{Bmatrix} \mathbf{u}_1 \\ \boldsymbol{\tau}_1 \end{Bmatrix} \qquad (9)$$

By solving Equations (8) and (9) on radiating displacements $\mathbf{u}_1$ get Dirichlet's boundary conditions to solve the exterior problem as an interior one. Computed amplitudes A_n are the parameters of the wave motion showing exactly how much distinct wave-modes contribute to the displacement and stress field. Finally, it is obvious that various combinations of excitations and boundary conditions can be solved by the same approach as presented above.

3 Numerical Example of Interaction of Foundation and Two-layered Soil

The analyzed case is symbolically presented in Fig. (1). The soil consists of two layers over rigid subsoil. The contact between layers and the sub-soil is considered as firm. Each layer is 10 meters deep, the lower one has the wave number k_1=2, while the upper one is softer and has k_2=1. The shear module of the upper layer is two times greater than the lower layer. Excitation is given by linearly distributed displacements with the amplitude a unit, as suggested in Fig. (1). The frequency of excitation is a unit. The rigid foundation dimensions are 4mx5m with the material density two times greater than that of the layers. The fictive boundary is 30m from the excitation cross-section, which makes the considered section 30 meters long. The width of the cell is 0,3 meters and is chosen arbitrarily. Finite elements are simple linear ones. The mesh has 46x41 nodal points, which can be observed in figures.

The solutions of Equation (7) are presented in Figures (2) and (3), some of the eigenvalues in Fig. (2), and some of the eigenvectors in Fig. (3). Comparison to exact values demonstrates excellent coincidence. All eigenvalues that belong to radiating wave modes are situated on the real axis between zero and one, they are diminishing standing waves, or have a negative imaginary part, propagating waves. The eigenvalues are marked by numbers, separately for standing and propagating wave-modes and according to the direction of propagation. Some of these numbers occur also in Fig. (3) to see to which eigenvalue a wave-mode belongs.

For the case of absence of the foundation, an analytical solution is computed in order to verify the numerical results. In the left graph in Fig. (4) radiating displacements are presented, while the graph on the right shows the absolute values of modal weighting factors (amplitudes) of constituent wave-modes – it represents the spectrum of displacements on the fictive boundary. It shows that the third mode is dominant, while the standing modes have vanished. Both figures demonstrate a considerably good agreement between analytical and numerical results.

The displacement field of foundation-soil interaction is presented in Fig. (5), and the absolute values of displacement profiles in Fig. (6). Fist three graphs show the displacements in the cross-sections of excitation, foundation site and fictive boundary, respectively. The fourth graph presents maximal displacements occurring outside of the analyzed segment at various cross-sections.

4 Conclusion

The presented theory and examples demonstrate that the computing approach is considerably simple. Already by simple finite elements and a rather coarse mesh we get excellent results. An advantage of the approach is that various engineering aspects can be analyzed with the aid of wave-modes and their weighting factor. Unfortunately, due to the limited length of this paper, only some advantages are presented.

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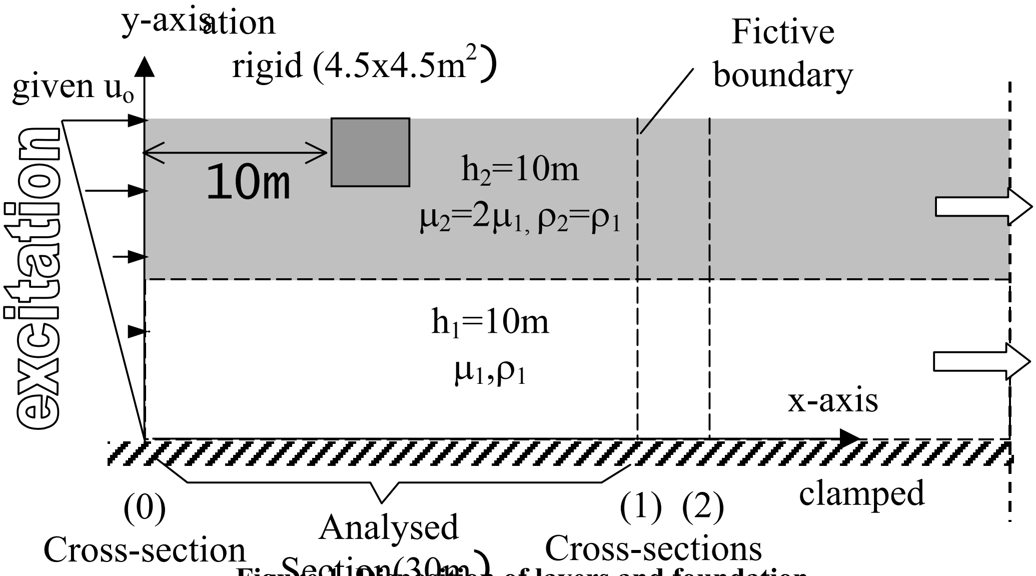

Figure 1. Disposition of layers and foundation.

