Bragg grating synthesis by Fourier transform

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Abstract: - We present a method for the synthesis of Bragg gratings using a Fourier Transform-based algorithm. The algorithm has an easy implementation, yields to reasonably accurate results and is fast due to the use of FFTs. Moreover, it resolves partially some of the problems faced by other Fourier transform methods when applied to high reflection problems.

Key-Words: - Bragg gratings, inverse problems, optical fiber devices

1 Introduction

Bragg gratings play a major role in lightwave devices. The simplest Bragg grating has a periodic uniform variation of the coupling function and closed equations can be found to relate the coupling function and the spectral response. In many applications, this kind of devices needs an aperiodic slow-varying coupling function to obtain a particular spectral response. In this case there are not closed equations to relate the coupling and spectral functions and so the problem of inferring the grating profile from a specified spectrum is not easy. This problem is often referred to as the inverse scattering of grating synthesis problem.

Electromagnetic inverse scattering (IS) techniques offer a great variety of possibilities for the design of gratings. The simplest approach exploits the approximated relationship between the Fourier Transform of the coupling grating function and the spectral grating reflectivity (see for example [1]). This approach is called first order Born approximation and it only takes into account the first reflection inside the grating. So, for high reflectivity this leads to poor approximation results. Other methods based on Fourier Transform use iterative calculation [2], but that increases the computational cost. There are also other algorithms based on Gel'Fand-Levitan-Marchenko Coupled Equations [3-4], on layer peeling techniques [5] or on direct integration of the coupled mode equations [6]. Many of them are also based on iterative techniques and so they are not cost-effective.

In this paper, we propose a straight-forward and very fast Fourier Transform algorithm resolving the high reflectivity problems of other Fourier Transform methods by means of a non-linear renormalization technique.

2 Theory

It is well known that the reflectivity of a Bragg filter can be solved from the Ricatti equation [1]:

$$\frac{d\rho}{dz} = -j2\Delta\beta\rho - j\kappa(z)\left(1+\rho^2\right) \tag{1}$$

where ρ is the local reflectivity, $\kappa(z)$ is the coupling function and $\Delta\beta$ is the detuning from the central Bragg propagation constant.

We can separate the phase information from the module information in (1) yielding to

$$\frac{d\rho}{dz} = \left(|\rho|^2 - 1\right)\kappa(z)\sin[\varphi(\Delta\beta, z)] \tag{2.a}$$

$$\frac{d\varphi}{dz} = -\Delta\beta z - \kappa(z)\frac{1+|\rho|^2}{|\rho|}\cos[\varphi(\Delta\beta, z)], \tag{2.b}$$

where the local reflectivity has been expressed as

$$\rho(\Delta\beta, z) = |\rho(\Delta\beta, z)|e^{j\varphi(\Delta\beta, z)}. \tag{3}$$

The equation system (2) is not easily solved, but if the second term of the differential equation (2.b) is neglected the phase can be calculated as

$$\varphi(\Delta\beta, z) = 2\Delta\beta z + \varphi_0. \tag{4}$$

This approximation is correct for low reflectivity as the variation of the phase is very fast and therefore there is no coupling between forward and backward waves. Only when the local reflectivity is high the second term of equation (2.b) must be taken into account because it represents a dispersion inside the grating. This dispersion is related with the optical band-gap of the grating. Inside this band-gap the grating acts as a distributed mirror [8].

Using equation (4), the first differential equation of the system (2) can be resolved as [7]:

$$|\rho| = tanh\left(\left|\int_{-\infty}^{+\infty} \kappa(z) e^{-j2\Delta\beta z} dz\right|\right) \qquad (5)$$

This approximation is correct while the FWHM bandwidth of the approximation is wider than $2\kappa_0$, where κ_0 is the maximum value of the coupling function [7]. This limit is related with the optical band-gap and the dispersion. While the dispersion inside the band-gap is low the approximation is correct, but when the dispersion is high the grating acts as a mirror so inside the band-gap there is high reflectivity. In this case the Fourier Transform predicts a narrower bandwidth than the real one.

