Robot Hybrid Position/Force Control for Non-Cartesian Constraints

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Abstract:- Hybrid position/force control enables a robot arm to apply specified forces to a constraint while moving on the constraint surface. The standard approach is to find a Cartesian coordinate frame (a *task frame*) in which the constraint is naturally decomposed into constrained coordinates and unconstrained coordinates. Force is then controlled in constrained directions, while position is controlled in unconstrained directions. However, this approach assumes that it is possible to decompose the constraint in this way using Cartesian coordinates, which is rarely the case. For example, Cartesian coordinates cannot be used to decompose a contour tracking task involving a curved surface. To deal with such *non-Cartesian constraints*, some researchers have proposed using a rotating task frame that keeps constrained axes pointing in constrained directions. However, it is explained in this paper that this approach leads to an over-specified control problem. A proper approach is to use non-Cartesian coordinates, as described by Yoshikawa in [1]. The present paper provides theoretical justification for Yoshikawa's approach. First, task coordinates are formally defined in terms of their position/force decoupling properties. Next, it is proved that Yoshikawa's method of choosing coordinates for a given holonomic constraint always yields coordinates with these properties. Finally, the method is demonstrated on two example non-Cartesian constraints, one of which corresponds to a six degree of freedom hybrid task.

Key-Words:- Robot control, constrained manipulators, hybrid position/force control.

1 Introduction

The goal of hybrid position/force control, or hybrid control for short, is to control simultaneously the position of a manipulator and the force it applies to a constraint. Research on hybrid control has been based largely on Raibert and Craig's seminal work [2], in which hybrid tasks are described with respect to a Cartesian *constraint frame*. Positions/rotations are controlled along/about certain axes, while forces/moments are controlled along/about others. The orientation of the constraint frame is chosen such that velocities are constrained to be zero in force-controlled directions, while constraint forces (not including friction forces) are zero in position-controlled directions. In this way, the constraint frame decouples the robot's tool location into velocity-constrained coordinates and force-constrained coordinates. Based on this framework, a number of hybrid controls have been proposed in the literature to regulate position and force simultaneously.

In [3], it is shown that the concept of *orthogonal complements* is incorrectly applied in the framework of [2]. In Section 3 of the present note, we identify two more problematic issues with the framework of [2] which limit the generality of the hybrid control approach. The first problem is that the constraint frame is defined according to velocity constraints, whereas hybrid controls are expressed in terms of position commands. This leads to the erroneous notion of a moving constraint frame [4]. The second problem is that most constraints cannot be decomposed in Cartesian coordinates, as we will demonstrate with some examples. We shall refer to such constraints as *non-Cartesian constraints*.

The problem of force and position control of robots in contact with non-Cartesian constraints was first addressed in [5], where the constraint was modelled as an algebraic equation in robot joint coordinates (i.e. as a holonomic constraint). There, the position was specified via joint coordinates and the force specified via Lagrange multipliers. A similar formulation is presented in [6] using world coordinates. Yoshikawa proposes a hybrid control formulation in [1, 7] that specifies the position and force using generalized coordinates that are directly related to the constraint geometry and the associated generalized forces (as opposed to Lagrange multipliers). This approach is an intuitive generalization of Raibert and Craig's hybrid position/force control to holonomic constraints.

In Section 4 of this note, we present theoretical justification for Yoshikawa's generalised hybrid control formulation. Specifically, we provide a constructive proof that his task coordinates exist for any holonomic constraint. We then apply this result to two examples.

2 The Hybrid Control Problem

In this section, we formulate the hybrid control problem in an arbitrary coordinate system. The resulting coupling between position- and force-controlled coordinates motivates the use of a special coordinate system related to the constraint geometry.

A constrained manipulator is depicted in Figure 1. An *end effector* mounted on the distal link of the manipulator is in contact with a constraint. The end effector of the planar manipulator shown in Figure 1 is simply a point in two dimensional space, and is constrained to a line. More generally, we model the end effector as a rigid body in three dimensional space and the constraint as a mechanism that allows certain end effector positions and orientations but not others.

