Phonon Spectra in Crystalline Nanostructures

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Abstract: — Considering the models of spatially bounded structures and structures with broken translational symmetry (ultrathin films, quantum wires and dots), we have found the phonon spectra using the method of two-time commutator Green's functions. The phonon spectra are distinctly discrete and possess gaps (which are the consequence of the spatial boundedness of the system). Using the definition of the activation temperature of the phonon subsystem we connected these results with better superconductive properties of micro samples, which can be a possible explanation of high critical temperature of HTC ceramics.

Key-Words: - Phonons, energy spectra, thin films, quantum wires, quantum dots

1 Introduction

As quantum low-dimensional structures are of current interest [1-7], we tried to study the basic microscopic behavior of phonon subsystem in quantum wires and dots which are of particular importance regarding its influence onto macroscopic properties that are connected with their possible application in nanotechnology. This paper is based on our previous works which dealt ultrathin films [8-12]. Contrary to thin films which are quasi 2D systems bounded by two surfaces parallel to XY planes, quantum wires and dots are quasi 1D and 0D crystalline systems bounded in two or three directions (y and z, or x, y and z) respectively.

2 Equation of motion

The phonon dispersion law can be derived using the Green's function method [13-17]. We use the following two-time temperature dependent retarded Green's function:

$$\begin{aligned}
G^{\alpha\alpha}_{\vec{n},\vec{m}}(t) &\equiv \langle \langle u^{\alpha}_{\vec{n}}(t) | u^{\alpha}_{\vec{m}}(0) \rangle \rangle = \\
&= \Theta(t) \langle [u^{\alpha}_{\vec{n}}(t), u^{\alpha}_{\vec{m}}(0)] \rangle_{0} ,
\end{aligned}$$
(1)

where $u_{\vec{n}}^{\alpha}$ is the displacement of an atom at the site $\vec{n} \equiv (n_x n_y n_z)$ in the α direction ($\alpha \in \{x, y, z\}$). Finding the second derivative, we get the equation of motion for Green's function:

$$M_{\vec{n}} \frac{d^2}{dt^2} G^{\alpha \alpha}_{\vec{n},\vec{m}}(t) = -i\hbar \delta_{\vec{n}\vec{m}} \delta(t) +$$

$$+ \frac{\Theta(t)}{i\hbar} \langle \left[\left[p^{\alpha}_{\vec{n}}, H(t) \right], u^{\alpha}_{\vec{m}}(0) \right] \rangle_0 .$$

$$(2)$$

Hamiltonian of the phonon subsystem in harmonic and nearest neighbor approximation (neglecting torsion constants $C^{\alpha \neq \beta}$, see [8-10]) is:

$$H = \frac{1}{4} \sum_{n_x n_y n_z; \alpha} \left[2M_{n_x n_y n_z} \dot{u}_{n_x n_y n_z; \alpha}^2 + (3) \right]$$

$$+C^{\alpha\alpha}_{n_{x}n_{y}n_{z};n_{x}\pm1,n_{y}n_{z}}\left(u^{\alpha}_{n_{x}n_{y}n_{z}}-u^{\alpha}_{n_{x}\pm1,n_{y}n_{z}}\right)^{2}+\\+C^{\alpha\alpha}_{n_{x}n_{y}n_{z};n_{x}n_{y}\pm1,n_{z}}\left(u^{\alpha}_{n_{x}n_{y}n_{z}}-u^{\alpha}_{n_{x}n_{y}\pm1,n_{z}}\right)^{2}+\\+C^{\alpha\alpha}_{n_{x}n_{y}n_{z};n_{x}n_{y}n_{z}\pm1}\left(u^{\alpha}_{n_{x}n_{y}n_{z}}-u^{\alpha}_{n_{x}n_{y}n_{z}\pm1}\right)^{2}\right].$$