On the other hand, it has been demonstrated [9] that this high reflectivity effect can be corrected by modifying the phase function, and this way the reflectivity can be calculated as:

$$|\rho(\Delta\beta)| = tanh\left(\left|\int_{-\infty}^{+\infty} \kappa(z) e^{-jf(\Delta\beta)z} dz\right|\right) \qquad (6)$$

where $f(\Delta\beta)$ is the modified phase function taking into account the dispersion of the optical band-gap. Thus, for high reflectivity, equation (5) provides the spectral reflectivity but it is compressed due to the dispersion. This is shown in Fig. 1 where the spectral response of a high reflectivity uniform grating calculated with a transfer matrix method and with the unmodified Fourier transform method are displayed. The Fourier transform method predicts a bandwidth narrower than the transfer matrix method, although the general shapes of the spectral function are equal.

Equation (6) can be used to calculate the inverse problem. First of all we need to know the phase function $f(\Delta\beta)$ to be used in the Fourier Transform.

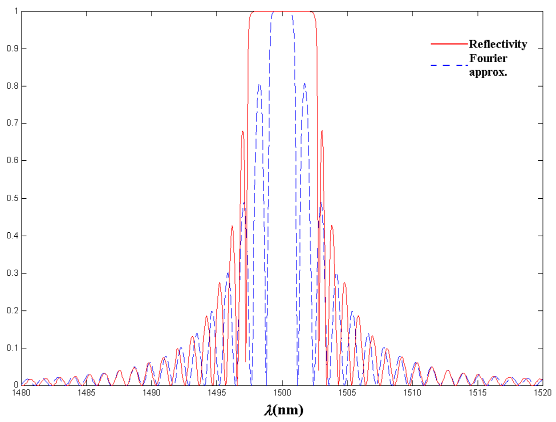

Fig 1. Reflectivity obtained with Fourier transform method versus reflectivity obtained with transfer matrix method. The Fourier transform method predicts the same function shape but compressed.

This phase function is the phase of the spectral reflectivity as shown in the system equation (2). So if the module and phase spectral information are given we can solve the inverse problem because the phase information will be used to compress the reflectivity and this way to obtain that reflectivity obtain with the equation (5).

3 Problem Solution

The Bragg gratings are causal filters and, in this way, there is a relationship between the module and the phase of the spectral filter response. Thus we cannot select any phase for a module. So, it is necessary to calculate the phase information from the module function. We can use for this task the algorithm presented in [10]. Since we have all the necessary information we can resolve the problem in the next five steps:

1. The phase function is calculated from the desired module reflectivity. The phase information represents the phase along the length of the filter, that is:

$$\varphi(\Delta\beta, 0) = f(\Delta\beta)L \qquad (7)$$

where L is the length of the filter.

2. The phase information is given between $-\pi$ and π. But we need a monotonically increasing function to renormalize (compress) the reflectivity spectrum. As we say before, the important information of the phase is located inside the optical band-gap. Thus, the central band-gap remains unaltered, and out of the central band-gap it is established as an asymptotic function to the straight line $2\Delta\beta L$. Since L is a free

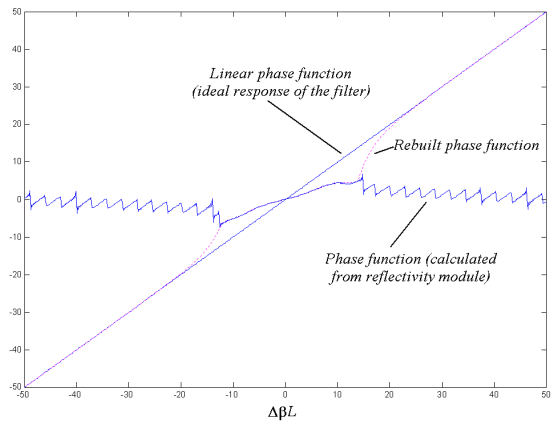

Fig. 2 Phase of the filter calculated from reflectivity module. Ideal phase function (when there is no dispersion) and rebuilt of the phase function to obtain a monotonically increasing function.

