Nonlinear Control of Cyclic Processes Using Fourier Coefficient Feedback

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Abstract: - In this paper we propose an enhanced control structure for a special class of nonlinear SISO time invariant plants. This shows how the combination of Fourier transform and classical control structure can be applied to iterative feedback control for cyclic processes

Key-Words: - Fourier, control, nonlinear cyclic process

1 Introduction

Nowadays, most of the control methods base on feedback control. However, for cyclic processes it is useful to apply some specific methods. Actually, adaptive control and iterative learning algorithms are quite popular [1]. Alternatively, good results can be achieved by combination of classical and intelligent control [2]. In this paper we will demonstrate an enhanced control structure for a special class of nonlinear SISO timeinvariant plants.

2 Background

In approximation, many plants can be expressed by a structure according to figure1.

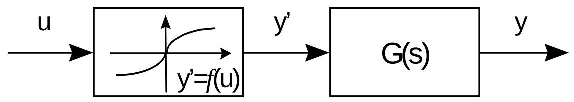

Figure 1: Plant structure.

Generally, the controller must be specially designed depending on the nonlinear function. However, to use an automatic algorithm for cyclic processes we can take advantage of the cyclic characteristic.

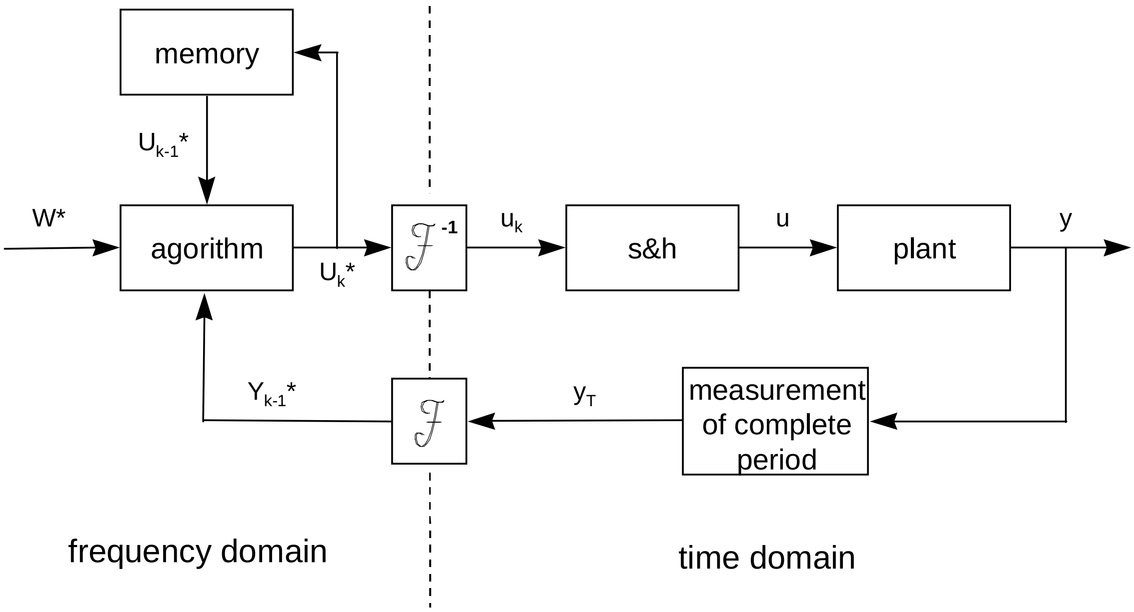

Figure 2: Control structure.

The control error can be reduced by modification of the input signal per period. Usually the variation method is an iterational process in time domain. That implies the drawback of slow improvement rates and substantial remaining offset basing on the lack of any relational description of input and output.
Variation methods in frequency domain can achieve good results more rapidly. But for rapid converging these methods imply the need on complicated algorithms. The structural implementation is shown in figure 2.

The setpoint signal and the output signal are transformed to complex Fourier coefficient vectors $W^*=FFT\{w(0 \dots T)\}$ and Y_{k-1}^* of one single period k within time T. Out of the coefficient ratios U_{ik-1}^*/Y_{ik-1}^* a frequency depending gain vector U_k^* can be calculated. For a linear plant this vector is corresponding to the frequency response. Therefore, if we multiply setpoint coefficients with inverse gain values the first manipulated signal $u(t)$ will almost compensate linear damping, especially for low frequencies (figure 3).