Performing time Fourier transform and simplifying the model, assuming $M_{\vec{n}} \equiv M$ and $C^{\alpha\alpha}_{\vec{n},\vec{n}+\vec{\lambda}_{x/y/z}} \equiv C^{\alpha\alpha}_{x/y/z}$, and calculating commutators $[p^{\alpha}_{n_x n_y n_z}, H]$, the equation of motion for Green's function turns into:

$$M\omega^{2}G_{n_{x}n_{y}n_{z};m_{x}m_{y}m_{z}}^{\alpha\alpha}(\omega) = \frac{i\hbar}{2\pi}\delta_{n_{x},m_{x}}\delta_{n_{y},m_{y}}\delta_{n_{z},m_{z}} + \left[C_{x}^{\alpha\alpha}\left(G_{n_{x}n_{y}n_{z};m_{x}m_{y}m_{z}}^{\alpha\alpha} - G_{n_{x}\pm1,n_{y}n_{z};m_{x}m_{y}m_{z}}^{\alpha\alpha}\right) + C_{y}^{\alpha\alpha}\left(G_{n_{x}n_{y}n_{z};m_{x}m_{y}m_{z}}^{\alpha\alpha} - G_{n_{x}n_{y}\pm1,n_{z};m_{x}m_{y}m_{z}}^{\alpha\alpha}\right) + C_{z}^{\alpha\alpha}\left(G_{n_{x}n_{y}n_{z};m_{x}m_{y}m_{z}}^{\alpha\alpha} - G_{n_{x}n_{y}n_{z}\pm1;m_{x}m_{y}m_{z}}^{\alpha\alpha}\right)\right)^{4}$$

(see in [12,16]). One can use this equation for analyzing phonon subsystem both in ideal infinite crystals and in bounded crystalline systems (films, quantum wires and dots), because of its general character.

3 Green's functions

The equation for Green's function must be amended with the conditions which described spatial constraints of these systems:

$$u^{\alpha}_{n_x n_y n_z} = 0$$
, $G^{\alpha \alpha}_{n_x n_y n_z; m_x m_y m_z}(\omega) = 0$ (5)

a) films: for {
$$n_z < 0$$
 and $n_z > N_z$ }
b) wires: for { $n_y < 0$ and $n_y > N_y$
 $n_z < 0$ and $n_z > N_z$ }
c) dots: for { $n_x < 0$ and $n_x > N_x$
 $n_y < 0$ and $n_y > N_y$
 $n_z < 0$ and $n_z > N_z$ }

One gets systems of (N_z+1) , $(N_y+1) \times (N_z+1)$ or $(N_x + 1) \times (N_y + 1) \times (N_z + 1)$ equations, respectively. Performing partial spatial Fourier transform (due to the breaking of translational invariance along (z), (y, z) and (x, y, z) directions, see in [12], [16], [17]), we obtain the systems of $(N_z + 1)$, $(N_y + 1) \times (N_z + 1)$ or $(N_x + 1) \times (N_y + 1) \times (N_z + 1)$ inhomogeneous algebraic difference equations:

a) films:

$$c_f \left(G_{n_z-1;m_z}^{\alpha\alpha} + G_{n_z+1;m_z}^{\alpha\alpha} \right) + (6) + \rho_f G_{n_z;m_z}^{\alpha\alpha} = \mathcal{K}_{n_z;m_z} ,$$

where:

$$\rho_f = -2 + \frac{1}{\left(\Omega_z^{\alpha\alpha}\right)^2} \left\{ \omega^2 - (7) \right\}$$

$$-4\left[\left(\Omega_x^{\alpha\alpha}\right)^2 \sin^2 \frac{a_x k_x}{2} + \left(\Omega_y^{\alpha\alpha}\right)^2 \sin^2 \frac{a_y k_y}{2}\right]\right\} ;$$

