An Engineering Model of Coils and Heat Exchangers for HVAC System Simulation and Optimization

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Abstract: - This paper presents a technique for developing a simple yet practicable model that can accurately replicate equipment performance of cooling coil units and heat exchangers. The modeling technique is based on energy balance and heat transfer principles. Only three or two parameters need to be fit in the developed model. Catalog or commissioning information is used to estimate the best value of the model parameters by least squares methods. The technique is illustrated for the heat transfer properties of a specific set of chilled water coiling coils. It is shown that the method is robust and gives a good match to the real performances over the entire operating range compared with the existing methods. This model is expected to have wide applications in real-time control and optimization of Heating, Ventilating, and Air-Conditioning (HVAC) systems.

Key-Words: - Coil model; Parameter identification; HVAC system; Simulation; Optimization

1 Introduction

Air-Conditioning systems provide a specified ambience for the occupants with comfortable temperature, humidity, etc. In such systems, cooling coils play an essential role (skimin 1995, Kreith & West 1997). The performance of the cooling coils, which is embodied through their heat transfer properties in practice, directly influences the performance of HVAC (Heating, Ventilating and Air-Conditioning) systems. Therefore, a simple, practicable yet accurate engineering model is very important for system control and optimization applications.

Considerable effort has been directed toward the development of models for coils or heat exchangers. A wide range of models for heat exchangers is currently available. By the degree of complexity and empiricism incorporated, these models can be classified as theoretical or designing models, and empirical or engineering models. The theoretical or designing models usually are complex and detailed, which are based on fundamental heat and mass transfer relations and may require details on construction of a cooling coil unit that are not often available from manufacturers' product catalogs. This type of models is usually used in the coil or heat-exchanger designing process, and can be a theoretical basis for designing engineer. One of the cooling coil models in the ASHRAE HVAC 2 Toolkit (Brandemuehl 1993) requires the dimensions of the fin and the tube thickness, diameter, and spacing as inputs in order to calculate the heat transfer coefficients. This model is only useful in providing performance over a limited range of the geometric parameters. Another fundamental approach to fitting catalog data is to calibrate a fundamental model using the catalog performance and then use heat transfer relations to estimate performance at other conditions. The effectiveness model of Braun et al. (1989) is one of such model. However, the formulations limit the prediction to the fluid types and temperature ranges used in calibration. In contrast to theoretical models, empirical or engineering models are simple and require few geometric specifications. In the empirical model proposed by Stoecher (1975), the performance is given by a second order correlation using fifteen empirical constants. In order to estimate values for these empirical constants, a large number of catalog data points are required, often more than those given in typical product catalogs. In order to relax some of the assumptions and complications of theoretical models, Rabehl (1997,1999) proposed a model of heat transfer coefficient-area product, which is given in terms of the mass flow rate and fluid properties using three empirical constants. The three constants are determined by using at least sixteen performance data. The heat transfer equation derived through the heat transfer coefficient-area product is also very complicated; make it difficult for engineering applications.

This paper presents technique for modeling the performance of coils or heat exchangers. Based on heat transfer mechanism and energy balance principle, a model with only three unknown parameters representing the lumped geometric terms has been developed. The proposed model gives a good match to the real performances over the entire operating range compared with the existing methods.

2 Theoretical Analysis and Model Development

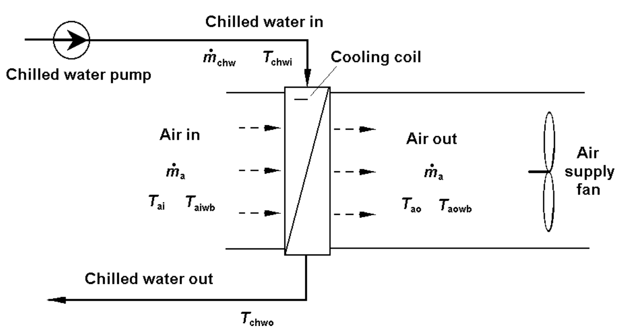

Fig. 1 Schematic of cooling coil

The schematic of AHU (Air Handling Unit) system is shown in Fig. 1. There are two loops in the AHU: chilled water loop and air loop. Chilled water flows from the cooling coil inlet to outlet forced by the chilled water pump with chilled water inlet temperature T_{chwi} and flow rate $\dot{m}_{chw}$. Through heat transfer with on coil air outside the cooling coil pipes, the temperature of the chilled water rises to T_{chwo}. Supply air flows from the air inlet to outlet of the cooling coil forced by the supply air fan. The dry-bulb temperature, web-bulb temperature and air flow rate of the on coil air are T_{ai}, T_{aiwb} and $\dot{m}_a$, respectively. Likewise, the off coil dry-bulb and wet-bulb air temperatures descends to T_{ao} and T_{aowb}, through heat transfer with chilled water in the cooling coil pipes.

