Image Content Described by Fractal Parameters

IRINI RELJIN (*), BRANIMIR RELJIN (**) , IVANA RAKOCEVIC (***) and NIKOS MASTORAKIS (****)
(*)PTT College, Belgrade
Zdravka Celara 16, 11120 Belgrade
(**)Faculty of Electrical Engineering, University of Belgrade, Bulevar Kralja Aleksandra 73, 11120 Belgrade
(***)TMS project, Ustanicka 63, 11000 Belgrade
YUGOSLAVIA
(****)Military Inst.of University Education, Hellenic Naval Academy, Terma Hatzikyriakou, Piraeus, GREECE

Abstract: - The analysis of images from the fractal viewpoint is very complex and it is not completely explored. It deals with fractal compression techniques, mainly. In order to explore another aspect of this problem, this paper suggests to estimate Hurst indices for scanned images and for their coded versions. Their variations represent an indication of the type of a process obtained in data transfer, i.e. in communication traffic. The results show that this process should be modeled as a multifractal one.

Key-Words: - Digital image, self-similarity, multifractal process, Hurst index. CSCC'99 Proc., Pages:7231-7234

1 Introduction

The fractal analysis of traffic in the communication networks became one of the hottest research topics. It is evident that data traffic exhibits a long range dependency (LRD), and is well modelled by an ON/OFF process whose ON and OFF periods are heavy tailed. Usual representations of those processes are given by their Hurst exponents, *H*. The estimation of *H* may be obtained applying different procedures [1]. As it is known, the LRD process is governed by spectral components at low frequencies. For general processes, such as traffic streams obtained in traffic control, short-term variations are very important, too. This is the reason for intensive investigations in the multifractal domain, accounting for the short time behaviour [2-4] of processes.

In order to decrease the problems in the network design, as well as in creating the applications, it is important to understand the reasons for the fractal property. It is interesting to research the influence of encoding techniques to contents of data, and find whether it reduces the fractal property or not [5]. Among others, describing of images by the LRD parameters, may be useful [6].

This paper concerns various images from the fractal viewpoint. The scanned versions of images were analyzed. Hurst indices for two types of coding schemes have been estimated. The aim was to investigate how the value of *H* changes. According to the obtained results, a fractal-multifractal dilemma for images is discussed.

2 Problem Statement

The main problem in estimating the fractal dimension of images is how to find a proper procedure for the gray scale ones. Having in mind the traffic obtained in transfer of still images and exploring the influence of different coding techniques, it is convenient to use scanned images. It should be noted that the image signal magnitude is a positive real value, leading to the non-zero mean value. As a consequence [7,8], such a process, if fractal, could be only multifractal.

It is worth mentioning that fractal means “breaking in order to create irregular fragments” [7]. It can be described as a process of repeating an identical motif on a diminishing scale. As it is irregular, it is present in nature. Self-similarity is defined in sets and, strictly speaking, represents a union of distinct, non-overlapping copies of itself, each of which has been scaled down, by a ratio smaller than unity. Working with gray scale images, it is possible to define a measure, in this case determining the image magnitude in the space domain. This is the way of extending self-similarity from sets to measures, i.e. towards multifractals.

As multimedia traffic is expected to be fractal, the Hurst index is chosen as an indication for the LRD property. Among the most popular estimation procedures for it are the R/S statistics, the index of dispersion (IDC) and the periodogram [1]. So, the three were used here. It is known that the LRD property is usually investigated in rather long sequences. It should be noted that a short-term behaviour points out to local singularities, large rapid variations in magnitude, and thus it is important in the traffic analysis.

Taqqu, Teverovsky and Willinger investigated how to determine whether a process is accurately modelled by self-similar or by multifractal one [4]. They gave the explicit relations describing how to make a decision. For the non-negative stationary sequence X, the corresponding aggregated sequence, with level of aggregation m, obtained by dividing the original series into non-overlapping blocks of size m, and averaging over each block, is:

$$X^{(m)}(k)=\frac{1}{m}\sum_{i=(k-1)m}^{km}X(i)$$

Process X is multifractal if the logarithms of the absolute moments scale linearly with the logarithm of the aggregation level, m. In order to discover whether a process is multifractal, it is sufficient to examine first four moments. The additional condition, that scaling exponent in that relation is linear in moment order, assures self-similarity property.

3 Simulation Results

The simulations have been done with various types of images, Fig. 1. The first image is artificially generated according to Polish mathematician Sierpinski, and is known by the name the Sierpinski carpet. The second and the third simulations are gray scale versions of the fractal forgery of the planet rise and the landscape from Santorini island. The image named Santorini 2 is obtained as a contour version of a similar landscape. Three frequently used images, medical (representing cells), baboon and Lena, have been examined, too. The dimensions of all images are 256x256 pixels, magnitudes spans on (0-black,1-white) interval, with 64 gray levels. Scanned versions are used for estimation of the Hurst exponents.