The position of the end effector is parameterized by a vector of generalized coordinates $p \in R^n$, where n is the number of degrees of freedom (DOF) of the end effector when unconstrained. For example, $n = 6$ for a three dimensional rigid body, $n = 3$ for a planar rigid body, and $n = 2$ for a single point in a plane (as in Figure 1). In

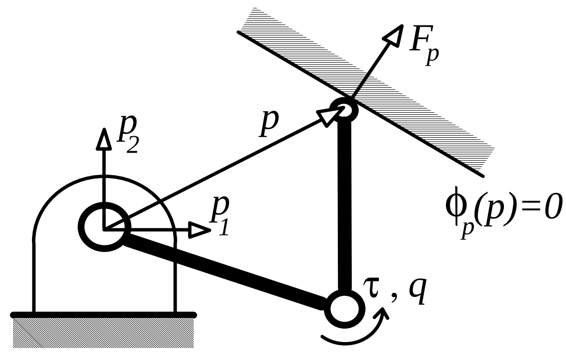

Figure 1: Planar Constrained Manipulator

the case of a three dimensional rigid body end effector, p is usually chosen as $p = \begin{pmatrix} p_t \\ p_r \end{pmatrix}$, where p_t is the three Cartesian coordinates of a point on the end effector, and p_r is the roll, pitch, and yaw angles of the end effector. In general, the coordinates p are unrelated to the constraint geometry.

The constraint is modeled as a set of smooth algebraic equations in p. That is, we assume there is a C^∞ function $\Phi_p : R^n \to R^m$ such that $\Phi_p(p) = 0$. The constraint shown in Figure 1 is actually of the form $\Phi_p(p) \leq 0$, since it permits loss of contact, but we shall assume that the robot is always in contact with the constraint. As well as moving on the constraint, the end effector can apply a generalized force $F_p \in R^n$ to the constraint. The subscript p signifies that F_p is the generalized force corresponding to the generalized coordinates p. For example, if $n = 6$ and p is chosen as a set of three Cartesian position coordinates and three orientation angles, then F_p consists of three force components and three torque components. We do not include frictional forces in the constraint force F_p. Therefore, F_p is normal to the hypersurface $\Phi_p(p) = 0$.

A *hybrid task* is specified by desired values of p and F_p. However, it is not possible to specify independently all $2n$ position and force components. It is shown in [8] that $n - m$ components of p, denoted by p_1, and m components of F_p, denoted by F_{p2}, are independent. This motivates us to define the *task vector* $r_p = \begin{pmatrix} p_1 \\ F_{p2} \end{pmatrix}$. Since the components of the task vector are independent, a hybrid task may be specified by a desired value r_d of the task vector, given as a function of time. The m dependent generalized coordinates, denoted by p_2, and the $n-m$ dependent generalized forces, denoted by F_{p1}, define the *reflected task vector* $\bar{r}_p = \begin{pmatrix} F_{p1} \\ p_2 \end{pmatrix}$. The general form of the functional dependence of $\bar{r}_p$ on r_p is derived in [8].

As an example, consider the simple constraint of Figure 1, and assume the constraint equation is given as

$$\Phi_p(p) = p_1 + 2p_2 - 1 = 0. \tag{1}$$

The end effector can move along the line while applying a force F_p normal to the line. By inspection, the dependence among the position and force components is $F_{p1} = \frac{1}{2}F_{p2}$ and $p_2 = \frac{1}{2}(1-p_1)$. Since p_1 and F_{p2} are independent variables, they may be freely specified and uniquely determine F_{p1} and p_2.

Now let us consider the control of p_1 and F_{p2} using the robot arm shown in Figure 1. The generalized actuator forces $\tau \in R^n$ applied at the joints of the arm are computed by a hybrid control based on feedback of p_1 and F_{p2} . Typically, p_1 is infered from measurements of joint coordinates $q \in R^n$ while F_{p2} is measured using a wrist-mounted force sensor. The *general hybrid control problem* is as follows:

> Given a hybrid task $r_d(t)$ and position and force measurements $p_1(t), F_{p2}(t)$, how should we command the manipulator actuator forces $\tau(t) \in R^n$ in order to keep the error $e = r_p(t) - r_d(t)$ small?

A block diagram associated with the general hybrid control problem is shown in Figure 2. It shows the interaction between the constraint, the robot arm, and the hybrid control. The input to the constraint is the task vector r_p while the output of the constraint is the reflected task vector $\bar{r}_p$. The input to the robot arm is the reflected task vector $\bar{r}_p$ and the actuator forces τ, while the output is the task vector r_p. The input to the hybrid control is the desired task vector r_d and measurements of the actual task vector r_p, while the output is the vector of actuator forces τ.

