Soliton Dynamics in Dispersion-Varying Compensating Fibers (DVCF)

Alessandro M. Melo[1], M. C. Gouveia, and Henrique J. A. da Silva
Instituto de Telecomunicações, Faculdade de Ciências e Tecnologia,
Universidade de Coimbra, Polo 2, P-3030-290 Coimbra
PORTUGAL

Abstract: - We investigated analytically and by numerical simulations the possibility of dispersion managed solitons in links with dispersion varying compensating fibers. We present an analysis where the conventional DCF (Dispersion Compensating Fiber) is replaced by a fiber with varying dispersion. We have studied the case of DDF (Dispersion Decreasing Fiber) and DIF (Dispersion Increasing Fiber), for the DVCF dispersion profiles. Results show that the soliton dynamics depends on the DVCF profile providing an evidence of system performance improvement.

Key-Words: - Soliton transmission, Dispersion management, Variational method, Soliton dynamics, System performance, DDF, Chirp.

1 Introduction

Dispersion-Managed solitons have been proven to be a potential alternative for the implementation of high-bit-rate long-haul optical communication systems [1-3].
The dynamics of the soliton pulse in such systems leads to an oscillating behavior of the pulse amplitude and width. This is sometimes referred to as breathing solitons [4]. Dispersion managed solitons have been shown to possess some advantages compared to conventional uniform fiber soliton systems (a fundamental soliton with the same time width corresponding to the same path-average dispersion). These include the energy enhancement factor [5] (that leads to an increase in signal-to-noise ratio), automatic soliton control [6] (due to reduced soliton interactions) and low FWM efficiency [7] (due to high local dispersion). In this paper we analyze a new scheme where the DCF is replaced by a DVCF with decreasing and increasing profiles. It will be shown analytically and by numerical simulations that the dynamic behavior of soliton pulses in such systems depends on the fiber profile which can be related to an improvement of the system performance. In our study we consider three cases for the dispersion coefficient of the compensating fiber: uniform (conventional DCF), exponential decreasing, and exponential increasing. In order to do a fair comparison as general as possible, the average dispersion of the DVCF is set to be equal to the dispersion of the uniform fiber.

2 Dispersion-managed Soliton Transmission

The propagation of solitons in optical fibers is described by the well-known Nonlinear Schrödinger Equation (NSE) that, for the case of periodically amplified systems, takes the form:

$$i\frac{\partial u}{\partial z}+\frac{1}{2}d(z)\frac{\partial^2 u}{\partial \tau^2}+|u|^2u= =-\frac{i}{2}\Gamma+i\left(\sqrt{G}-1\right)\sum_{m=1}^{N_A}\delta\left(z-mz_A\right)u \tag{1}$$

With the change of variables $u(z,\tau)=a(z)v(z,\tau)$, this equation can be rewritten in the form:

$$i\frac{\partial v}{\partial z}+\frac{1}{2}d(z)\frac{\partial^2 v}{\partial \tau^2}+a^2(z)|v|^2v=0 \tag{2}$$

$$\frac{da}{dz}=-\frac{1}{2}\Gamma a+\left(\sqrt{G}-1\right)\sum_{m=1}^{N_A}\delta\left(z-mz_A\right)a \tag{3}$$

where $a(z)$ takes into account the periodic loss-amplification dynamics, NA is the number of amplifiers, G is the gain in each amplifier, Γ is the normalized loss coefficient ($\Gamma=\left(\alpha_{dB}/4.343\right)L_D$), and $d(z)$ is the normalized dispersion coefficient. Here we use a normal dispersion-coefficient fiber to compensate (compensating fiber) a dispersion-shifted fiber (DSF) used as transmission fiber. We have therefore:

$$d(z)=\begin{cases}\beta_2^{DSF}/\beta_2^{ave} & \text{for the case of DSF,}\\ \beta_2^{DCF}/\beta_2^{ave} & \text{for the case of DCF and}\\ \beta_2^{DVCF}(z)/\beta_2^{ave} & \text{for the case of DVCF.}\end{cases} \tag{4}$$

with $\beta_2^{ave}=\left(\beta_2^{Comp}z_{Comp}+\beta_2^{Tr}z_{Tr}\right)/\left(z_{Comp}+z_{Tr}\right)$, where *Comp* and *Tr* stand for Compensating and Transmission fibers, respectively.
Figure 1 shows the dispersion profiles studied in our analysis.
The DVCF is defined as:

