Need of Knowing Fiber Non-linear Coefficient in Optical Networks

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Abstract: - This paper describes the need of knowing fiber non-linear coefficient in global optical networks. The basic Kerr non-linearities that appear in silica-based optical fibers have been summarized, together with their implications for optical communication systems. In this paper, the non-linear coefficient has been discussed and its measurement techniques reviewed. The brief description of each method is given. This paper focuses on the comparison of different interferometric and non-interferometric measurement schemes in terms of their simplicity and versatility of testing various types of fibers. Accurate determination of fiber non-linearities is an important issue in the design of optical systems.

Key-Words: - optical fiber, fiber non-linearities, fiber measurements, non-linear coefficient, self-phase modulation, modulation instability, cross-phase modulation, four-wave mixing

1 Introduction

Non-linear effects have become significant at high optical power levels and have become even more important since the development of the erbium-doped fiber amplifier (EDFA) and wavelength division multiplexed (WDM) systems. By increasing information spectral efficiency [1], which can be done by increasing channel bit rate, decreasing channel spacing or the combination of both, the effects of fiber non-linearity come to play even more decisive role.

Although the individual power in each channel may be below the one needed to produce non-linearities, the total power summed over all channels can quickly become significant. The combination of high total optical power and a large number of channels at closely spaced wavelengths is ideal for many kinds of non-linear effects. For all these reasons it is important to understand non-linear phenomena and to be able to simply and accurately measure fiber non-linearities.

2 Non-linear effects in optical systems

The refractive index of silica, the major material of optical fiber, has a slight dependence on the intensity of the optical field. This dependence is known as the optical Kerr effect. The general expression for the refractive index *n* of silica includes a constant term n_0 and a power density dependent term n_2S , where n_2 is known as second-order refractive index.

$$n = n_0 + n_2 S \tag{1}$$

Refractive index is a dimensionless parameter, optical power density is measured in Watts per square meter and therefore the second-order refractive index has units of square meter per Watts. Typical values of n_0 and n_2 are 1,5 and 2,5 $\cdot 10^{-20}$ m²/W, respectively. Silica has one of the lowest n_2 of any optical material. It can easily be shown that high intensities are required to make the intensity dependent term comparable to the constant one. In spite of this, appearance of non-linear phenomena in single or multi channel communication systems is frequent. Actually, non-linearities can occur at reasonable powers of few dBm in the fiber because of large distances and small effective core area.

2.1 Self-phase modulation (SPM)

The SPM refers to the self-induced phase shift experienced by an optical field during its propagation in fiber [2]. The non-linear phase shift ϕ_{NL} is given by

$$\varphi = \frac{2\pi}{\lambda} \cdot \frac{n_2}{A_{eff}} \cdot L_{eff} \cdot P \cdot m , \qquad (2)$$

where *P* is optical power, A_{eff} is the effective fiber core area, and λ is the vacuum wavelength of the signal. The effective fiber length L_{eff} determines the distance where non-linear effects are stronger. The L_{eff} is given as $L_{eff} = (1 - e^{-\alpha L})/\alpha$, where *L* is the fiber length, α is the fiber attenuation coefficient. The polarization parameter *m* depends on the polarization characteristics of the fiber and the input signal polarization state.

For pulses in digital communication systems the phase is delayed at the pulse maximum relative to the wings. The effect of the non-linear phase shift is producing new frequencies and the power spectrum is broadening during signal propagation. The amount of broadening depends on the fiber length, peak input power and fiber dispersion. Greater frequency width then increases pulse spreading through group-velocity dispersion.

2.2 Modulation instability (MI)

The modulation instability is a phenomenon of spontaneous modulation of the continuous-wave (CW) laser. It refers to the selective amplification of noise and it occurs only in the anomalous dispersion regime (D>0).