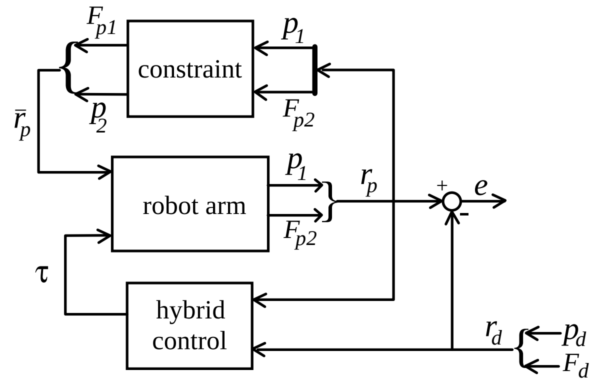

Figure 2: Block Diagram of Hybrid Control Problem

3 Task Frames

The drawback of expressing the hybrid task of Figure 1 in terms of p -coordinates and forces is that F_{p1} is proportional to the controlled force F_{p2}. This is undesirable because F_{p1} acts as a disturbance to the controlled position p_1 and tends to drive the end effector away from the desired value of p_1. If separate controls are used to control p_1 and F_{p2}, they must work against each other to some extent.

In Figure 3, a new coordinate frame O_x has been introduced, and hybrid tasks may be specified via x_1 and F_{x2}. Since the x_1 axis is parallel to the (frictionless) constraint, we have $F_{x1} = 0$. Thus, a desired force F_{x2} does not produce a disturbance force F_{x1}. A frame O_x whose orientation is chosen such that $F_{x1} = 0$ is referred to as a *task frame* (also called a *constraint frame* in [2]). The task frame is the foundation of Raibert and Craig's hybrid control framework [2], wherein hybrid tasks are specified via x_1 and F_{x2}.

A fundamental problem with Raibert and Craig's approach to hybrid task specification is that task frames do not exist for certain constraints. For example, consider the curved constraint of Figure 4. When the end effector is at the point shown, we have $F_{x1} = 0$. However, when the end effector is at any other point on the constraint, F_{x1} is proportional to F_{x2}. Therefore O_x is not a task frame for this constraint. Moreover, this constraint has no task frame be-

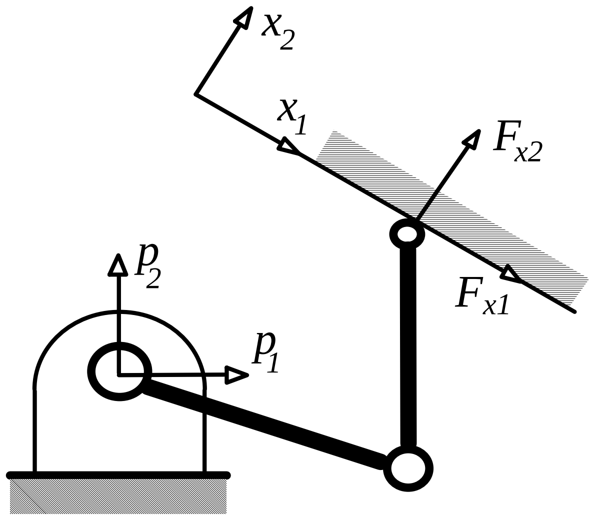

Figure 3: Task Frame for a Linear Constraint

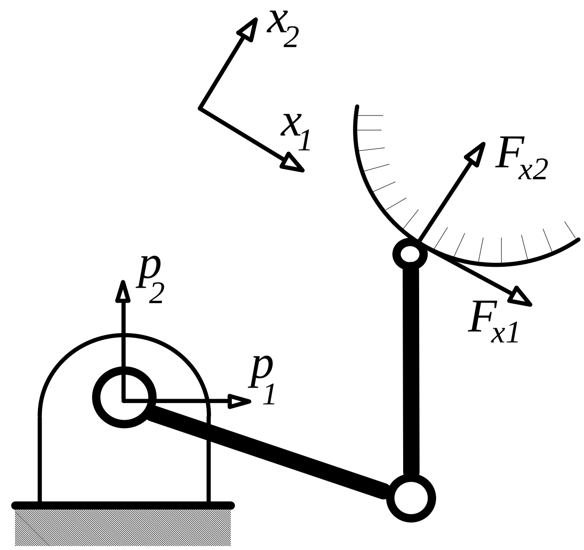

Figure 4: A Constraint That Has No Task Frame

cause it is curved.