It is worth mentioning that the Sierpinski carpet is a binary image. It is present in almost every paper dealing with fractals. It is strictly self-similar, with dimension D_S=1.89 [8]. This result is obtained by analyzing the procedure of making the scaled copies of a rectangular. It is hard to put self-similarity dimension into connection with fractal dimensions obtained in other procedures [7].

Table 1 contains the values for the R/S statistics (H_{RS}), periodogram (H_{per}) and the index of dispersion (H_{IDC}) method. Differences in the estimated values obtained by the three methods are common in this domain. It should be noted that H_{RS} for the Sierpinski carpet is rather low compared to two other results. This is a binary image, so the R/S method is based on statistics of only two levels, which may be the explanation for a non-reliable result. This image is strictly self-similar, while the others used here are fractal, which means that they contain some random structure.

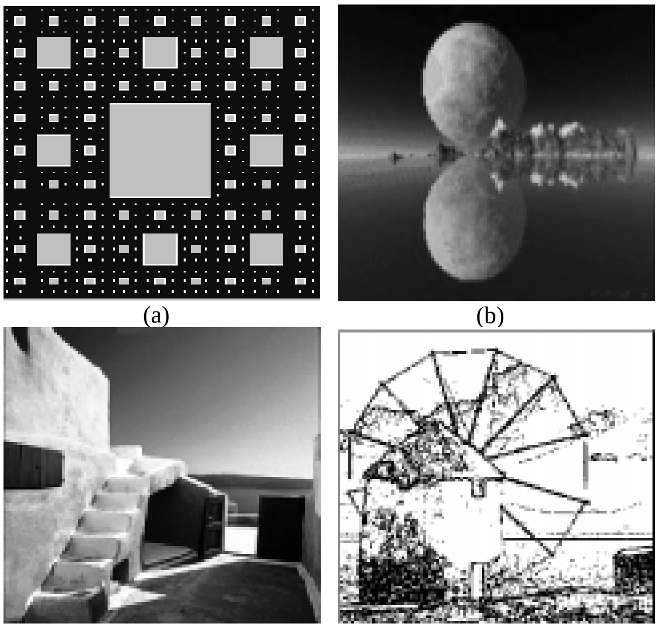


Fig. 1. (a) Sierpinski carpet; (b) fractal forgery of planet rise; (c) Santorini 1; (d) Santorini 2.

Table 1. Hurst index estimations.

	H_{RS}	H_{per}	H_{IDC}
Sierpinski carpet	0.52260	0.87880	0.91442
Santorini 1	0.95104	0.98158	0.80590
Santorini 2	0.90782	0.71580	0.73196
‘planet rise’	0.68300	*	0.66560
medical image	0.80916	0.92752	0.73196
baboon	0.89445	0.83348	0.83165
Lena	0.77947	0.70510	0.83696

- estimated value H >1.

In order to make an insight in how common coding techniques change fractal properties, discrete cosine transform (DCT) has been applied to each image. The spectrum obtained is split into 16 blocks according to Fig. 2. Blocks are successively removed in the designed order: 1-9. For such images, Hurst exponents are estimated. So, H_j denotes the exponent for the image with all blocks removed from 1 to j (j=1,2,…,9), according to Fig. 2.

L L	0	9	8	7	
	9	6	5	4	
	8	5	3	2	
	7	4	2	1	HH

Fig. 2. Splitting of the DCT spectrum of images into blocks.

Table 2 contains the estimations for Hurst indices (R/S statistics) for the original image, as well as for the reduced spectrum images. With the exception of the Santorini 1 image, differences are negligible. We can conclude that these images have significant low frequency components. A low frequency spectrum determines the fractal property, so it should be pointed out that this is the domain of the self-similarity. The Santorini 1 image has small details, which are the scaled versions of the big ones and thus are important for the fractal behaviour. These details have spectral components located at high frequencies. By removing them image becomes (almost) unchangeable, from the fractal property viewpoint. The variations of the Hurst exponent at high frequencies indicate the multifractal property of an image. The last row in Table 2 contains the absolute error due to the coding procedure, normalized to the image size (256x256 pixels).

Table 2. Hurst indices for reduced spectrum images.

	Sierpin. carpet	Santorini No 1	'planet rise'	Santorini No 2
H orig	0.52260	0.95104	0.68300	0.90782
H 1	0.52236	0.81321	0.68305	0.90838
H 2	0.52144	0.81410	0.68333	0.90998
H 3	0.52057	0.80051	0.68544	0.91003
H 9	0.52122	0.76988	0.68920	0.91038
error (9)	2.5E-2	5E-3	6E-4	1E-1

Fig. 3. contains images obtained by removing the block denoted as ‘1’ (the first row) in the DCT domain. The second row in Fig. 3 contains two images obtained if only the block denoted as ‘0’ remained. Errors for few images produced keeping only that part of spectrum, are given in Table 2.