Actually, the MI originates from the interplay between Kerr effect and anomalous dispersion, and gives rise to two spectral gain bands, symmetrically located with respect to the pump frequency $\omega_0 \pm \Omega$. The MI spectral gain coefficient is given by [3]

$$g(\Omega) = |\beta \Omega| \sqrt{2\omega_{\rm MI}^2 - \Omega^2} , \qquad (3)$$

where $\beta = -(\lambda^2/2\pi c)D$, $\omega_{\rm MI} = \sqrt{2\gamma P/|\beta|}$ (the modulation instability frequency), and *P* is optical power.

2.3 Cross-phase modulation (XPM)

When two or more optical waves propagate inside the fiber, the refractive index seen by a particular wave depends not only on the intensity of that wave but also on the intensity of other copropagating waves. The non-linear phase shift for the *j*th channel depends on the power of that and other channels [4] and is given by [3]

$$\varphi_{NL,j} = \frac{2\pi}{\lambda} \cdot \frac{n_2}{A_{eff}} \cdot L_{eff} \cdot m \cdot \left(P_j + 2\sum_{m \neq j}^M P_m\right), \quad (4)$$

where P_j is the channel power and M is the total number of channels. The factor 2 indicates that XPM is twice as effective as SPM for the same amount of power. Similar as SPM, XPM manifests as an alteration of the optical phase of a channel, which translates into intensity distortion through group-velocity dispersion.

2.4 Four-wave mixing (FWM)

Four-wave mixing is another effect produced by the intensity-dependent refractive index. It occurs when two or more wavelengths of light propagate together through an optical fiber. Providing a condition known as phase matching [5] is satisfied, light is generated at new frequencies using optical power from the original signals. The FWM generated power is given by [6]

$$P_{i,j,k} = \left(\frac{D}{3}\right)^2 \left(\frac{2\pi n_2 L_{eff}}{\lambda A_{eff}}\right)^2 P_i P_j P_k e^{-\alpha L} \eta, \quad (5)$$

where D is degenerescency factor and η stands for FWM efficiency. FWM has very serious implications for

multichannel WDM communication systems. The power in existing signals will be reduced as some is transferred to the new mixing products. If the new frequencies also happen to fall on allocated channels, then the overall system performance will be degraded by crosstalk.

Phase matching only occurs for particular combinations of fiber dispersion and signal frequencies. In particular, phase matching is achieved for signals with very similar frequencies propagating near the zero dispersion wavelength of the fiber.

When N channels are launched in the fiber, the number of generated mixing products is [7]

$$M = \frac{N^2}{2} \cdot (N-1). \tag{6}$$

3 Non-linear coefficient measurement methods

The parameter that is normally measured when investigating fiber non-linearities is ratio of the secondorder refractive index and the effective core area. The ratio is known as the non-linear coefficient and is defined as [3]

$$\gamma = \frac{2\pi \cdot n_2}{\lambda \cdot A_{eff}},\tag{7}$$

where n_2 is the fiber non-linear refractive index introduced in Eq. (1). The determination of second-order refractive index n_2 usually involves the measuring of the non-linear coefficient and the effective area first and then using Eq. (7) to calculate n_2 .

Several methods have been proposed for the measurement of non-linear coefficient. It can be measured by using a number of interferometric or non-interferometric techniques based on fiber non-linear effects. SPM, MI, XPM and FWM are all used.

Heretofore, European COST 241 Action (Characterization of Advanced Fibers for the New Photonic Network) has dealt with non-linear refractive measurements in dispersion shifted fibers [8]. Recently, the ITU-T (International Telecommunication Union) non-linear coefficient round Robin measurements has reported intercomparison of (n_2/A_{eff}) measurement in various optical fibers [9]. It uses different SPM and XPM methods and it is planning to expand on other measurement methods in the future.

3.1 Interferometric methods

Interferometric methods are based on interferometric detection of the phase shift caused by SPM or XPM in Fiber Under Test (FUT) of which non-linear coefficient is to be measured. The disadvantage of interferometric detection schemes is related to its susceptibility to the environmental perturbations that lead to a poor stability. The measurement accuracy of these schemes strongly depends on the measurement conditions.