To extend the notion of task frames to constraints such as that of Figure 4, it was suggested by way of example in [2] that the task frame could move with the end effector and rotate so that F_{x1} is always zero. However, it is meaningless to specify the position x_1 of the end effector with respect to such a moving frame since this distance would always be zero.

Another approach, proposed in [4], is to have the task frame move with the *desired* position of the end effector and rotate so that F_{x1} is always zero. The desired force is then expressed with respect to this rotating task frame via F_{x2}, while the desired position is given by the trajectory of the task frame itself. For the constraint of Figure 4, the task would be specified by the position and orientation of the task frame and the force F_{x2}. However, this specification requires four variables (two position coordinates, a rotation angle, and force) instead of two. This redundancy is due to the fact that p_2 and the rotation angle are each functions of p_1 (which depend on the constraint equation). Hence, the task is overspecified by this approach.

We conclude, therefore, that the hybrid control framework of [2] properly applies only to constraints that have stationary task frames. We will refer to such constraints as 'Cartesian constraints'. For end effectors represented as points in R^2 or R^3, Cartesian constraints include only straight lines and flat surfaces (if the decoupling condition $F_{x1} \equiv 0$ is to always hold).

4 Generalized Task Coordinates

In this section, we generalize the hybrid control approach to apply to non-Cartesian constraints. Given a general holonomic constraint, the goal is to find a set of generalized coordinates x such that F_{x1} is always zero. We refer to such coordinates as *task coordinates*. Yoshikawa [1, 7] illustrated how task coordinates could be chosen for a number of holonomic constraints. We will prove that task coordinates can be found for any holonomic constraint.

Let us begin by finding task coordinates for the constraint of Figure 4. Given a point p in the plane, let x_2 be the distance from the constraint to p, measured along a line normal to the constraint, and let x_1 be the arc length along the constraint to the normal (from some reference point on the constraint). Figure 5 shows curves of constant x_1 and curves of constant x_2, which together yield a coordinate chart. Let $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$. Since

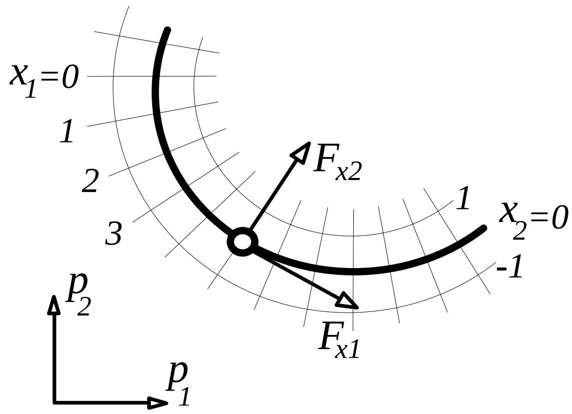

Figure 5: Task Coordinates for a Curved Constraint

x uniquely determines p in the region shown, x is a set of generalized coordinates for the end effector. Let F_{x1} be the generalized force associated with x_1 and let F_{x2} be the generalized force associated with x_2. Clearly, F_{x1} is the tangential force and F_{x2} is the normal force. Since F_{x1} is always zero, the coordinates x are task coordinates for the constraint. Note that there is no task frame associated with x. In this example, where the end effector is modelled simply as a point, the task coordinates x are generally referred to as *curvilinear coordinates*.

Let us formulate a general definition of task coordinates, given a set of generalized coordinates p in R^n, a smooth holonomic constraint $\Phi_p(p) = 0$, and a point p_0 on the constraint. We assume that rank$\frac{\partial \Phi_p}{\partial p}(p_0) = m$. Then the partition of p can be chosen such that $\frac{\partial \Phi_p}{\partial p_2}(p_0)$ is invertible.