$$c_f = \frac{\left(\Omega_z^{\alpha\alpha}\right)^2}{\left(\Omega_z^{\alpha\alpha}\right)^2} \equiv 1 ; \quad \mathcal{K}_{n_z;m_z} \equiv \frac{i\hbar \,\delta_{n_z;m_z}}{2\pi \cdot \left(\Omega_z^{\alpha\alpha}\right)^2} ,$$

b) wires:

$$b_w \left(G^{\alpha\alpha}_{n_y-1,n_z;m_ym_z} + G^{\alpha\alpha}_{n_y+1,n_z;m_ym_z} \right) + + c_w \left(G^{\alpha\alpha}_{n_yn_z-1;m_ym_z} + G^{\alpha\alpha}_{n_yn_z+1;m_ym_z} \right) + (8) + \rho_w G^{\alpha\alpha}_{n_yn_z;m_ym_z} = \mathcal{K}_{n_yn_z;m_ym_z} ;$$

where:

$$\rho_w = -2 + \frac{\omega^2 - 4\left(\Omega_x^{\alpha\alpha}\right)^2 \sin^2 \frac{a_x k_x}{2}}{\left(\Omega_y^{\alpha\alpha}\right)^2 + \left(\Omega_z^{\alpha\alpha}\right)^2} ; \qquad (9)$$

$$b_{w} = \frac{\left(\Omega_{y}^{\alpha\alpha}\right)^{2}}{\left(\Omega_{y}^{\alpha\alpha}\right)^{2} + \left(\Omega_{z}^{\alpha\alpha}\right)^{2}};$$

$$c_{w} = \frac{\left(\Omega_{z}^{\alpha\alpha}\right)^{2}}{\left(\Omega_{y}^{\alpha\alpha}\right)^{2} + \left(\Omega_{z}^{\alpha\alpha}\right)^{2}};$$

$$\mathcal{K}_{n_{y}n_{z};m_{y}m_{z}} \equiv \frac{i\hbar \, \delta_{n_{y}n_{z};m_{y}m_{z}}}{2\pi \cdot \left[\left(\Omega_{y}^{\alpha\alpha}\right)^{2} + \left(\Omega_{z}^{\alpha\alpha}\right)^{2}\right]},$$

c) dots:

$$a_d \left(G^{\alpha\alpha}_{n_x-1,n_yn_z;m_xm_ym_z} + G^{\alpha\alpha}_{n_x+1,n_yn_z;m_xm_ym_z} \right) + \\ + b_d \left(G^{\alpha\alpha}_{n_xn_y-1,n_z;m_xm_ym_z} + G^{\alpha\alpha}_{n_xn_y+1,n_z;m_xm_ym_z} \right) + \\ + c_d \left(G^{\alpha\alpha}_{n_xn_yn_z-1;m_xm_ym_z} + G^{\alpha\alpha}_{n_xn_yn_z+1;m_xm_ym_z} \right) + \\ + \rho_d G^{\alpha\alpha}_{n_xn_yn_z;m_xm_ym_z} = \mathcal{K}_{n_xn_yn_z;m_xm_ym_z} , \quad (10)$$

where:

$$\rho_d = -2 + \frac{\omega^2}{\left(\Omega_{xyz}^{\alpha\alpha}\right)^2} ; \qquad (11)$$

$$a_{d} = \left(\frac{\Omega_{x}^{\alpha\alpha}}{\Omega_{xyz}^{\alpha\alpha}}\right)^{2}; \quad b_{d} = \left(\frac{\Omega_{y}^{\alpha\alpha}}{\Omega_{xyz}^{\alpha\alpha}}\right)^{2}; \quad c_{d} = \left(\frac{\Omega_{z}^{\alpha\alpha}}{\Omega_{xyz}^{\alpha\alpha}}\right)^{2};$$
$$\mathcal{K}_{n_{x}n_{y}n_{z};m_{x}m_{y}m_{z}} \equiv \frac{i\hbar \,\delta_{n_{x}n_{y}n_{z};m_{x}m_{y}m_{z}}}{2\pi \left(\Omega_{xyz}^{\alpha\alpha}\right)^{2}};$$
$$\left(\Omega_{xyz}^{\alpha\alpha}\right)^{2} \equiv \left(\Omega_{x}^{\alpha\alpha}\right)^{2} + \left(\Omega_{y}^{\alpha\alpha}\right)^{2} + \left(\Omega_{z}^{\alpha\alpha}\right)^{2}.$$

