

# Bandwidth Tolerances of Cascaded Filters in WDM Optical Networks

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**Abstract:** - In this contribution we develop a theoretical statistical approach to predict the acceptable bandwidth tolerances for optical filter cascades in wavelength division multiplexing (WDM) networks. We also establish tolerances for the center frequency misalignments considering a statistical approach presented in a previous work. The optical filters analyzed have a trapezoidal shape, and different bandwidths are considered for the study. The analytical results are validated with system simulations.

**Key-Words:** - WDM optical network, optical filtering, filter cascadability, bandwidth variation, frequency misalignment, system performance.

## 1 Introduction

WDM systems are evolving from point-to-point systems to transparent optical networks, in which wavelength channels are routed without opto-electronic conversion. All-optical WDM networks with add-drop, routing and cross-connecting capabilities include several wavelength selective components. The concatenation of optical filters makes the system susceptible to filter pass-band misalignments arising from device imperfections, temperature variation and aging. The emission spectrum of the laser source may also be misaligned with the effective center frequency of the optical filters owing to manufacturing tolerances, aging, or operating conditions (for example temperature). Performance degradation in WDM systems may arise owing to optical filter misalignments and concatenation, combined with laser misalignments and chirp [1]. Another important degradation factor is the reduction of the filters bandwidth due to operation conditions and aging.

In a previous work we have developed a theoretical formulation to predict the allowable tolerances for the center frequency, considering cascades of different types of filters [2]. In this contribution we complement that study, presenting an analytical solution to establish tolerances for the optical filters bandwidth variation. The analytical results are validated with system simulations, considering optical filters with trapezoidal transfer functions.

## 2 Theoretical Models

In this section we present the theoretical statistical formulations to predict the allowable tolerances for the bandwidth and center frequency, considering optical filter cascades.

### 2.1 Bandwidth Variation

We consider that the filters used have, by means of the temperature distribution of their locations and/or by means of their fabrication tolerances, a bandwidth stochastic distribution that meets a Gaussian probability density function (PDF) with standard deviation  $\sigma$  (Hz).

The change in quality factor,  $Q$ , as the filter bandwidth varies, is assumed to follow the function presented in Fig. 1. Considering signal and filter bandwidths to be  $B$  and  $L_B$  respectively, if the filter bandwidth is higher than  $f_{mi}$  (with  $f_{mi}=L_B/2-B$ ), the signal of the selected channel fits totally within the filter pass-band, and  $Q$  is maximum ( $Q_{max}$ ). For lower bandwidths than  $f_{mi}$ ,  $Q$  decreases, with a variation that depends on the filter shape, the signal format, but for simplicity we will assume it is linear.  $Q$  is null for  $\Delta f < L_B/2$ , corresponding to unreal negative bandwidths, and therefore those values should not be considered; nevertheless the Gaussian approximation is assumed as a valid option, since we will consider standard deviation values that lead to a allowable maximum of bandwidth realizations in the negative part.

The evolution of the average  $Q$ ,  $Q_{avg}$ , as a function of  $\sigma$ , can be obtained by solving the integral:

$$Q_{avg} = \int_{-\infty}^{+\infty} Q(\Delta f, Q_{max}, B, L_B) \cdot PDF(\Delta f, \sigma) d\Delta f \quad (1)$$

For the assumed  $Q(\Delta f)$ ,  $Q_{avg}$  can be obtained in the following closed form:

$$Q_{avg} = \frac{Q_{max}}{2} - \frac{Q_{max} L_B}{4(L_B/2 - f_{mi})} \operatorname{erf}\left(-\frac{L_B}{2\sqrt{2}\sigma}\right) + \left(\frac{Q_{max} L_B}{4(L_B/2 - f_{mi})} - \frac{Q_{max}}{2}\right) \operatorname{erf}\left(-\frac{f_{mi}}{\sqrt{2}\sigma}\right) + \frac{Q_{max}\sigma}{\sqrt{2\pi}(L_B/2 - f_{mi})} \left[ \exp\left(-\frac{L_B^2}{8\sigma^2}\right) - \exp\left(-\frac{f_{mi}^2}{2\sigma^2}\right) \right] \quad (2)$$

## 2.2 Center Frequency Misalignment

For the center frequency distribution we have also considered a Gaussian PDF.

The change in  $Q$  factor when the filter center frequency is detuned by  $\Delta f$ , is assumed to follow the function presented in Fig. 2. If the signal of the selected channel fits totally within the filter pass-band, corresponding to values of detuning  $\Delta f$  in the interval  $[-f_{mi}; f_{mi}]$ , then  $Q$  is maximum. On the other hand, if the signal is totally out of the pass-band ( $|\Delta f| \geq f_{ma}$ ), then  $Q$  is null. In between, the variation depends on the filter shape, the signal format, so for simplicity we will assume it is linear.