Suppose that x in R^n and p are related by a smooth diffeomorphism $x = X_p(p)$ in some neighborhood of p_0 so that x is locally a set of generalized coordinates for the end effector. In x coordinates, the constraint manifold is expressed as

$$\Phi_x(x) \equiv \Phi_p[X_p^{-1}(x)] = 0. \tag{2}$$

We assume the vector partition $x = (x_1, x_2)^T$ with $x_1 \in R^{n-m}$ and $x_2 \in R^m$. Let $x_0 = X_p(p_0)$, and assume x_{10} is the value of x_1 when $x = x_0$.

Definition 1 *Generalized coordinates x are task coordinates if $F_{x1} = 0$ and $x_2 = 0$ for all x_1 in some neighborhood of x_{10}.*

The following theorem states that task coordinates can be found for any holonomic constraint.

Theorem 1 *Let $p_0 \in \Theta_p$ such that $\Phi_p(p_0) = 0$, and assume that $\frac{\partial \Phi_p}{\partial p_2}(p_0)$ is invertible. Then there exists a set of task coordinates x defined on some neighborhood of p_0.*

Proof Define the coordinate transformation $x = X_p(p)$, where $X_p(p) = \begin{pmatrix} p_1 \\ \Phi_p(p) \end{pmatrix}$. Then we have

$$\frac{\partial X_p(p)}{\partial p} = \begin{bmatrix} I_{n-m} & 0 \\ \frac{\partial \Phi_p(p)}{\partial p_1} & \frac{\partial \Phi_p(p)}{\partial p_2} \end{bmatrix}. \tag{3}$$

As $\frac{\partial \Phi_p}{\partial p_2}(p_0)$ is invertible, $\left(\frac{\partial X_p}{\partial p}(p_0)\right)^{-1}$ exists and is given by

$$\begin{bmatrix} I_{n-m} & 0 \\ -\left(\frac{\partial \Phi_p}{\partial p_2}(p_0)\right)^{-1} \frac{\partial \Phi_p}{\partial p_1}(p_0) & \left(\frac{\partial \Phi_p(p)}{\partial p_2}\right)^{-1} \end{bmatrix}. \tag{4}$$

Because $X_p(p)$ is C^∞ and has an invertible Jacobian at $p = p_0$, the inverse function theorem [9] states that there exist neighborhoods Θ_p of p_0 and Θ_x of $x_0 = X_p(p_0)$, and a C^∞ function $X_p^{-1} : R^n \to R^n$, such that for each $x \in \Theta_x$, there exists a $p \in \Theta_p$ given by $p = X_p^{-1}(x)$. Hence, $X_p(p)$ is a smooth diffeomorphism, and thus x is a set of generalized coordinates. Since $\Phi_p[X_p^{-1}(x)] = x_2$, the constraint equation $\Phi_p(p) = 0$ becomes