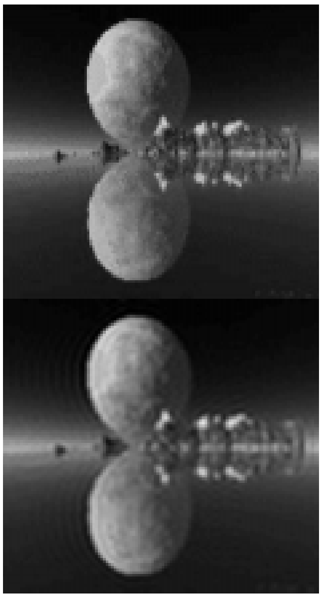
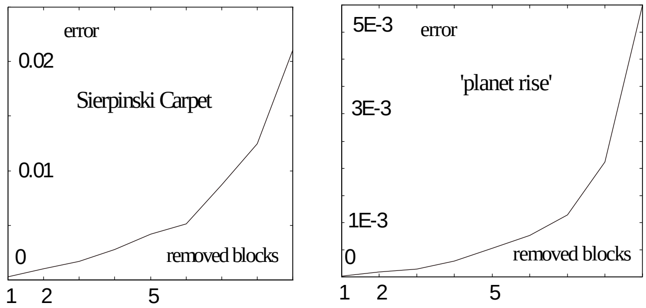


Fig. 3. Two images after removing the spectral blocks denoted by '1' (the first row) in Fig. 2, and denoted by '1-9' (the second row).

Fig. 4. represents plots of errors due to removing the DCT blocks ordered as indicated in Fig.2, for the two pictures: Sierpinski carpet and the ‘planet rise’. The error increase depends on the spectral content of an image. Comparing the results from Table 2 and Fig. 4, no connection of the Hurst exponent and the error may be derived. The question is whether a scanned image is self-similar, and thus the Hurst exponent is sufficient for its representation, or is it necessary to use multifractal model.



(a) (b)

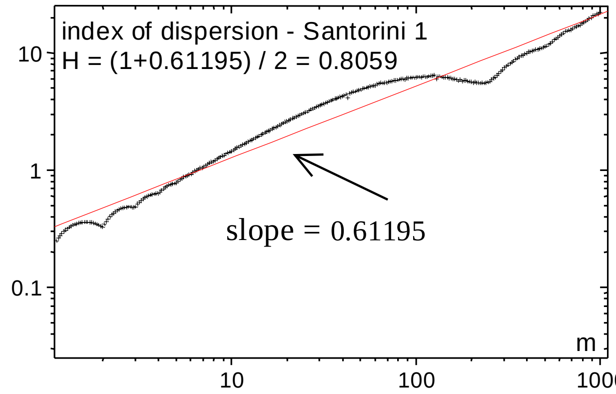
Fig. 4. Normalized absolute error for the two images.

Criteria for distinguishing the self-similar and multifractal processes, as described in Section 2 of this paper, are based on absolute moments and the scaling exponent [4]. The linear dependency in the log-log version of moments as functions of aggregation levels, is crucial. It is easy to prove that linear property in the second moment produces the same dependency in the IDC on the logarithm of the aggregation level.

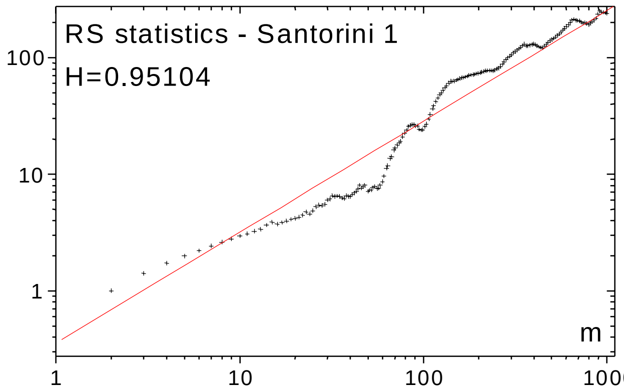
In order to investigate the multifractal property, the log-log version of the IDC, Fig. 5.a, is presented. A linear slope is evident for aggregation levels in range (10,100). At the ends, the plot is non-linear. According to the IDC, the scanned image Santorini 1, may be multifractal, as well as self-similar, too. From the slope of the scattered plot, obtained by linear regression method, it is possible to estimate the Hurst exponent [1], as it is labelled on the plot.

The behaviour of the Hurst exponent should be investigated through the log-log version of the R/S plot, Fig. 5.b, too. Its slope determines the Hurst exponent. Scattered points indicate high irregularities in *H*, especially for high aggregation levels. This points out that the Santorini 1 image should be modelled as multifractal rather than self-similar.