During the course of the heat transfer, cooling load moves from hot air through metal pipe wall (usually the materials are copper or aluminum) into chilled water. As shown in Fig. 2, the heat transfer process can be classified as three parts: chilled water convection, metal conduction, and hot air convection. The cooling load can be calculated by overall heat resistant according to heat transfer theory and energy balance.

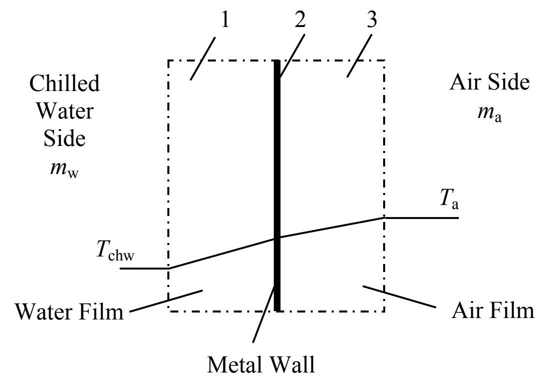

Fig. 2 heat transfer model of coil
1 Chilled water film: Convection; 2 Metal wall: Conduction; 3 Air film: Convection

The heat or cooling load moved by cooling coil can be given by:

$$Q = \frac{T_a - T_{chw}}{R_h}, \tag{1}$$

where Q is the cooling load or heat to be transferred by cooling coil;
T_a is the temperature of hot air;
T_{chw} is the temperature of chilled water;
R_h is the overall heat resistant of heat transfer;

Theoretically, the overall heat resistant R_h consists of the three parts. However, as the materials of metal wall used in the cooling coil are a good conductor of heat, the conduction effect can be neglected. Therefore:

$$R_h = R_1 + R_3, \tag{2}$$

where R_1 is the heat resistant of the chilled water convection;
R_3 is the heat resistant of the air convection.

The term convection heat transfer refers to the heat exchanged between a surface and a fluid moving over the surface. The amount of heat thus transferred depends on the nature of the surface and fluid, its geometry, its velocity over the surface as well as the temperature differences.

The water and air in the chilled-water coil is moved mechanically by pump and fan. Therefore, both are forced convections. The value of the film coefficient, h, for forced convection depends on the diameter of the passage D, the fluid velocity v, the mean stream density ρ, and the following properties of the mean film temperature: viscosity μ, specific heat c_p, and thermal conductivity k.

As a result of dimensional analysis, the following equation has been developed [12]:

$$\frac{hD}{k} = C\left(\frac{D\rho v}{\mu}\right)^e \left(\frac{c_p \mu}{k}\right)^f , \quad (3)$$

The values of the constant C and the exponents e and f are difficult to determine exactly but the most widely accepted engineering values are: C=0.023, e=0.8, and f=0.4.

For steady stable flow, it is reasonable to assume that both the fluid density ρ, velocity v, and the product ρv (or the quality flow rate $\dot{m}$) remains a constant, provided that the flow area is constant. Moreover, μ, k are approximately be constant if the temperature difference is not too big. Consequently, for the specific chilled-water coil, the equation (3) can be turned to:

$$h = C\left(\frac{4\dot{m}}{\pi\mu D}\right)^e \left(\frac{c_p \mu}{k}\right)^f \frac{k}{D} = b\dot{m}^e , \quad (4)$$

where $b = \dfrac{C4^e c_p^f k^{1-f} \mu^{f-e}}{\pi^e D^{1+e}}$.

Therefore, for chilled-water coil:

$$R_h = \frac{1}{h_{chw} A_{chw}} + \frac{1}{h_a A_a} = \frac{c_{chw}\dot{m}_{chw}^e + c_a \dot{m}_a^e}{c_{chw} m_{chw}^e c_a m_a^e} , \quad (5)$$

where h_{chw} is the film coefficient of chilled water;
h_a is the film coefficient of air;
A_{chw} is the chilled-water-side convection heat transfer area;
A_a is the air-side convection heat transfer area;
$c_{chw} = b_{chw} A_{chw}$;
$c_a = b_a A_a$.

The heat transfer equation of the cooling coil is derived from equation (1) and equation (5):

$$Q = \frac{c_1 \dot{m}_a^e}{1 + c_2\left(\dfrac{\dot{m}_a}{\dot{m}_{chw}}\right)^e} (T_a - T_{chw}) , \quad (6)$$

where $c_1 = c_a$; $c_2 = \dfrac{c_a}{c_{chw}}$.