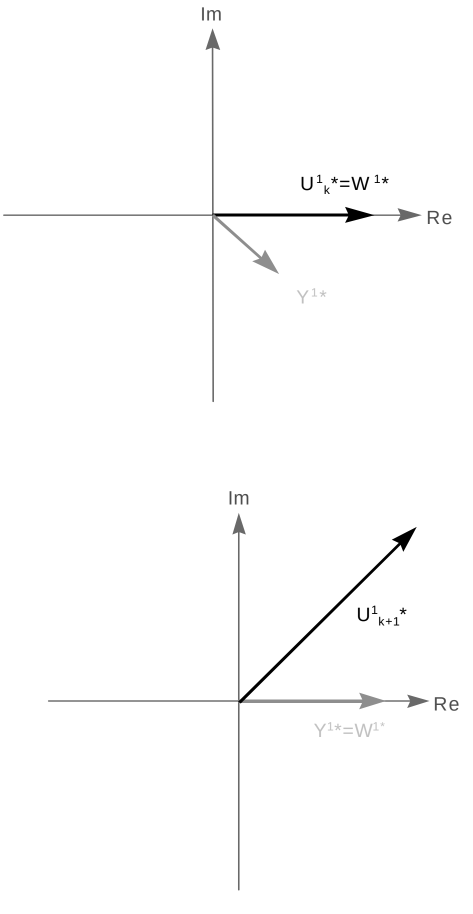

Figure 3: Multiplying linear correction.

For nonlinear plants the nonlinearity will result in additional higher order output coefficients, for example $Y^2{}_k^*$, $Y^3{}_k^*$, depending on lower order input coefficients, for example $U^1{}_k^*$. To remove their influence to the output signal we have to subtract them from the input signal, for example using $U^2{}_{k+1}^*$ and $U^3{}_{k+1}^*$ (figure 4).

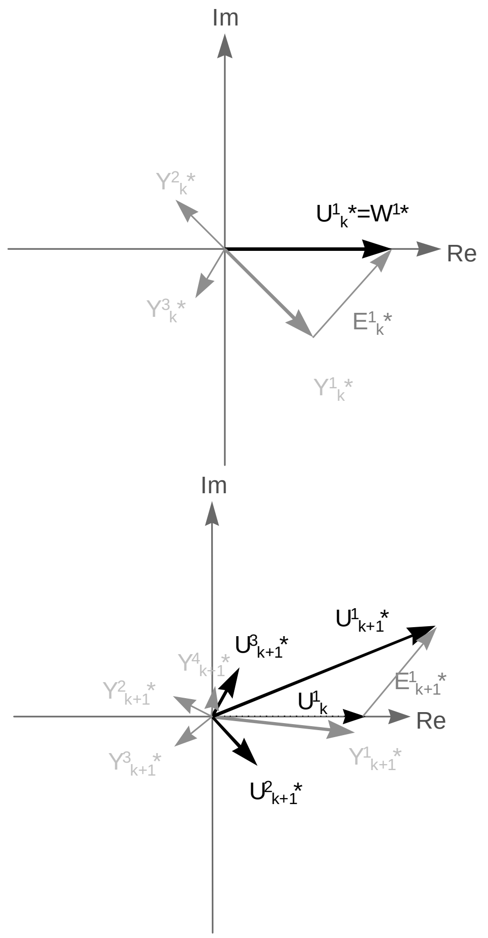

Figure 4: Additive nonlinear correction.

It is obvious, that every modification of input coefficients will lead to a change of many higher order output coefficients. Therefore, the compensation of those coefficients must be done by iteration. The lower frequencies will iterate first, iteration of higher order frequencies will last for longer because they are effected meanwhile by the changes of the lower coefficients. To prevent the system to start swinging from period to period it is necessary to define a damping factor [3]. By using a combination of multiplicative and additive correction it is possible to enhance the results starting from the linear to the nonlinear plant depending on this damping multiplier (figure 5).