For the assumed  $Q(\Delta f)$ ,  $Q_{avg}$  can be obtained in the following closed form [2]:

$$Q_{avg} = \frac{Q_{max}}{f_{ma} - f_{mi}} \left\{ f_{ma} \operatorname{erf}\left(\frac{f_{ma}}{\sqrt{2}\sigma}\right) - f_{mi} \operatorname{erf}\left(\frac{f_{mi}}{\sqrt{2}\sigma}\right) + \sqrt{2/\pi} \sigma \left[ \exp\left(-\frac{f_{ma}^2}{2\sigma^2}\right) - \exp\left(-\frac{f_{mi}^2}{2\sigma^2}\right) \right] \right\} \quad (3)$$

## 2.3 Filter Cascading

At the output of the  $i^{th}$  filter, the signal presents, for a given  $\sigma$ , a degradation relatively to its quality at the input. At the input of the cascade,  $Q$  is maximum, and assuming that all the cascaded filters have the same characteristics, then  $Q_{max\ i} = Q_{max}$  and  $Q_{avg\ i} = \eta Q_{avg}$ . The parameter  $\eta$  represents some insertion penalty that the filter may introduce, by means of its transfer function shape and/or dispersion properties, relatively to an ideal filter (rectangular shape and linear phase) with the same bandwidth.

Thus, the average  $Q$  after a cascade of  $n$  filters,  $Q_{avg\ n}$ , is given by [2]:

$$Q_{avg\ n} = Q_{max} [\eta(Q_{avg}/Q_{max})]^n \quad (4)$$

## 3 Analytical and Simulation Results

To test the presented analytical formulations we have simulated an 8×20 Gbit/s WDM system, with an optical path (single mode fiber,  $D=17$  ps/(nm.Km)) of length 5 Km, and filtered one channel considering trapezoidal shaped filters with

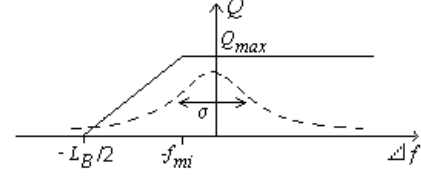


Fig. 1 – Representation of the model's  $Q$  factor for one channel (continuous line) and of a Gaussian distribution with standard deviation  $\sigma$  (dashed line) as a function of the bandwidth variation  $\Delta f$  around  $L_B/2$ .

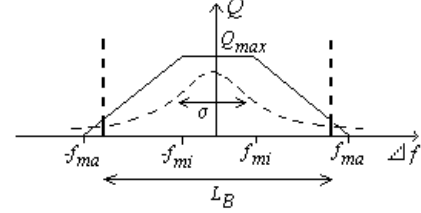


Fig. 2 – Representation of the model's  $Q$  factor for one channel (continuous line) and of a Gaussian distribution with standard deviation  $\sigma$  (dashed line) as a function of the center frequency detuning  $\Delta f$  (after [2]).

variable bandwidth. The trapezoidal filters considered had no insertion loss and bandwidths (at 0 dB) of 50 GHz and 80 GHz. The bandwidths at -20 dB were 5 GHz higher than the 0 dB bandwidths in all cases. To observe the effect of filter cascading we have considered the isolated filter and 9 cascaded filters, for each case.

Each test was performed 80 times and at each run the center frequency or bandwidth of the filter was changed randomly following a Gaussian distribution of standard deviation  $\sigma$ . The simulations were performed using PTDS from Virtual Photonics ©.

To obtain analytically the bandwidth allowable tolerances, we have used equation (2) with  $f_{mi} = L_B/2 - B$  as referred. To the center frequency allowable tolerances, we have considered  $f_{mi} = L_B/2 - B$  and  $f_{ma} = L_B/2 + 0.7B$  in equation (3). In all studied cases we considered the ideal value  $\eta=1.0$ .

As referred, for the Gaussian approximation (bandwidth variation PDF) to be valid, we have to use  $\sigma$  values that lead to an allowable maximum of bandwidth realizations in the negative part. For a maximum of 1%, considering the lowest filter bandwidth (50 GHz),  $\sigma_{max} \approx 10.7$  GHz.

We have considered maximal  $\sigma$  of 10 GHz and 100 GHz, respectively for the bandwidth variation and center frequency misalignment PDFs. This factor of 10 between the maximal  $\sigma$  considered is based on experimental results obtained for a particular kind of optical filter, the fiber Bragg grating [3].