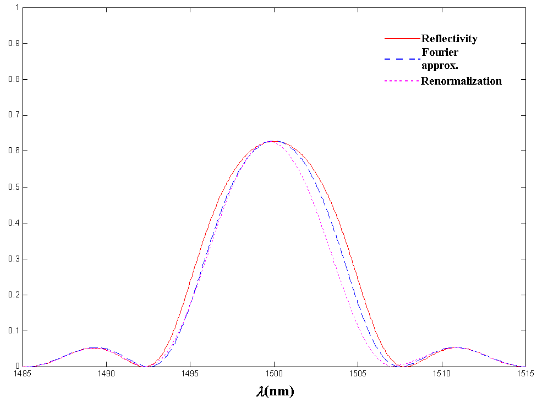

Fig 3. Compression from the given reflectivity to the Fourier transform method prediction expected for a uniform low reflectivity filter.

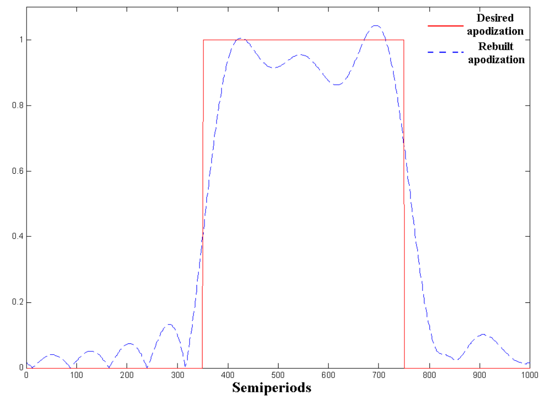

Fig 4. Desired coupling function and rebuilt coupling function for a uniform low reflectivity filter.

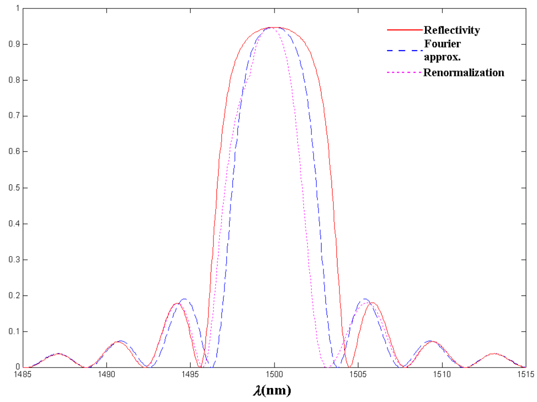

Fig 5. Compression from the given reflectivity to the Fourier transform method prediction expected for a uniform medium reflectivity filter.

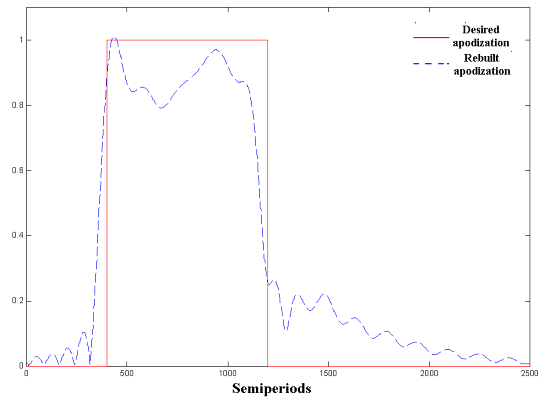

Fig 6. Desired coupling function and rebuilt coupling function for a uniform medium reflectivity filter.

design parameter, the slope of the line can be calculated from the phase function outside of the optical band-gap, because in that region the waves propagate (inside they are evanescent [8]) and so the phase can be expressed by equation (4). In Fig. 2 it is shown an example of the reconstruction of the phase function.

3. Once this phase function has been calculated, the spectral reflectivity module must be compressed to approximate the real response to that predicted by the Fourier transform method. This way we obtain:

$$\left|\int_{-\infty}^{\infty} \kappa(z) e^{-j2\Delta\beta' z} dz\right| = tanh\left(\left|\rho(\Delta\beta')\right|\right) \qquad (8)$$

$$2\Delta\beta' = f(\Delta\beta) \qquad (9)$$

4. Now, in order to apply the inverse Fourier transform, it is necessary to calculate the phase information. Again we use the causality property. It must be noted that it is not required to find a monotonic phase function.