$$x_2 = 0,\ \forall x_1 \in \Theta_{x1}. \tag{5}$$

Since the work W done by the end effector on the constraint is zero, we have

$$\delta W = F_{x1}^T \delta x_1 + F_{x2}^T \delta x_2 = 0 \tag{6}$$

From (5) we have $\delta x_2 = 0$, which together with (6) gives

$$F_{x1}^T \delta x_1 = 0. \tag{7}$$

Now as (7) must hold for arbitrary δx_1, we have

$$F_{x1} = 0. \tag{8}$$

$\square$

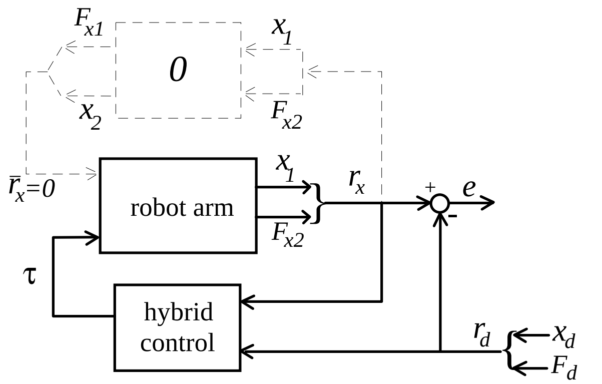

Figure 6: Hybrid Control Problem in Task Coordinates

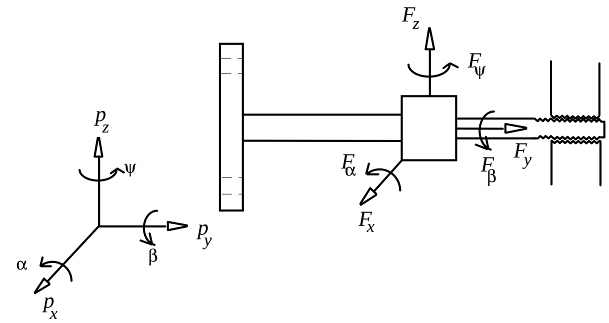

Figure 7: A Bolt-Tightening Task

Figure 6 shows the hybrid control paradigm cast in terms of task coordinates. Note that since $\bar{r}_x = h_x(r_x) = 0$, there is no feedback through the constraint block. In particular, the applied force F_{x2} does not produce a disturbance force F_{x1}, which might otherwise drive the end effector away from a desired value of x_1.

Theorem 1 lends theoretical justification to the method of hybrid task specification proposed in [1, 7], where the idea of using curvilinear coordinates instead of task frames was proposed. In [1, 7], x_2 was chosen as $x_2 = \Phi_p(p)$, but it was not proved that x_1 can then be chosen so that x is a set of generalized coordinates. Also, it was not proved that $x_2 = \Phi_p(p)$ implies $F_{x1} = 0$.

We conclude this section with a 6-DOF example of task coordinates for which a task frame does not exist. In Figure 7, the end effector is a socket wrench, assumed to be rigidly fixed to a bolt which is threaded into a hole. As the constrained end effector has just one remaining degree of freedom, we have $m = 5$. The world coordinates p consists of the three linear translation components (p_x, p_y, p_z) and the three orientation angles (α, β, ψ) applied in turn about the fixed axes.

The constraint equation is

$$\Phi_p(p) = \begin{pmatrix} 2\pi\mu p_y - \beta \\ p_x \\ \alpha \\ p_z \\ \psi \end{pmatrix} = 0, \qquad (9)$$

where μ is the pitch of the thread.

Note that there is no task frame for the constraint of Figure 7 since it is not possible to reorient the p-frame such that one of the generalized forces is zero. If the bolt were simply a peg with no threads, the p-coordinate frame shown in Figure 7 would be a task frame for this constraint, since then all coordinates except p_y and β would be constant. The additional constraint introduced by the threads makes p_y and β mutually dependent. Clearly, reorienting the frame does not eliminate this coupling of rotational and translational motion.

Let us now find a set of task coordinates for this constraint. Note that Φ_p is smooth and that for each $p_0 \in R^6$, rank $\left(\frac{\partial \Phi_p}{\partial p}(p_0)\right) = 5$. If we let $p_1 = p_y$ and $p_2 = \begin{pmatrix} \beta & p_x & \alpha & p_z & \psi \end{pmatrix}^T$, then for each $p_0 \in R^6$, $\frac{\partial \Phi_p}{\partial p_2}(p_0)$ is invertible. From the proof of Theorem 1 we may thus choose the task coordinates x as $x_1 = p_1 = p_y$ and $x_2 = \Phi_p$. This transformation is a global smooth diffeomorphism whose inverse transformation is

$$\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{bmatrix} x_1 \\ x_2 + \begin{pmatrix} 2\pi\mu x_1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \end{bmatrix}. \qquad (10)$$

Therefore x is *globally* a set of task coordinates for this constraint.

Note that another valid choice of task coordinates is $x_1 = \beta$, $x_2 = \Phi_p$. Thus, the position of the end effector can be uniquely specified by either its travel or its rotation.

5 Summary and Conclusion

In this paper, we have explained why a Cartesian task frame cannot be used to decompose hybrid tasks that involve contact with non-Cartesian constraints. We have also provided theoretical justification for Yoshikawa's more general notion of task coordinates by proving that such coordinates can be found for any holonomic constraint. We have demonstrated this approach for the case of a planar manipulator tracking a curved constraint and for the case of a 6-DOF robot tightening a bolt.

This paper provides a formal theoretical framework for specifying hybrid tasks and for designing and analysing hybrid controls. For example, specific hybrid controls that were originally formulated in Cartesian task coordinates (such as the original control of [2]) can be recast in terms of the generalized task coordinates described here and applied to non-Cartesian constraints. This framework has in fact been applied in [8] to examine the stability of three established hybrid controls generalized in this fashion.

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