# ADJOINT TRANSFORMATIONS IN OTA-C FILTERS USING NULLORS

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## ABSTRACT

A method focused on transforming an OTA-C filter working in voltage-mode to current-mode using nullors, is presented. First, the ideal behavior of the OTA is modeled using nullors. Second, the transformation process is done by applying four basic rules. Third, the nullors of the transformed circuit are synthesized by OTAs, which have been implemented by two CCII- made up using a CMOS technology of 0.3$\mu$m of AMS. Finally, the computation of the transfer function in voltage- and current-mode, along with a simulation result using HSPICE, demonstrates the suitability of the proposed method.

## 1. INTRODUCTION

Nowadays, an analog system is described at various levels of abstraction where the more detailed the structure, the less abstract the description [1]. Furthermore, the modeling level of detail can vary from idealized models to very accurate ones. In particular, it has been shown that the *nullor* can be used to model the ideal behavior of the Operational Transconductance Amplifier (OTA) [2]-[6]. That way, any OTA circuit can be represented as a nullor circuit, from which the transformation technique to convert circuits working in voltage- to current-mode, and vice-versa can be done. To describe the transformation process, the modeling approach using nullors is given in section 2. In section 3, four basic transformation rules are introduced. An illustrative example is presented in section 4, from which simulation results of an implemetation with CCII-s using HSPICE is given. Finally, the conclusions are listed in section 5.

## 2. ANALOG MODELING USING NULLORS

Tellegen introduced the concept of ideal amplifier [2], and ten years later, Carlin modeled it using the nullor concept, as shown in Fig. 1 [3]. The nullor is composed of two elements: The *nullator* connected at its input-port where both $V_1$=$I_1$=0, and the *norator* connected at its output-port where both $V_2$=$I_2$ are undefined.

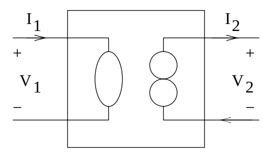


**Fig. 1**. The two-port nullor element

By handling nullator-norator pairs in a special way, the ideal behavior of some commonly used analog circuits can be well-modeled with them. It has been shown that this modeling-process does simplify several analysis and synthesis methods [4]-[9].

### 2.1. Modeling building-blocks

The accurate quantitative modeling and characterization of active devices is an essential prerequisite for the successful design and optimization of high performance analog systems. Despite such considerations, we can model the ideal behavior of some analog building-blocks using the nullor.

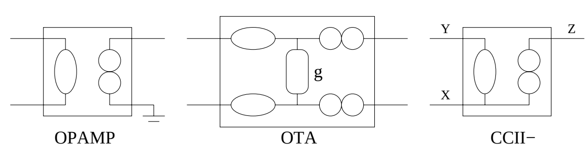


**Fig. 2**. Basic nullor-based analog building blocks.

For instance, the nullor equivalent of the OpAmp, the OTA and the CCII-, are shown in Fig. 2 [4, 5, 10]. Their corresponding mathematical relationships, modeling their ideal behavior can be described as follows:

The ideal OpAmp accomplishes the following properties: At its input-port: v=i=0, and at its output port: v=i=$\infty$. Note that the norator is always-grounded!.

The ideal OTA accomplishes the following properties: Since the voltage across a *nullator* equals to zero,

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the differential input voltage is bypassed directly to the conductance **g**, and since the current through a norator remains unchanged, the output current becomes to be $i_o = g_m(v_i{}^+ - v_i{}^-)$.

For the ideal CII- the following properties can be established [6]: across the nullator $v_x = v_y$ and $i_y = 0$, and through the norator $i_x = i_z$.

## 3. THE TRANSFORMATION METHOD

The transformation process of a nullor circuit from voltage-to current-mode and vice-versa, by assuming that the circuit is driving a single and independent voltage [resp. current] source, is done by applying the following four basic rules [4]:

*Rule 1*: Given a nullor-based circuit, set the nullator and norator interconnection-relationships references.

*Rule 2*: For each nullor, interchange the interconnection-location of the nullator [resp. norator] by the norator [resp. nullator], while maintaining intact the nullor external circuitry.

*Rule 3*: At the input port of the given nullor-circuit: the independent voltage [resp. current] source is short [resp. open] circuited. Now it becomes to be the output port of the nullor circuit, where the current [resp. voltage] is measured.

*Rule 4*: At the output port of the given nullor-circuit: if the output port-variable is voltage [resp. current], connect an independent current [resp. voltage] source. Now this port becomes to be the input-port of the nullor circuit where the current [resp. voltage] is supplied.