3.1.1 Sagnac interferometer

The measurement technique shown in Fig. 1 uses the principle of the Sagnac interferometer [10, 11]. After amplification in EDFA the laser pulse is split into two equal parts by a 50:50 splitter. The two pulses created in this way are counterpropagating through the FUT. One of the pulses is attenuated before entering the fiber and therefore induces a smaller non-linear phase shift. The polarization controller (PC) inserted in the interferometer has to be adjusted in such a way that induced non-linear phase shift is maximized.



Fig. 1 Setup based on SPM in Sagnac interferometer

3.1.2 Michelson interferometer

The FUT and the reference fiber with the same length constitute the two arms of a Michelson interferometer [12]. The continuous wave (CW) probe feeds both arms of the interferometer. The output interference is detected by the oscilloscope for pulse width measurement. The FUT is fed by the pump beam, which induces a phase shift by XPM on the probe signal. The pump signal is modulated by a square wave and amplified by EDFA. At the end of FUT the probe beam is reflected and propagates backwards through the test fiber. The advantage of this method is intrinsical insensitivity to the polarization state [13]. The necessity for difficulty associated with balancing of the interferometric arms is the disadvantage.



Fig. 2 Setup based on XPM in Michelson interferometer

3.1.3 Self-aligned interferometer

This method is based on interferometric detection of phase shift using a self-aligned interferometer with a

Faraday mirror, which completely removes the fluctuations due to the environmental perturbations [14, 15, 16]. Amplified laser pulses are split into interferometer arms, which are different in length. One of the exit arms of the second coupler is connected to the FUT. After being reflected at the Faraday mirror, the pulses return back towards the first coupler. Owing to the difference in path length, three different arrival times can be discerned. The power of middle pulse, which is due to the interference between the short-long and long-short pulses, depends on the non-linear phase shift experienced in the FUT. This method is independent of the FUT length even in the presence of large group-velocity dispersion.



Fig. 3 Setup based on SPM in self-aligned interferometer

3.1.4 Mach-Zehnder interferometer

In this scheme FUT is inserted in one arm of the Mach-Zehnder interferometer [17, 18]. The pump signal is split by a coupler and then recombined by a polarizing beam combiner (PBC). The pump signal is composed of two incoherent optical beams with the same intensity and orthogonal polarization. Both pumps copropagate with the probe in the FUT inducing a non-linear phase shift which is detected by the oscilloscope. This method is intrinsically insensitive to the polarization state.



interferometer

3.2 Non-interferometric methods

3.2.1 Self-phase modulation method

Basically, there are two non-iterferometric measurement techniques involving the use of SPM [19]. One uses relatively low average power in the form of very short pulses, produced by a pulsed laser source with high peak intensity [20, 21, 22, 23, 24], while the other uses relatively high power beat signal between the two CW lasers [25, 26, 27].

The non-linear phase shift introduced to the pulse by SPM in the FUT is measured and used to derive the nonlinear coefficient by Eq. 2. Its accuracy can be affected by pulse broadening due to chromatic dispersion. So, the measurements are made only in fibers that are short enough to neglect loss and dispersion. Alternatively, it is possible to use a numerical simulation to include those effects in the analysis.



Fig. 5 SPM method with pulsed laser

In SPM method with pulsed laser the spectral broadening factor for a Gaussian pulse is given by [24]

$$\frac{(\Delta\omega)_{out}}{(\Delta\omega)_{in}} = \sqrt{1 + \frac{4}{3\sqrt{3}} \left(\gamma \cdot P \cdot L_{eff}\right)^2} , \qquad (8)$$

where $\Delta \omega$ is the RMS spectral width and *P* is the peak power.