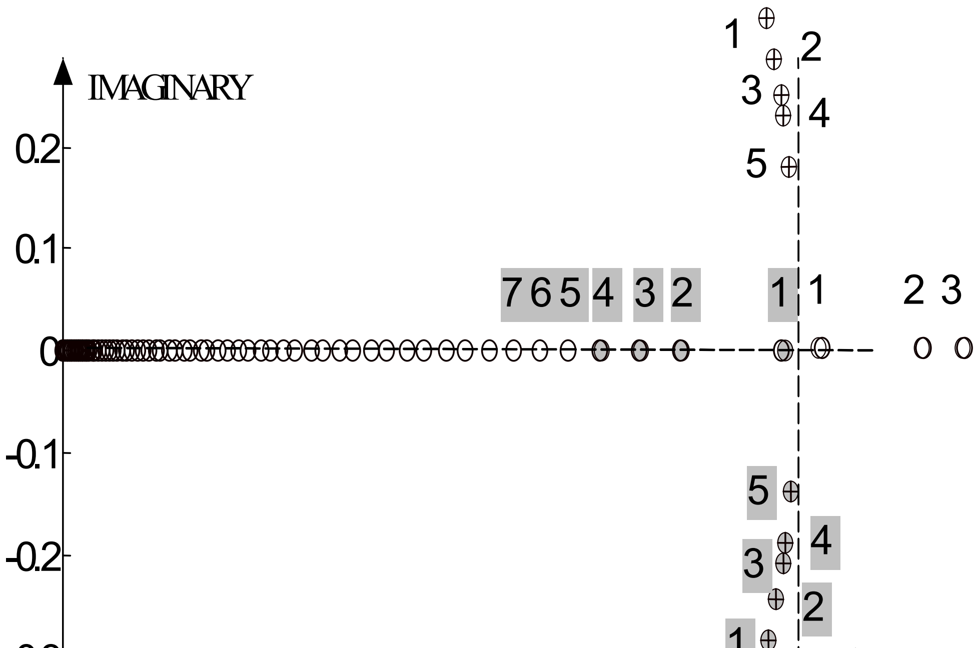

Figure 2. Eigenvalues. Explanation of the graph - eigenvalues belong to: radiating modes (numbers on patches, only few leading of them are marked), exact values (filled circles, only few leading are presented), computed standing wavemodes (void circles), computed propagating wavemodes (“ +” signs).