[1] Now with Technische Universität Berlin,
Fachgebiet Hochfrequenztechnik, Berlin - Germany

$$\beta_2^{DVCF}(z)=\begin{cases} C_F \exp\left(-\frac{\ln(\delta)}{L}z\right), & \text{decreasing profile} \\ C_F \exp\left(\frac{\ln(\delta)}{L}z\right), & \text{increasing profile} \end{cases} \quad (5)$$

where $z_{Tr} < z < z_A$, indicating that the DVCF is placed before the amplifiers, which can be considered as the worst case [8], and C_F is a correction factor responsible for adjusting the proper DVCF average dispersion. Without the correction factor (C_F = 1), the dispersion coefficient would decrease (DDF) or increase (DIF) monotonically from 1 to the final value $1/\delta$, after a length L of fiber.

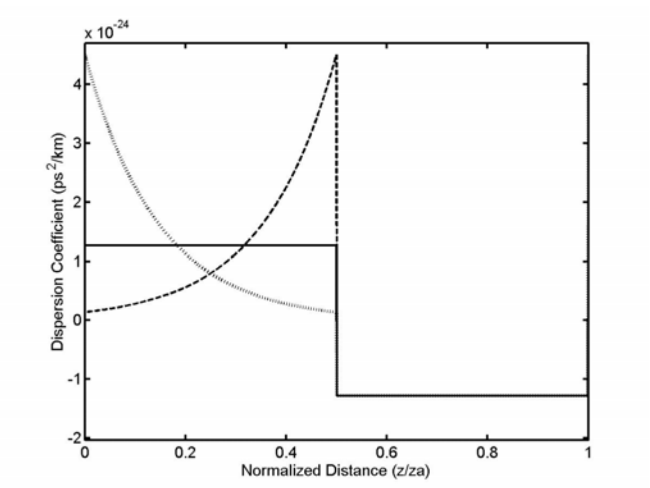

Fig.1 Plots of the dispersion profiles considered. (solid): uniform profile, (dotted): decreasing profile, (dashed): increasing profile.

In order to analyze the soliton dynamics in a fiber with varying dispersion, we have substituted the DCF (or DVCF with uniform profile) by a DVCF with both exponential decreasing and exponential increasing profiles. The DVCF dispersion profile is chosen in order to obtain an average value of the dispersion coefficient equal to the uniform dispersion coefficient of the DCF:

$$\frac{\int_{\xi_1}^{\xi_2} \beta_2^{DVCF}(z)dz}{\xi_2-\xi_1}=\beta_2^{DCF} \quad (6)$$

3 Results and Discussion

Equation (2) may be solved by using a variational approach [9]. We have chosen a general ansatz of the type [10]:

$$U(z,t)=A(z)f\left[t/B(z)\right]\exp\left[iP(z)+iC(z)t^2\right] \quad (7)$$

where $A(z)$, $B(z)$, $P(z)$ and $C(z)$ account for the complex amplitude, pulse width, phase and pulse chirp, respectively.

By applying the variational principle, the NSE reduces to a set of ordinary differential equations:

$$A(z)^2 B(z)=\text{constant} \quad (8)$$

$$\frac{dB}{dz}=2d(z)BC \quad (9)$$

$$\frac{dC}{dz}=\frac{d(z)K_1}{2B^4}-\frac{c(z)A^2K_2}{B^2}-2d(z)C^2 \quad (10)$$

where K_1 and K_2 are constants that depend on the shape of the input pulse. For *sech* pulses, $K_1=2K_2=4/\pi^2$.

We integrated equation 2 numerically, using the Split Step Fourier Method (SSFM). The system parameters used here were the full width at half maximum (FWHM) of the input pulse, $T_S=10$ ps, the amplifier spacing, z_A=100 km, the length of the compensating and transmission fibers, $z_{Comp}=z_{Tr}=50$ km, the dispersion coefficient of the transmission fiber, $\beta_2^{Transm}=-1.28\text{ ps}^2\text{/km}$, the dispersion coefficient of the compensating fiber with uniform dispersion, $\beta_2^{DCF}=1.26\text{ ps}^2\text{/km}$ and the fiber loss coefficient $\alpha_{dB}=0.25\text{ dB/km}$. The input pulse considered was $u(0,\tau)=\sec h(\tau)$.