The advantage of SPM method using CW lasers, which is shown in Fig. 6, is the fact that this method does not require special pulse source. The output from two single frequency lasers is combined to generate an optical beat signal in the fiber. This beat signal is then amplified in an EDFA and propagated down the sample of FUT. On traveling through the FUT the optical signal experiences SPM, which leads to the generation of sidebands in the optical spectrum. The ratio of intensity of the first-order sideband to the spectral intensity at the fundamental frequency can be expressed as [25]

$$\frac{I_0}{I_1} = \frac{J_0^2(\varphi_{\text{SPM}}/2) + J_1^2(\varphi_{\text{SPM}}/2)}{J_1^2(\varphi_{\text{SPM}}/2) + J_2^2(\varphi_{\text{SPM}}/2)},$$
(9)

where I_0 and I_1 are the intensities of the zero- and firstorder harmonics and J_n is the Bessel function of the *n*th order. Neglecting dispersion, the non-linear phase shift is a function of I_0/I_1 only, which can be readily measured.



Fig. 6 SPM method with two CW lasers

3.2.2 Modulation Instability method

The non-linear coefficient can be determined by the measuring of modulation instability gain, the maximum value of which is given by [28]

$$g_{\max} = 2\gamma P. \tag{10}$$

This method was extended into method which includes the simultaneous determination of the average values of non-linear coefficient, zero-dispersion wavelength and dispersion [29]. The method relies on modulation instability amplification. The setup, shown in Fig. 7, consists of only one CW tunable laser source, EDFAs for boosting laser power, optical variable attenuator, FUT, and optical spectrum analyzer. When the laser wavelength is in the anomalous dispersion region (D>0), the spectral sidelobes are clearly observed.



3.2.3 Cross-Phase Modulation method

A non-interferometric measurement system based on XPM [30, 31] is shown in Fig. 8. Pump signal is amplified and is relatively strong in comparison with probe signal power so that the effect of SPM is negligible. Since the pump source is modulated in its intensity, probe signal is modulated in its phase through XPM. Non-linear coefficient is determined by measuring frequency components induced by phase modulation with the self-delayed heterodyne detection technique while changing pump power.



Fig. 8 XPM method

3.2.4 Four-Wave Mixing method

Recent attention has been focused on FWM measurement techniques using either two CW lasers [32, 33, 34] or one modulated laser source [35, 36].

In the first case, the non-linear coefficient of FUT is measured by FWM method using two 1550 nm DFB lasers, which generate two continuous waves of wavelengths separation $\Delta\lambda$. The two waves of equal power are sent to two EDFAs, combined and sent to FUT. The polarizations of the waves are adjusted using PCs and a polarizer until they become linear and parallel to each other. After propagation, the output signal is fed to an optical spectrum analyzer to get the power ratio between the pumps power and the harmonics power, generated by FWM.



Fig. 9 FWM method with two CW lasers

In FWM method, presented recently, two CW lasers are replaced by one externally modulated laser source. Carrier suppressed amplitude modulation (AM) with a train of RF pulses of given width and repetition period gives two sidebands, separated by twice the RF modulation frequency. The sidebands, having the same polarization and equal power, are amplified using one EDFA only. High peak powers are obtained by using low duty cycles. This simple measurement scheme employs one laser source only. It is polarization independent and enables high sensitivity, which leads to higher accuracy.



Fig. 10 FWM method with externally modulated laser

4 Conclusion

Non-linear refraction leads to a large number of nonlinear effects such as SPM, which permits the existence of optical solitons in single channel optical transmission systems, XPM and FWM, which is detrimental in WDM networks. The SPM and XPM cause spectral broadening within optical pulses, which then interacts with the dispersion of the fiber. This can be beneficial or detrimental to optical communication systems depending on whether the dispersion is normal or anormalus.

When two or more wavelengths of light are propagating along a fiber, FWM can cause waves to be generated at new frequencies. This effect can be particularly detrimental in WDM systems where each channel has its own wavelength and any new signal generated at this wavelength will appear as noise and it will degrade system performance.

All of the proposed measurement techniques provide indirect estimation of the second-order refractive index

through one of the Kerr-related phenomena. Currently, there is a debate as to which method provides the most accurate values of non-linear coefficient and which measurement technique provides the most appropriate figures for optical communication systems. The argument depends on how accurately the launch power can be measured in each case and also on how important the effect of electrostriction on the measurement is.

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