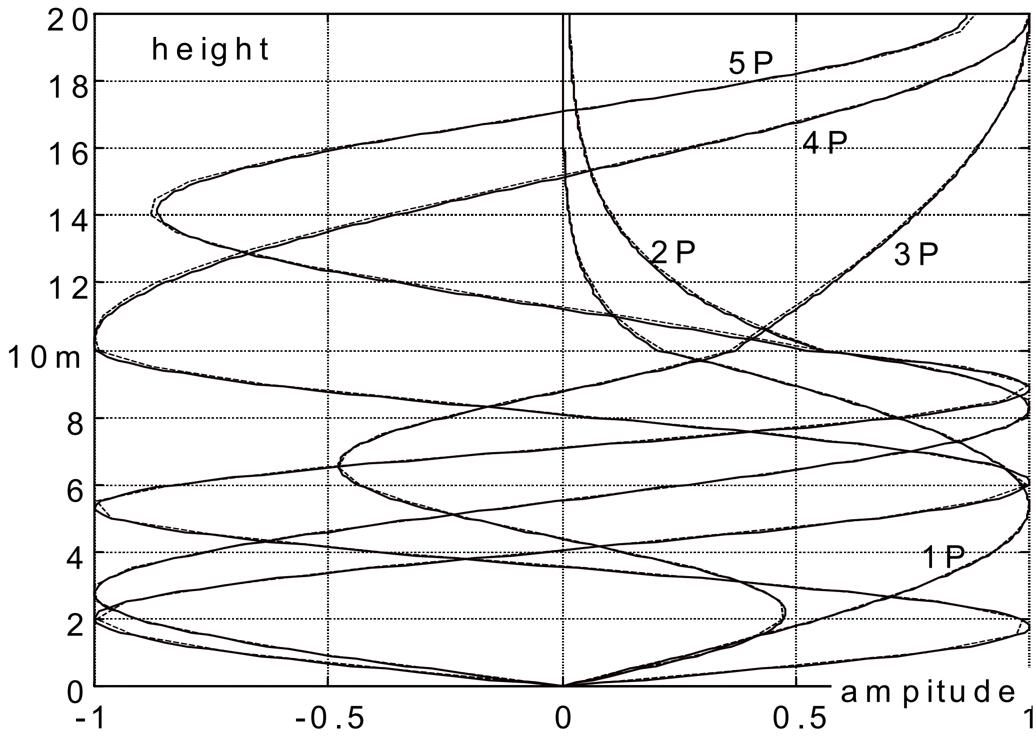

Figure 3. Firs five propagating wave-modes. Exact values -solid lines, computed - dashed lines.

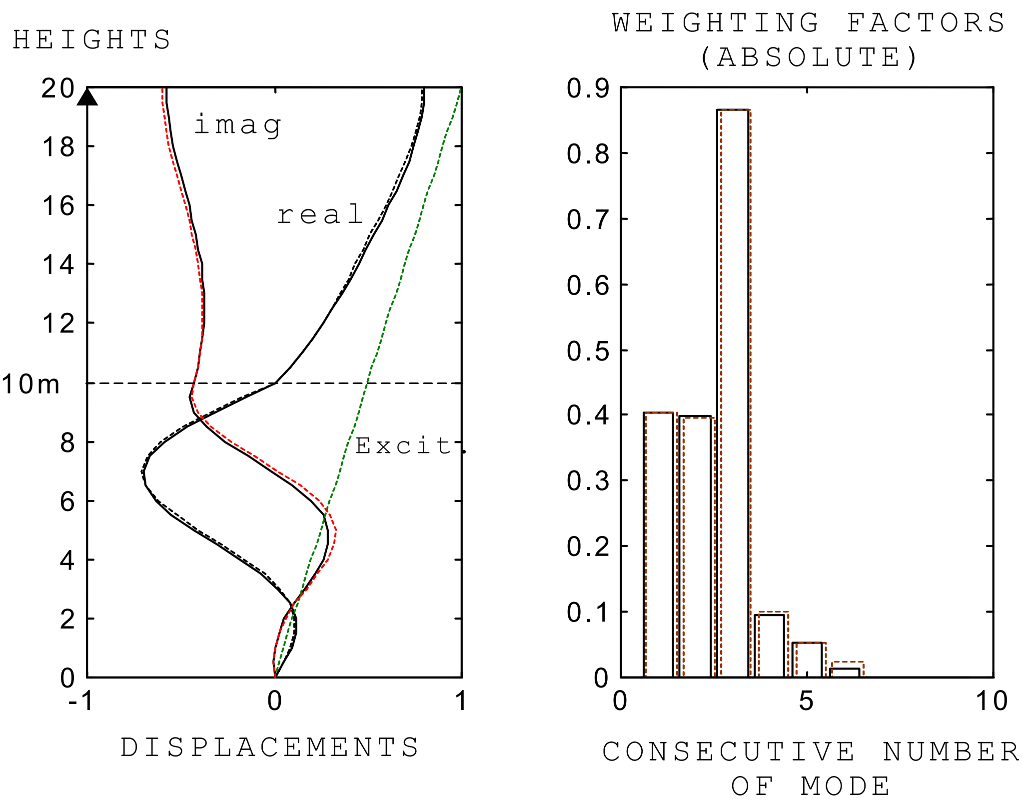

Figure 4. Displacements on the fictive boundary and their weighting factors. Exact values - solid lines, computed - dashed lines.