In the cases of quantum wires and dots we replace two and three index counting by single index counting using the following substitutions:

a) films:
$$l_f = n_z + 1$$
,
b) wires: $l_w = n_z + 1 + (N_z + 1) n_y$,
c) dots: $l_d = n_z + 1 + (N_z + 1) [n_y + (N_y + 1) n_x]$

The systems of equations for determining Green's function transform into more practical forms:

a) films:
$$l_f \in [1, N_z + 1]$$

 $c_f \left(G_{l_f-1}^{\alpha \alpha} + G_{l_f+1}^{\alpha \alpha} \right) + \rho_f G_{l_f}^{\alpha \alpha} = \mathcal{K}_{l_f} , \qquad (12)$

b) wires:
$$l_w \in [1, (N_y + 1) \times (N_z + 1)]$$

$$b_w \left[G_{l_w - (N_z + 1)}^{\alpha \alpha} + G_{l_w + (N_z + 1)}^{\alpha \alpha} \right] + (13)$$

+ $c_w \left(G_{l_w - 1}^{\alpha \alpha} + G_{l_w + 1}^{\alpha \alpha} \right) + \rho_w G_{l_w}^{\alpha \alpha} = \mathcal{K}_{l_w} ,$

c) dots: $l_d \in [1, (N_x + 1) \times (N_y + 1) \times (N_z + 1)]^{-1}$

$$a_d \left[G_{l_d - (N_y + 1) \times (N_z + 1)}^{\alpha \alpha} + G_{l_d + (N_y + 1) \times (N_z + 1)}^{\alpha \alpha} \right] +$$

+
$$b_d \left[G_{l_d-(N_z+1)}^{\alpha\alpha} + G_{l_d+(N_z+1)}^{\alpha\alpha} \right] + (14)$$

+ $c_d \left(G_{l_d-1}^{\alpha\alpha} + G_{l_d+1}^{\alpha\alpha} \right) + \rho_d G_{l_d}^{\alpha\alpha} = \mathcal{K}_{l_d}$.

We can express Green's function as $G_{l_{f/w/d}}^{\alpha\alpha} = \frac{D_{l_{f/w/d}}}{D_{f/w/d}}$ where $D_{f/w/d}$ is determinant of these systems of equations. As the poles of Green's function determine phonon excitation energies [12-17] the problem reduces to the finding of the roots of the characteristic polynomials of these determinants.

a) films:

$$D_{f} = \begin{vmatrix} \rho_{f} & 1 & 0 & \cdots & 0 & 0 & 0 \\ 1 & \rho_{f} & 1 & \cdots & 0 & 0 & 0 \\ 0 & 1 & \rho_{f} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \rho_{f} & 1 & 0 \\ 0 & 0 & 0 & \cdots & 1 & \rho_{f} & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 & \rho_{f} \end{vmatrix} \Big|_{N_{z}+1}$$
(15)