Figures 2 and 3 show the evolution of the envelope (with data sampled at each amplifier) of the amplitude and time width, respectively, for the three profiles, obtained both with the SSFM and with the variational analysis. It may be seen that there is a good agreement between the two approaches. The differences observed between the calculated values obtained with the two methods used here are related to the soliton-shape change as the pulse propagates along the fiber [see p.ex.5]. This effect was not taken into account in the numerical analysis, because we have used an indirect measurement of the FWHM of the pulse, based on the calculation of the RMS width of the pulse. We have used the relation:

$$\frac{T_{FWHM}}{\sigma_0}=\text{constant} \quad (11)$$

where

$$\sigma_0=\left[\frac{\int_{-\infty}^{\infty} t^2\left|\sec h(t)\right|^2 dt}{\int_{-\infty}^{\infty}\left|\sec h(t)\right|^2 dt}-\left(\frac{\int_{-\infty}^{\infty} t\left|\sec h(t)\right|^2 dt}{\int_{-\infty}^{\infty}\left|\sec h(t)\right|^2 dt}\right)^2\right]^{1/2} \quad (12)$$

is the RMS width of the pulse.

Therefore, the differences observed in Figure 3 are a direct consequence of the assumption of a constant in equation 11, instead of a parameter that is a function of z. This function of z would vary very slowly and would take into consideration the variation of the pulse shape as it propagates. However, the constant chosen here produced a good fitting to the dispersion increasing profile, meaning that for this profile the soliton shape does not change considerably from its original shape in the range of values considered. It was chosen according to the values of the T_{FWHM} and σ_0 at the input of the fiber. In spite of the relative inaccuracy of this indirect method, the results obtained showed very good qualitative agreement.

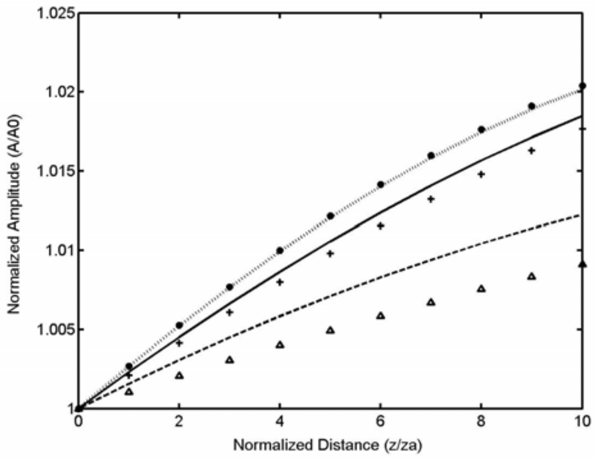

Fig.2 Amplitude envelope for the three dispersion profiles. (solid): uniform profile (variational method), (+): uniform profile (SSFM), (dotted): decreasing profile (variational method), (*): decreasing profile (SSFM), (dashed): increasing profile (variational method), (Δ): increasing profile (SSFM).

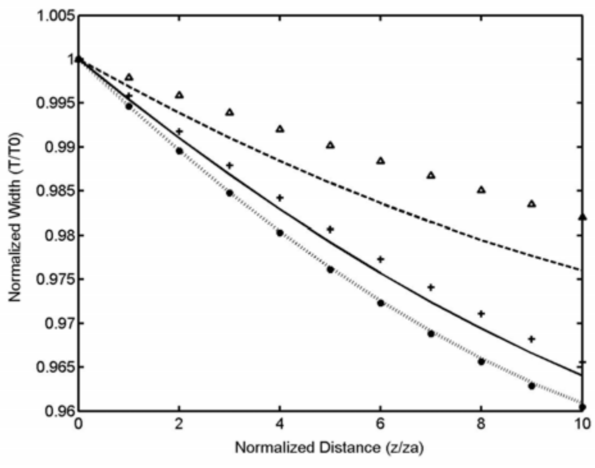

Fig.3 Width envelope for the three dispersion profiles. (solid): uniform profile (variational method), (+): uniform profile (SSFM), (dotted): decreasing profile (variational method), (*): decreasing profile (SSFM), (dashed): increasing profile (variational method), (Δ): increasing profile (SSFM).