The forced convection heat transfer is very complicated and influenced by many factors. The approach of combining the property factors and geometric factors into the constants ---- characteristic parameters was used in the equation (6). Compared with the existing cooling coil model, the equation (6) is characterized by fewer characteristic parameters and simplicity. If e =0.8 (engineering rule of thumb value) is adopted, the model has only two parameters c_1 and c_2 to be determined. The model is linear in terms of establishing the model parameters and the linear least-squares method can be used to estimate the model. Otherwise, the model has three parameters and a non-linear least-squares method is required to determine the model.

3 Model Identification

For the two-parameter model, the linear least-squares method proposed by Yong (1970) and Strejc (1980) can be directly used to estimate the model.

For three unknown parameter model, the non-linear least-squares equation can be used as follows:

$$f(c) = \sum_{i=1}^{m} r_i^2(c)$$

$$= \sum_{i=1}^{m} \left(\frac{c_1 \dot{m}_a^e}{1 + c_2\left(\dfrac{\dot{m}_a}{\dot{m}_{chw}}\right)^e} (T_a - T_{chw}) - Q_i \right)^2 , \quad (7)$$

where $f(c)$ = the sum of the squares of the residuals between evaluated and experiment data;
$c = \begin{bmatrix} c_1 & c_2 & e \end{bmatrix}^T$: the parameter vector to be determined;
$r_i(c)$: the residuals between evaluated and experiment data;
m: the number of fitting points;

Q_i : experiment data (cooling load).

To find optimal solution for Equation (7), Levenberg-Marquardt method (Wolfe, M.A., 1978; Nocedal, J. and S.J. Wright, 1999) can be used. The method incorporates a constant or variable scalar number to deal with problems related to singularity and is an effective algorithm for small residual problems. The downhill iteration direction can be obtained by solving the following equation:

$$(J^{(k)^T}(c)J^{(k)}(c)+\lambda^{(k)}I)P^{(k)}(c)=-J^{(k)^T}(c)r^{(k)}(c) \quad (8)$$

where $r(c)=\begin{bmatrix} r_1(c) & r_2(c) & \cdots & r_m(c)\end{bmatrix}^T$,

$\lambda^{(k)} \geq 0$ is a scalar, I is the identity matrix of order *3* for this particular three parameter model and Jacobian matrix J is defined as:

$$J=\begin{bmatrix} \frac{\partial r_1}{\partial c_1} & \frac{\partial r_1}{\partial c_2} & \frac{\partial r_1}{\partial e} \\ \frac{\partial r_2}{\partial c_1} & \frac{\partial r_2}{\partial c_2} & \frac{\partial r_2}{\partial e} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \frac{\partial r_m}{\partial c_1} & \frac{\partial r_m}{\partial c_2} & \frac{\partial r_m}{\partial e} \end{bmatrix}, \quad (9)$$

For a sufficiently large value of $\lambda^{(k)}$, the matrix $J^{(k)^T}J^{(k)}+\lambda^{(k)}I$ is positive definite and $P^{(k)}$ is then in a descent direction. Therefore, suitable values should be given to $\lambda^{(k)}$ in the course of iteration. For $\lambda^{(0)}=0.01$ and $\nu=10$, it can be specified as:

$$\lambda^{(k+1)}=\begin{cases} \lambda^{(k)}/\nu & \text{if} \quad f^{(k+1)}<f^{(k)} \\ \lambda^{(k)}\nu & \text{if} \quad f^{(k+1)}>f^{(k)} \end{cases}, \quad (10a)$$

$$c^{(k+1)}=c^{(k)}+P^{(k)}, \quad (10b)$$

The iteration ends if $|c^{(k+1)}-c^{(k)}|<\mu$, where μ is a predefined positive number (usually in the range of 1×10^{-6}-1×10^{-5}).

4 Experiments

The test is conducted on a pilot centralized HVAC system as shown in Picture 1. The system has three chillers, three zones with three AHUs, three cooling towers and flexible partitions up to twelve rooms. All motors (fans, pumps and compressors) are equipped with VSDs. The system is made very flexible to configure these three units to form different schemes.

Picture 1. Pilot plant of Centralized HVAC systems

The cooling coils for the system are two rows with the dimension of 25cm × 25cm × 8 cm. The measurement signals for the experiment are the water and air flow rates, on-coil air dry-bulb/web-bulb temperatures, coil inlet and outlet water temperatures. The experiment is conducted under the following conditions:

1) All fresh air is used to keep a relatively constant on-coil air temperature.

2) The chilled water supply temperature is fixed, the cooling load variation is achieved through the air and water flow rates.