b) wires

$$\begin{array}{c} \begin{array}{c} 1) n_{x} \end{array} \\ 1) n_{x} \end{array} \\ \begin{array}{c} 1 \\ nining \\ actical \end{array} D_{w} = \left| \begin{array}{c} R_{w} & B_{w} & O & \cdots & O & O & O \\ B_{w} & R_{w} & B_{w} & \cdots & O & O & O \\ O & B_{w} & R_{w} & \cdots & O & O & O \\ O & B_{w} & R_{w} & \cdots & O & O & O \\ \end{array} \right| \\ \begin{array}{c} 12 \\ (12) \end{array} \\ \begin{array}{c} (13) \\ (13) \\ (13) \\ (14) \\ (14) \\ \end{array} \\ \begin{array}{c} R_{w} = \\ (14) \\ \end{array} \\ \begin{array}{c} R_{w} = \\ (14) \\ \end{array} \\ \begin{array}{c} R_{w} = \\ (14) \\ R_{w} = \\ \end{array} \\ \begin{array}{c} R_{w} = \\ R_{w} = \\ (14) \\ \end{array} \\ \begin{array}{c} R_{w} = \\ \end{array} \\ \begin{array}{c} R_{w} = \\ \end{array} \\ \begin{array}{c} R_{w} = \\ \end{array} \\ \begin{array}{c} R_{w} = \\ R_{w} = \\$$

c) dots

$$D_{d} = \begin{vmatrix} \mathcal{R}_{d} & \mathcal{A}_{d} & \mathcal{O} & \cdots & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{A}_{d} & \mathcal{R}_{d} & \mathcal{A}_{d} & \cdots & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{A}_{d} & \mathcal{R}_{d} & \cdots & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{A}_{d} & \mathcal{R}_{d} & \cdots & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \cdots & \mathcal{A}_{d} & \mathcal{R}_{d} & \mathcal{A}_{d} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \cdots & \mathcal{O} & \mathcal{A}_{d} & \mathcal{R}_{d} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \cdots & \mathcal{O} & \mathcal{A}_{d} & \mathcal{R}_{d} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \cdots & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{R}_{d} & \mathcal{R}_{d} & \mathcal{B}_{d} & \cdots & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{R}_{d} & \mathcal{R}_{d} & \mathcal{B}_{d} & \cdots & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{R}_{d} & \mathcal{R}_{d} & \mathcal{B}_{d} & \cdots & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \cdots & \mathcal{R}_{d} & \mathcal{B}_{d} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \cdots & \mathcal{O} & \mathcal{B}_{d} & \mathcal{R}_{d} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \cdots & \mathcal{O} & \mathcal{B}_{d} & \mathcal{R}_{d} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \cdots & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{R}_{d} & \mathcal{P}_{d} & \mathcal{C}_{d} & \cdots & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \cdots & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \cdots & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \cdots & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \cdots & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \cdots & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \cdots & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \cdots & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \cdots & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \cdots & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} & \cdots & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} & \cdots & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} & \mathcal{O}$$

 $\mathcal{A}_d = \operatorname{diag} [a_d]_{(N_y+1)\times(N_z+1)}$; $B_d = \operatorname{diag} [b_d]_{N_z+1}$ where \mathcal{O} and O are 3D and 2D zero-matrixes, respectively.

4 Dispersion laws

The analytical results for phonon spectra in films:

$$\omega_{k_x k_y n_z/f}^{\alpha \alpha} = 2 \left[\Omega_x^{\alpha \alpha} \sin^2 \frac{a_x k_x}{2} + \right]$$
(18)

+
$$\Omega_y^{\alpha\alpha} \sin^2 \frac{a_y k_y}{2} + \Omega_z^{\alpha\alpha} \sin^2 \frac{\pi \nu_z}{2(N_z+2)} \bigg]^{1/2}$$

has already been found in our previous works [12,18]. By numerically analysis of quantum wires and dots problem we conjectured the analytical forms of the dispersion laws in these crystals structures:

$$\omega_{k_x k_y n_z/w}^{\alpha \alpha} = 2 \left[\Omega_x^{\alpha \alpha} \sin^2 \frac{a_x k_x}{2} + \right]$$
(19)