It can be seen from figures 2 and 3 that the amplitude and time width evolution of the pulse are dependent on the profile of the compensating fiber. The pulse is compressed as it propagates along the fiber. This compression results in a deviation from the initial amplitude and width values which can be detrimental for the pulse detection at the receiver (Intensity Modulation – Direct Detection). Our analysis showed that the dispersion increasing profile presents less amplitude (see Fig. 2) and width (see Fig. 3) deviations when compared with the other profiles considered. On the other hand, the DVCF with decreasing profile produces the opposite behavior, leading to a bigger compression and more amplitude deviation, when compared with the pulse dynamics with the other profiles. This behavior can be understood if we look at the chirp accumulation during propagation. Figure 4 shows the accumulated chirp for the three profiles, calculated with the variational method. It can be seen that the chirp parameter grows faster with the DDF-profile than with the DIF-profile. As the chirp parameter increases positively this leads to pulse compression. This pulse compression is observed for every profile, but depends on the chirp evolution for each profile. Therefore, the profile-dependent behavior of the pulse as it propagates in the dispersion-managed link is related to the chirp evolution for each dispersion profile. Since one can relate a smaller amplitude and width deviation from its initial value to the use of a DVCF, an evidence of a system performance improvement technique based on the use of DVCF's instead of DCF's is reported.

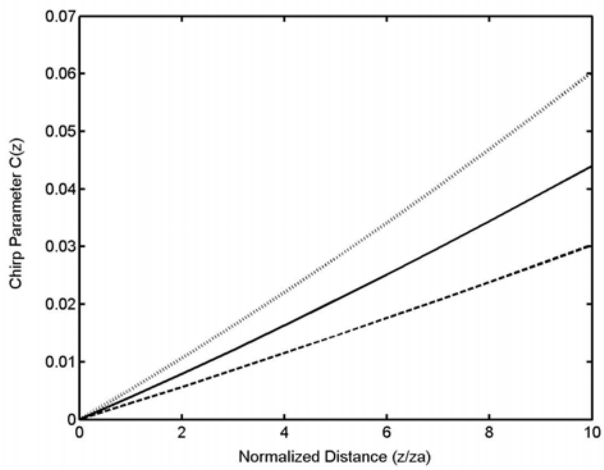

Fig.4 Accumulated chirp during propagation for three different dispersion profiles, calculated through the variational analysis. (solid): uniform profile, (dotted): decreasing profile, (dashed): increasing profile.

4 Conclusion

We performed a study of the dynamics of soliton pulses in dispersion-managed systems using a new scheme where the conventional compensating fiber (DCF with uniform dispersion profile) is replaced by a dispersion-varying compensating fiber (DVCF). The results obtained show that the pulse dynamic behavior depends on the dispersion profile of the compensating fiber, and that the chirp acquired by the pulse as it propagates is responsible for the different evolution of the three profiles considered. The chirp increases positively from the dispersion increasing profile to the dispersion decreasing one, with the constant profile in between. Therefore the preliminary analysis presented here indicates that it is possible to improve the system performance with a proper control of the chirp parameter evolution. This can be achieved through the use of dispersion-varying compensating fibers instead of conventional DCF's. One assumption concerning the system performance used here is that the pulse dynamics whose amplitude and width values deviates more to the input amplitude and width values would be more detrimental to the system performance in comparison to that one whose values are closer to the initial ones. The aim of this work is to provide evidence of a system performance improvement technique based on the use of DVCF. The quantitative influence of the DVCF profile on the system performance is subject of a future work. Furthermore, the use of an DVCF with average dispersion with the same value as to the DCF leads to a generalization of the procedure which can be applied to other dispersion maps. This would mean that for a conventional DCF with an arbitrary dispersion value the system performance could be always improved through the use of its DVCF counterpart (i.e., DVCF with average value of the dispersion coefficient equal to the uniform dispersion coefficient of the DCF) whose profile has to be chosen between an increasing or a decreasing one. Finally, the differences observed between the results obtained with the SSFM and with the variational analysis can be attributed to the assumption of a proportionality constant between the FWHM and the RMS pulse width. Despite of that, both methods showed very good qualitative agreement.

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