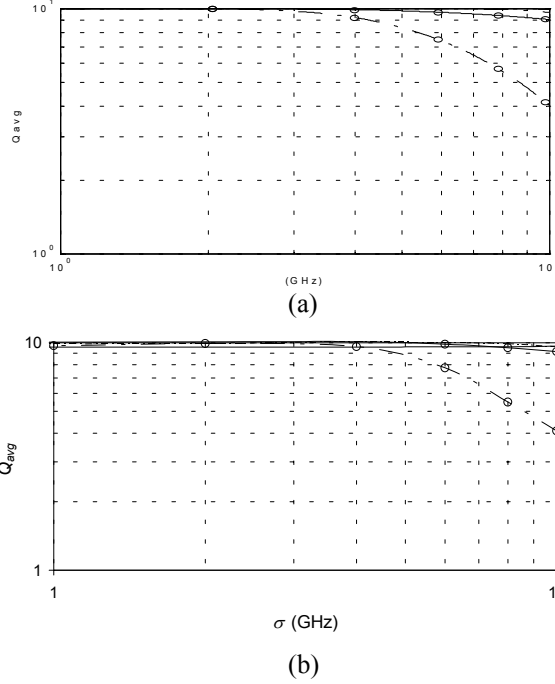


Fig. 3 – Theoretical results of  $Q_{avg}$  (a), with  $Q_{max}=10$  and  $B=20$  GHz, and the simulation results (b) for the referred setup, as a function of the allowed bandwidth variation standard deviation, for trapezoidal optical filters with bandwidths 80 GHz (no shape) and 50 GHz (circles), considering the single filter (heavy line) or a cascade of 9 (dashed line).

In Fig. 3 and in Fig. 4 we present the analytical (a) and simulated (b) results of  $Q_{avg}$ , as a function of the bandwidth variation and center frequency misalignment standard deviations, respectively.

## 4 Results Discussion

Comparing the results presented in Fig. 3 and Fig. 4, we notice that the simulated results follow closely the analytical ones, for the bandwidth variation and filter detuning allowable tolerances and curve tendencies, for single and cascaded filters.

The effect of cascading filters leads to a steeper dependence of  $Q_{avg}$  on  $\sigma$ , due to a steeper shape of the filters spectra after cascading and to the increased delay changes near the pass-band borders.

We also verify that the effect of the center frequency misalignment is more problematic than the bandwidth variation one, due to a wider range of the standard deviations to be considered.

## 5 Conclusions

We have presented a simple theoretical formulation to predict the allowable tolerances for the bandwidth variation, considering cascades of trapezoidal

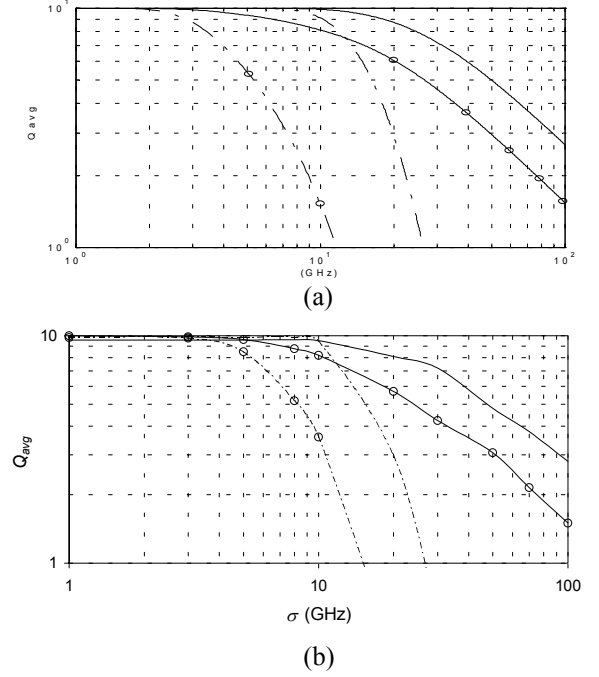


Fig. 4 – Theoretical results of  $Q_{avg}$  (a), with  $Q_{max}=10$  and  $B=20$  GHz, and the simulation results (b) for the referred setup, as a function of the allowed filter detuning standard deviation, for trapezoidal optical filters with bandwidths 80 GHz (no shape) and 50 GHz (circles), considering the single filter (heavy line) or a cascade of 9 (dashed line).

optical filters, with distinct bandwidths, in WDM systems. We also confirmed the theoretical formulation presented in a previous work for the center frequency misalignments tolerances considering the referred type of filters. The analytical results follow closely the simulated ones for both situations analyzed.

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