### 3.1. Applying the four rules

Lets consider the nullor-based circuit working in voltage-mode, which is shown in Fig. 3(a). The application of the four rules, in order to obtain the current-mode circuit equivalent, is done as follows:

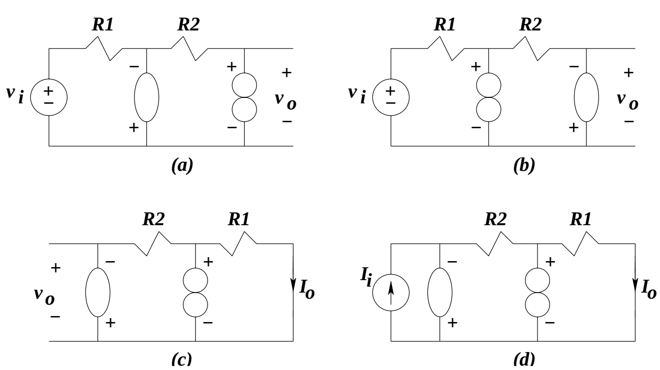


**Fig. 3**. Applying the transformation process: (a) rule 1, (b) rule 2, (c) rule 3 and (d) rule 4.

1. Rule 1 is applied in Fig. 3(a) by setting the nullator-norator interconnection-relationships references. The voltage transfer function of this circuit is computed using the method given in [7, 8, 9], however, the manner in which it is analytically calculated is done as follows:

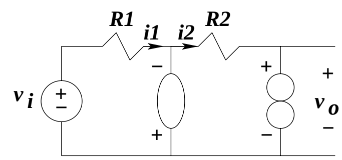


**Fig. 4**. Applying KCL

   (a) By applying the Kirchhoff's Current Law (KCL), as shown in Fig. 4, it results that
   $$i_1 = i_2 \tag{1}$$
   (b) Since the voltage across the nullator equals to zero, by applying Ohm's Law to equation (1) we get
   $$\frac{v_i}{R_1} = \frac{-v_o}{R_2} \tag{2}$$
   (c) Finally, the resulting voltage transfer function equals to
   $$\frac{v_o}{v_i} = -\frac{R_2}{R_1} \tag{3}$$

2. The application of Rule 2 is shown in Fig. 3(b), where the nullator is interchanged by the norator and viceversa.

3. Rule 3 is shown in Fig. 3(c), where the input port is short circuited in order to become to be the output port, where the output current is measured.

4. Rule 4 is shown in Fig. 3(d), where the output port becomes to be the input port, where an input current is supplied. Now, the current transfer function of the transformed circuit is analytically obtained as follows:

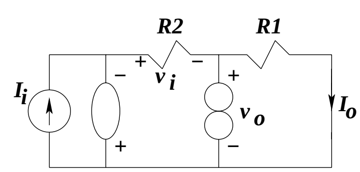


**Fig. 5**. Obtaining the current transfer function

   (a) Since the current through a nullator equals to zero, then $I_i$ is biasing the element $R_2$, generating a voltage given as
   $$v_i = I_i R_2 \tag{4}$$

(b) Since the voltage across a nullator equals to zero, then

$$v_o = -v_i \tag{5}$$

(c) By considering that there is a voltage $v_o$ across the norator, and by applying Ohm's law we obtain

$$I_o = \frac{v_o}{R_1} \tag{6}$$

(d) Finally, by using these last three equations, the current transfer function becomes to be

$$\frac{I_o}{I_i} = -\frac{R_2}{R_1} \tag{7}$$

As one sees, from equations (3) and (7) we can conclude that the circuit shown in Fig. 3(d), effectively becomes to be the current mode circuit-equivalent of the circuit shown in Fig. 3(a), which is working in voltage mode.

## 4. EXAMPLE

A tutorial focused on designing OTA-C voltage-mode circuits is given in [11], while an excellent book considering the design aspects of high performance OTAs is given in [10].

In order to show the usefulness of the transformation method, lets consider the OTA-C filter circuit shown in Fig. 6, which is working in voltage mode.

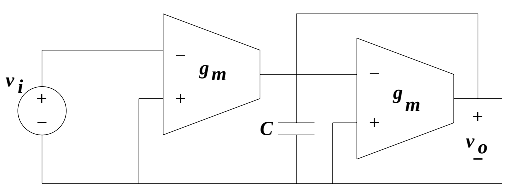


**Fig. 6**. $OTA - C$ filter working in voltage mode.

The application of the transformation process is sketched in Fig. 7. In Fig. 7a, it is shown the nullor circuit-equivalent of the OTA-C filter shown in Fig. 6. In Fig. 7b, it is shown the transformed nullor circuit working in current mode.