In order to obtain the best model by using the proposed algorithms and the experiment data, models with two parameters, variable two variable parameters and three parameters are analyzed. To compare the fitting results, the fitness of the model is evaluated through the relative error, which is the ratio of the root-mean-square error to the mean value of the heat transfer, that is:

$$R_e=\frac{RMS_e}{\frac{1}{N}\sum_{m=1}^{N}Q_{m,\exp er}}\times100\%, \quad (11)$$

$$RMS_e=\sqrt{\frac{\sum_{m=1}^{N}(Q_{m,cacul}-Q_{m,\exp er})^2}{N}}, \quad (12)$$

where RMS_e is root-mean-square error;
N is the number of fitted points;

$Q_{m,cacul}$ is the cooling load predicted by the model;
$Q_{m,exper}$ is the cooling load acquied by experiments;

Two parameters:

In the case of two unknown parameters, the energy balance equation is given as Equation (6b) with c_1 and c_2 are fitting parameters, and e=0.8. The fitting results are c_1= 0.6484 and c_2=0.5104.

The results of the parameter estimation are shown in Fig. 3, where the calculated heat transfer rate is plotted as a function of the experiment heat transfer rate. The data points used cover the entire operating range of the equipment cooling load capacity. R_e=7.25 %.

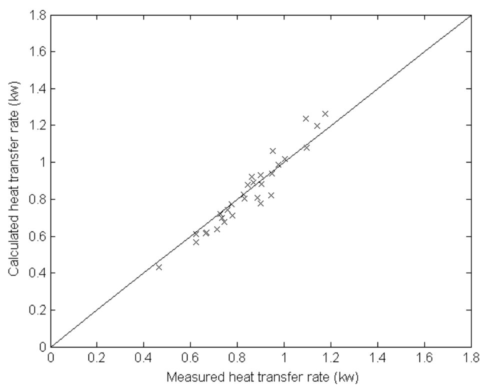

Fig. 3 Heat transfer rate calculated versus measured for two parameters case

Two variable parameters:

In order to improve the model accuracy, the overall operation range is divided into three regions and each region has a set of parameters as shown in Table 1 and Fig. 4.

Table 1. Two Variable Parameters Model

Minimum Cooling Load (kw)	Maximum Cooling Load (kw)	C_1	C_2
0.40	0.75	0.7132	0.5402
0.75	1.00	0.7076	0.6499
1.00	1.20	0.7403	0.9183

In this case, R_e=3.45%.

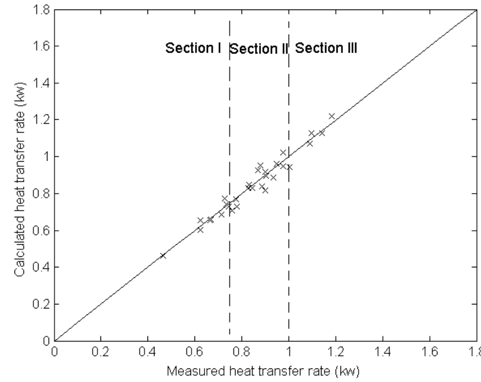

Fig. 4 Heat transfer rate calculated versus measured for two variable parameters case

Three Parameters:

The results of three fitted parameters, c_1, c_2, and e are c_1 = 0.4533, c_2=0.7021, e = 0.6074, respectively.

The results of the parameter estimation are shown in Fig. 5, where the calculated heat transfer rate is plotted as a function of the experiment heat transfer rate. R_e=2.62%.

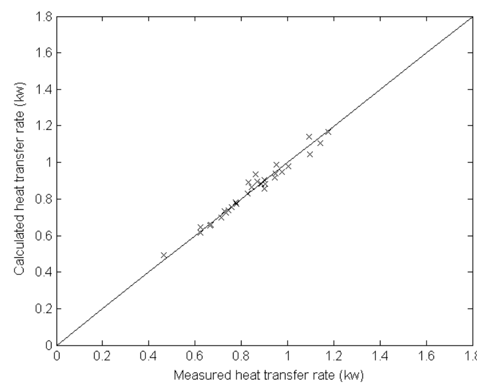

Fig. 5 Heat transfer rate calculated versus measured for three parameters case

Comparing Fig. 3, 4 and 5, it can be easily seen that:

1) If the cooling load variation is small, the simplest fitting model of two parameters gives a relatively reasonable answer.

2) For wide variation of cooling loads, the two variable parameters model provides a relatively simple and accurate solution.

3) The three-parameter model is the best in terms of minimum square error over entire operation range but with the price of more complicated calculation.

5 Conclusions

A simple engineering model for cooling coil unit based on the energy conservation and heat transfer principles was derived. Unlike the existing cooling coil model, the method captures the geometric effect without requiring geometric specifications. The proposed model using experiment data is simple yet accurate, and appropriate for application in engineering practice. Consequently, on-line determination of the model parameters becomes simple. The optimization problems of HVAC system actually are ones that set points are looked for. Cooling coils connect chilled water loop and air loop based on heat transfer and energy balance. Therefore, the cooling coil model is the constraint condition in the course of the energy minimization of the two loops. Further work in the area will concentrate on the test on the simulation and energy optimization of HVAC system based on the proposed model.

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