5. The inverse Fourier transform is calculated, and the result is the coupling function.

All this steps have low computational cost and this way the global algorithm is very fast.

4 Results

In order to prove the performance of the method we are going to try to rebuilt different coupling functions. The first is a uniform coupling function of low reflectivity. In Fig. 3 it is shown the reflectivity spectrum calculated for the desired coupling function (that is our desired reflectivity), the reflectivity spectrum predicted by the Fourier transform method and the approximation from the desired reflectivity to the Fourier reflectivity by the phase renormalization.

Since it is a low reflectivity case there is not a great deviation between them. Thus we can expect a good rebuilt of the coupling function as it is shown in Fig. 4.

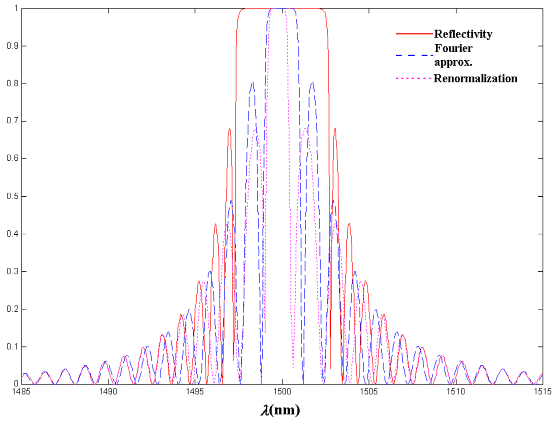

Fig 7. Compression from the given reflectivity to the Fourier transform method prediction expected for a uniform high reflectivity filter.

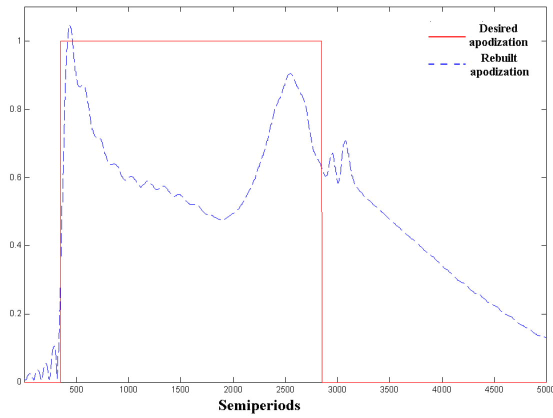

Fig 8. Desired coupling function and rebuilt coupling function for a uniform high reflectivity filter.

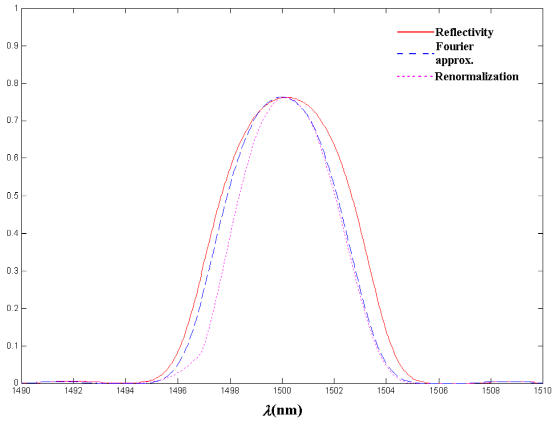

Fig 9. Compression from the given reflectivity to the Fourier transform method prediction expected for a triangular medium reflectivity filter.