By using the method given in [7, 8], the computation of the symbolic transfer functions of the circuits shown in Fig. 7a and Fig. 7b, have the following relationship:

$$\frac{V_o}{V_i} = \frac{I_o}{I_i} = \frac{1}{1 + \frac{sC}{G_m}} \tag{8}$$

By synthesizing each pair of nullors-arrangement from Fig. 7b with the OTA, the resulting OTA-C filter circuit working in current mode, is shown in Fig. 8.

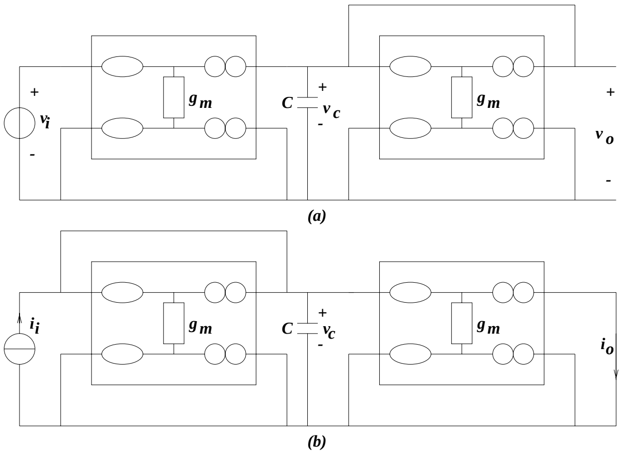


**Fig. 7**. Nullor-based $OTA - C$ filter working in (a) voltage and (b) current mode.

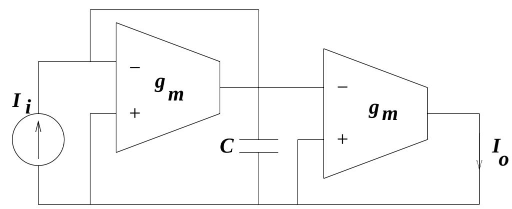


**Fig. 8**. $OTA - C$ filter working in current mode.

### 4.1. HSPICE simulations

In [12], it has been designed a CCII- using a CMOS technology of 0.3$\mu$m of AMS. HSPICE simulations of the CCII- shows that the parasitic impedance at node X, as shown in Fig. 9, has several hundreds of ohms in magnitude. The CCII- designed in [12], has a parasitic impedance equal to $R_X$=480 ohms at DC. The advantage of this impedance is that it can be used to implement the OTA by coupling two CCII- as shown in [4]-[6]. The resulting OTA has a transconductance equal to $g=\frac{1}{2R_X}$. Using the resulting OTA to implement the circuits shown in Figs. 6 and 8, in order to demonstrate the result given by equation (8), and by setting C=1.62nF, the lowpass filter has a dominant-pole at 100kHz (-3dB), as shown in Fig. 10. As one sees, both transfer functions in voltage- and current-mode present the same frequency behavior leading us to conclude about the suitability of the proposed adjoint-transformation technique.

## 5. CONCLUSIONS

A nullor-based method suitable for transforming circuits working in voltage mode to current mode and viceversa has been described. The transformation technique has been applied to OTA-C filter circuits.

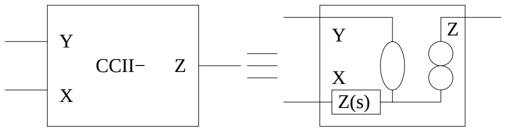


**Fig. 9**. CCII- representation and its nullor equivalent including its parasitic impedance at node X.

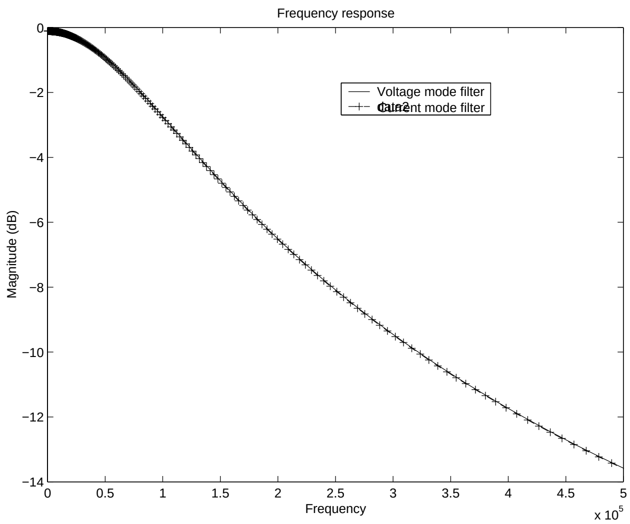


**Fig. 10**. HSPICE simulations of the adjoint lowpass filters

A set of four basic transformation rules has been established step by step.

From the example given in section 4, one can conclude on the appropriateness of this technique to be used as an analytical tool.

## 6. REFERENCES

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