+
$$\Omega_y^{\alpha\alpha} \sin^2 \frac{\pi \nu_y}{2(N_y+2)} + \Omega_z^{\alpha\alpha} \sin^2 \frac{\pi \nu_z}{2(N_z+2)} \bigg]^{1/2}$$
;

$$\omega_{k_x k_y n_z/d}^{\alpha \alpha} = 2 \left[\Omega_x^{\alpha \alpha} \sin^2 \frac{\pi \nu_x}{2(N_x + 2)} + \right]$$
(20)

$$+ \Omega_y^{\alpha\alpha} \sin^2 \frac{\pi\nu_y}{2(N_y+2)} + \Omega_z^{\alpha\alpha} \sin^2 \frac{\pi\nu_z}{2(N_z+2)} \bigg]^{1/2} .$$

Analytical procedure for evaluation of determinants (16) and (17) is given in [18]. The quantum numbers ν_{α} , $\alpha = (x, y, z)$ from expressions (18), (19) and (20) take the values: $\nu_{\alpha} \in [1, N_{\alpha} + 1]$. One can see that solutions have the form of the Pithagora's theorem and that the discretization of the spectrum along one direction is independent of the discretization along other two directions.

5 Conclusion

1. The dispersion laws (18-20) in thin films, quantum wires and dots are formally of the same form as in corresponding infinite ideal crystal. The main difference is that the appearance of discretization of the phonon energies along directions that are spatially bounded.

2. The most important result of these analyzes is the existence of energy gaps in bounded structures. Because its appearance is the consequence of the breaking of translational symmetries (along bounded directions), the gaps values are greater in quantum dots than i wires, i.e. thin films, while in unbounded crystals its values are neglected. From (18-20) one can write the expression for energy gaps in lowdimensional crystal systems as:

$$g_{f/w/d}^{\alpha\alpha} \approx \pi \left[\sum_{\beta \in \{x,y,z\}} \left(\Omega_{\beta}^{\alpha\alpha} \frac{\nu_{min}^{\beta}}{N_{\beta} + 2} \right)^2 \right]^{1/2} \quad (21)$$

where:

a) $\nu_{min}^{x} = \nu_{min}^{y} = 0$, $\nu_{min}^{z} = 1$ - films and b) $\nu_{min}^{x} = 0$, $\nu_{min}^{y} = \nu_{min}^{z} = 1$ - wires.

One can see from (21) that the values of the gaps decrease sharply (nearly hyperbolically) with the increase of the size of the system, i.e. with the increase of N_{β} .

3. The presence of the gaps in the phonon spectra cannot be explained by the disappearance of acoustic and appearance of optical phonons ([8-10,19,21]), for in our model we analyzed only the simple cubic lattice. The possible explanation is the appearance of acoustic phonons of optical type [12,19]. This explanation can be supported by the calculation of the statistical limit, i.e. $\lim_{N_{x/y/z}\to\infty}g_{f/w/d}^{\alpha\alpha}=0.$

4. The presence of phonon gaps can be interpreted as the existence of an activation temperature $T_{ac}^{f/w/d} = g_{f/w/d}/\hbar$. The phonon subsystem appears in these crystal structures (films, wires and dots) only if the temperature of the crystal is higher than $T_{ac}^{f/w/d} = g_{f/w/d}/\hbar$. Below this temperature the system is "frozen" and has ideal electric conductance. According to the results of our analysis (21), the best properties have the quantum dots.

5. On the other hand, these results can be connected with the fact that superconducting samples (such as ultrathin films, quantum wires and dots) are of micro-dimensions and possess better superconductive properties and higher critical temperatures than the corresponding bulk samples [22-24]. This can be very important in the explanation of superconducting mechanism of HTC ceramics [5,24,25-29]. Our preliminary results of the analysis of electric carriers spectra in low-dimensional systems [30-33] show that these spectra also possess energy gaps. This can result in the appearance of higher energy gaps in electron(hole)-phonon system and, naturally, in higher critical temperatures of these crystalline structures (than in the bulk) [33]. It is also very interesting to notice that high-temperature superconductors are of high granularity [5,24-29] and that quantum dots can be connected by Josephson tunnel junctions among themselves.

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