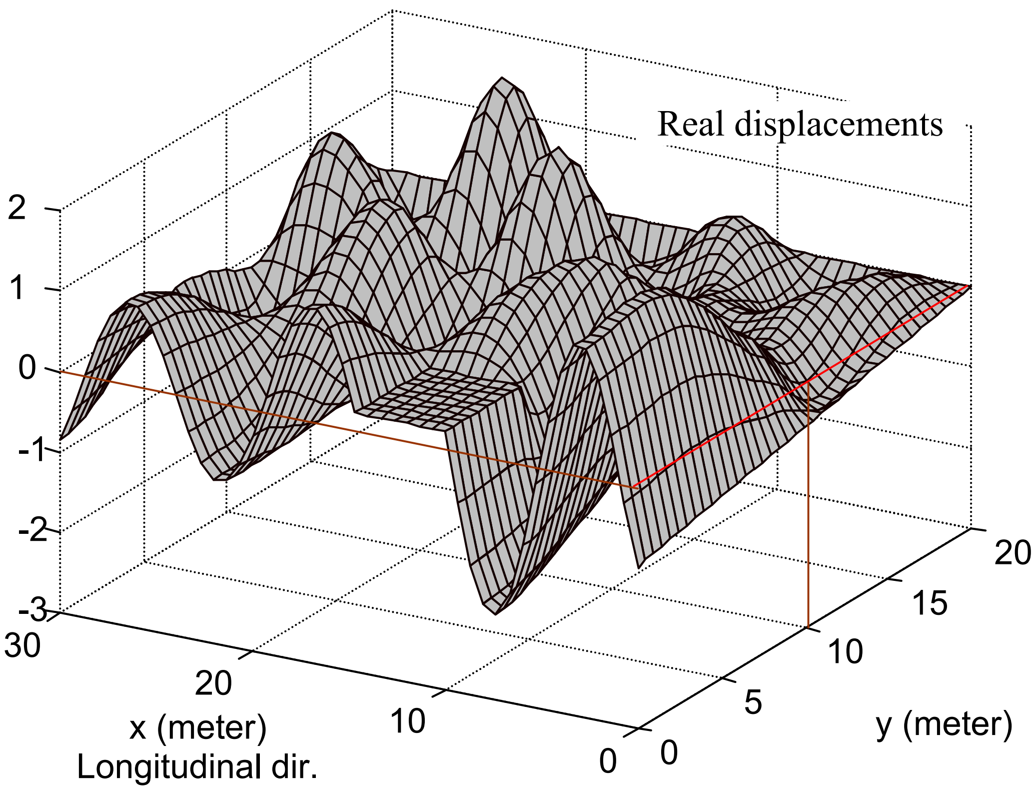

Figure 5. Displacements field.

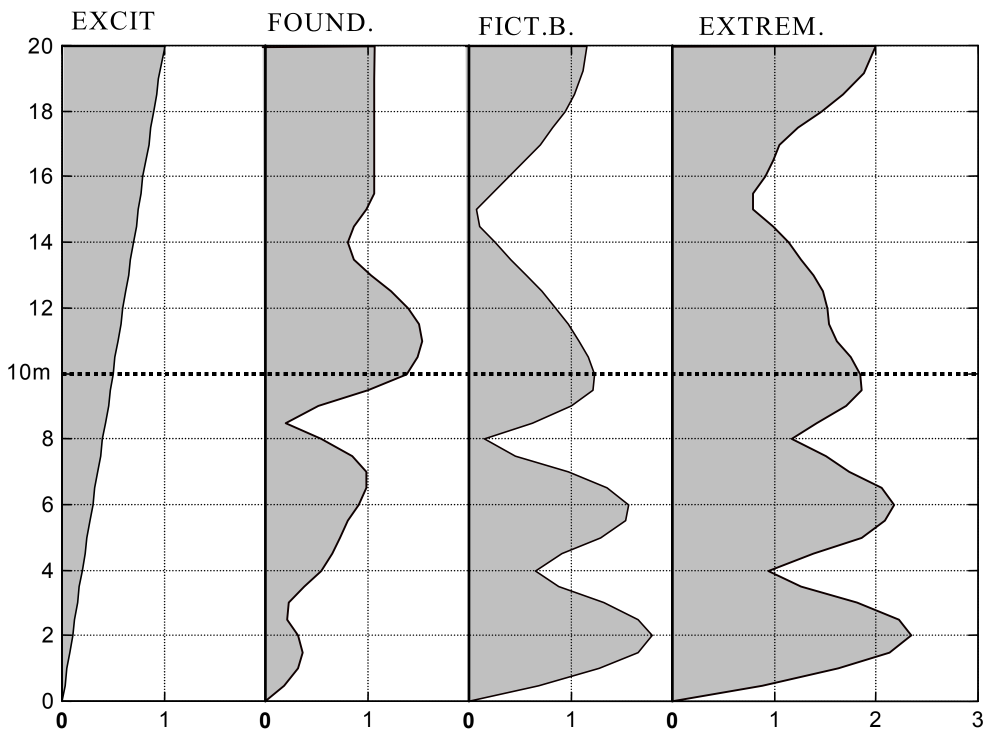

Figure 6. Absolute displacements profiles