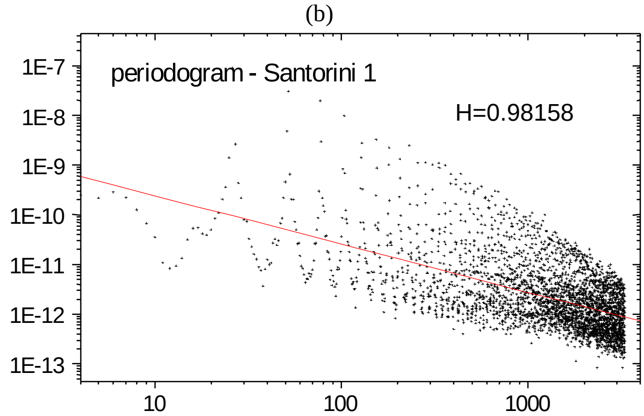
A log-log version of the periodogram for the Santorini 1 image is presented in Fig. 5.c. The Hurst exponent is obtained by applying the linear regression method, i.e. by estimating the slope [1]. Strong periodicity in the plot is a consequence of the scanning process. It should be noted that the estimation of the Hurst exponent from a periodogram may be polarized, especially in the case of strong LRD components.



(a)



(b)



(c)

Fig. 5. Santorini 1 image: (a) IDC, (b) R/S statistics, (c) periodogram.

Another coding technique, the pyramidal one, seems to be interesting for investigating from the fractal viewpoint. The R/S statistics was applied to the scanned version of pyramidal coded images. The first pyramidal level after the original one, produces the most important change in the Hurst exponent, Table 3. From that level variations of *H* are negligible.

Table 3. Hurst indices for pyramidal coded images.

index *H* for pictures		Sierpinski carpet	Santorini No 1	'planet rise'
RS stat	original	0.5226	0.95104	0.683
	128x128	0.77209	0.78977	0.84591
	64 x 64	0.81961	0.80645	0.84049
	32 x 32	0.82562	0.79778	0.8557

The R/S plots for the original image Santorini 1 and the three further pyramidal levels, are presented in Fig. 6.

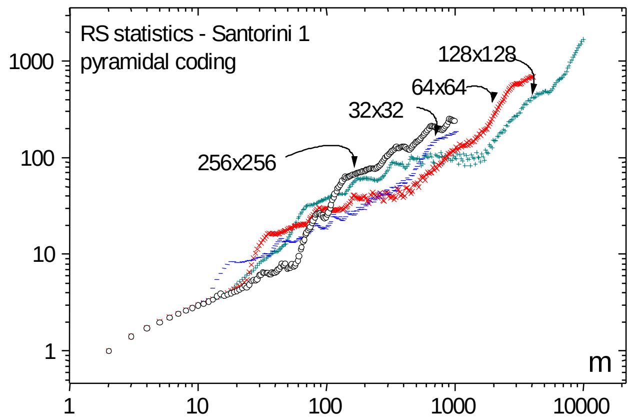


Fig. 6. R/S statistics for pyramidal coded Santorini 1 image.

4 Conclusion

This paper considers scanned images from the fractal viewpoint. By applying few methods for estimating the Hurst exponent, images have been analyzed. The image represents a non-negative process, so its magnitude was observed as a measure in the analysis of the fractal process.

The conclusions are based on the results for the IDC, the R/S statistics and the periodogram. The scanned images should be modelled as a multifractal, rather than a self-similar process. However, it is necessary to make additional multifractal analysis.

It should be noticed that a binary image, though it is a classic example of self-similar image (Sierpinski carpet), varies much for different estimation procedures.

References:

[1] W. Leland, M. Taqqu, W. Willinger, D. Wilson, On the self-similar nature of Ethernet traffic (extended version), *IEEE/ACM Tran. on Netw*, 1994, pp. 1-15.

[2] R. Riedi, J. L. Vehel, Multifractal properties of TCP traffic: A numerical study, *INRIA report*, N^{o} 3129, March 1997.

[3] J. L. Vehel, R. Riedi, Fractional Brownian motion and data traffic modeling: The other end of the spectrum, in *Fractals in Engineering*, Springer 1997.

[4] M. Taqqu, V. Teverovsky, W. Wllinger, Is network traffic self-similar or multifractal?, *Fractals*, 1996.

[5] W. Frohberg, Fractal behavior of video and data Traffic, *TR-96-027*, *Technical Report*, Berkeley, 1996.

[6] D. Chen, Multiresolution Models on analysis and classification, *http://www.geog.ucsb.edu/~dongmei*.

[7] M. Turner, J. Blackledge, P. Andrews, *Fractal Geometry in Digital Imaging*, Academic Press, 1998.

[8] H. O. Peitgen, H. Jurgens, D. Saupe, *Chaos and Fractals*, Springer, 1992.