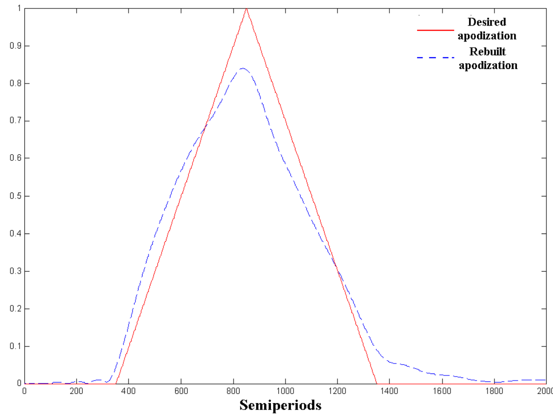

Fig 10. Desired coupling function and rebuilt coupling function for a triangular medium reflectivity filter.

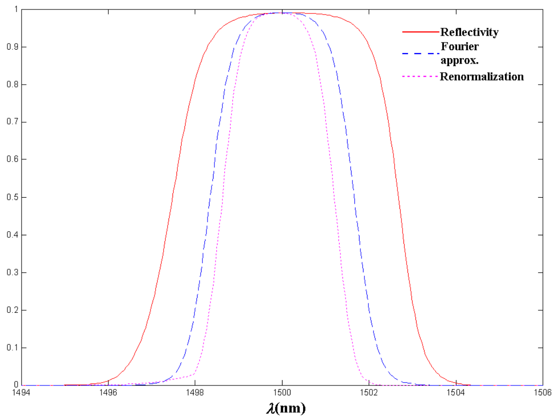

Fig 11. Compression from the given reflectivity to the Fourier transform method prediction expected for a gaussian high reflectivity filter.

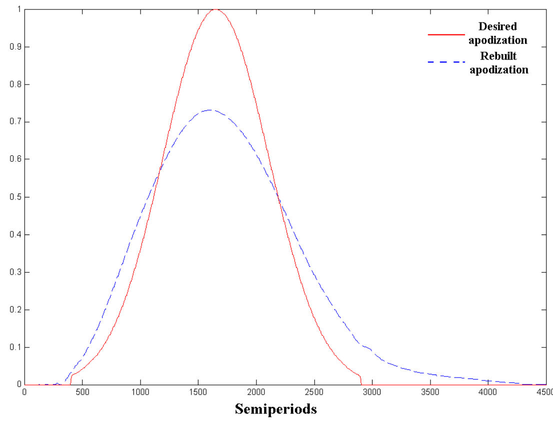

Fig 12. Desired coupling function and rebuilt coupling function for a gaussian high reflectivity filter.

Next we test with a uniform coupling function of medium reflectivity (the FWHM bandwidth of the Fourier transform is approximately the band-gap). In Fig. 5, we can see the calculated spectrum and in Fig. 6 the rebuilt coupling function. In this case there is a deviation from the ideal coupling function. This deviation cannot be attributed to a wrong renormalization but to the high sidelobe levels of this particular reflectivity function. As we will see in next examples, for high reflectivity the length of the filter is not calculated properly, however the shape is correctly obtained. Thus, when the specified reflectivity has high sidelobe level the rebuilt process will not work properly.

This is shown in Figs. 7 and 8. Now the coupling function is not correctly rebuilt although the reflectivity spectrum is correctly compressed.

Figs. 9 and 10 show the result for a triangular apodization for medium reflectivity. In this case, it can be seen that the shape of the apodization profile is correctly rebuilt, and that the main deviation is centered in the length and strength of the coupling function. This is due to the renormalization procedure, which leads to a narrower bandwidth than the Fourier transform approximation, affecting mainly to the length of the filter.

In Figs. 11 and 12 it is shown the case of a high reflectivity gaussian coupling function. It must be noted that, unlike the uniform coupling function, the shape of the coupling function is correctly calculated. This is due to gaussian reflectivity has a very low sidelobe level. However there is a deviation for the length of the filter.

The deviation on the length and strength of the coupling factor can be corrected with a more fine calculation of the limits of the optical band-gap.

5 Conclusions

A non-iterative algorithm for resolving the inverse scattering problem based in the Fourier transform has been presented. The algorithm resolve partially the high reflectivity problems found in other methods of the same kind. The method presented here (1) is very easy to implement (2) yields to reasonably accurate results and (3) it can take advantage from the numerous filter design techniques based on the Fourier Transform.

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