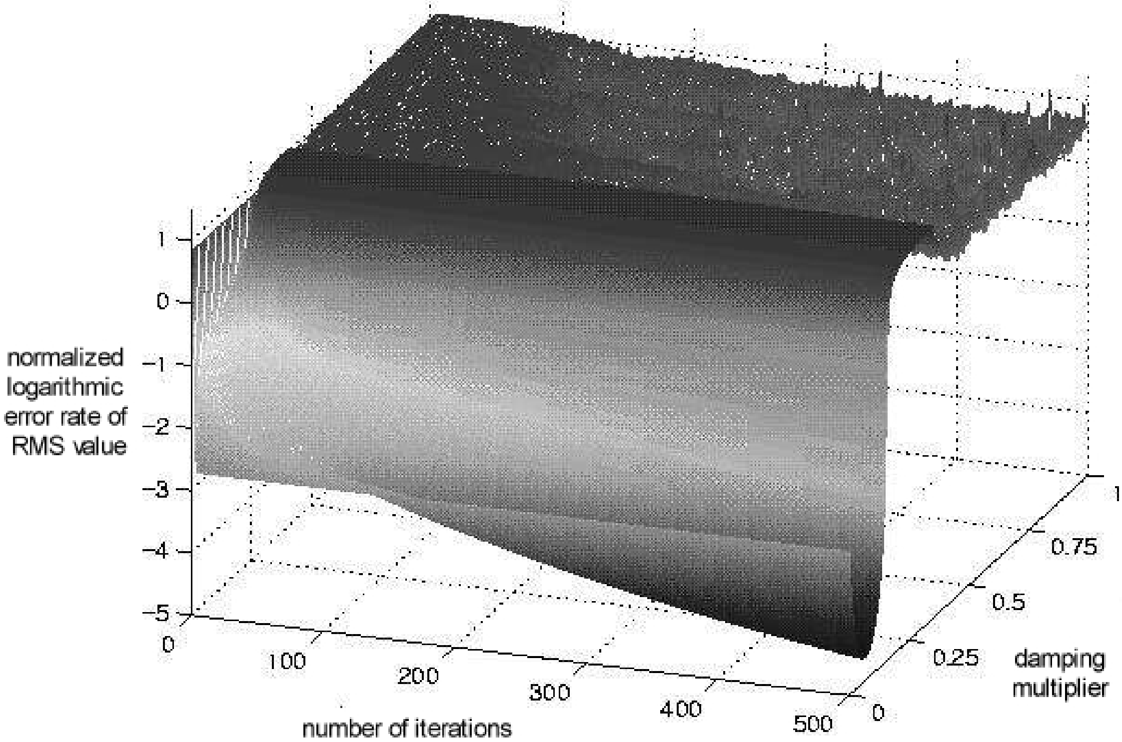

Figure 5: Maximum error depending on damping factor.

3 Restrictions

Using the combined control algorithm there are some restrictions for stability purposes. First of all, the plant itself must be BIBO stable. Furthermore, the linear plant must maintain at least first order lag system. Additionally, gain must be noticeable below one where phase angle is near 180°. Finally, in some cases it also depends on the nonlinear characteristic itself if the controlled system will be stable [4].

In addition to the inherent restrictions it is necessary to define some additional rules for the algorithm, too. To receive reasonable results for the multiplication all coefficients ratios without sufficient excitement have to be set to zero. Furthermore, the correcting coefficients must be limited to a value where the correcting signal u still remains within adjusting rage.

4 Results

Using the Fourier control algorithm for a structure like in figure 1 the results will depend on the plant and nonlinearity.
Exemplarily, the effects are demonstrated below utilising a bounding nonlinearity (figure 6) in combination with a first order lag system. Saturation curves are characteristic for a large number of nonlinear plants.

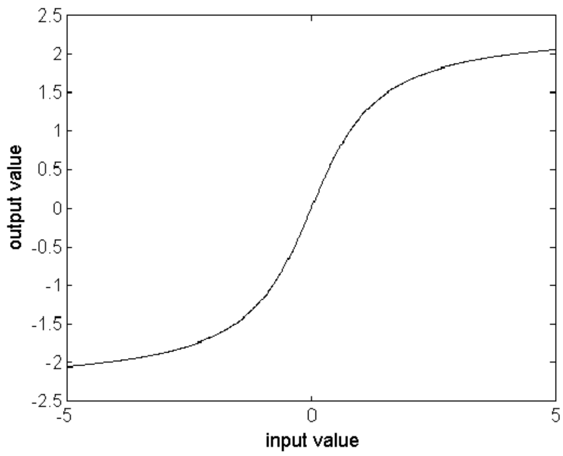

Figure 6: Nonlinearity.

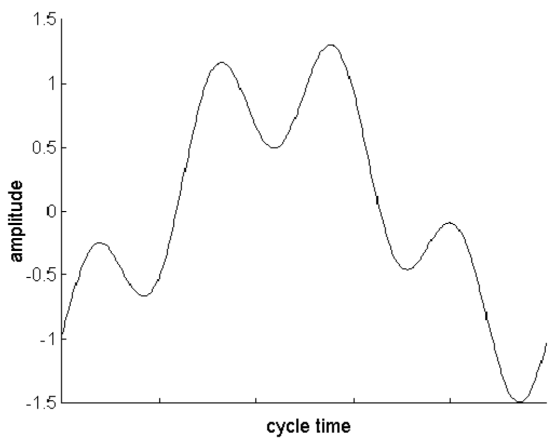

Figure 7: Setpoint value cycle.

The setpoint cycle only contains pretty low frequency coefficients, much lower than the maximum actuator frequency.
It is obvious, that the maximum error rate decreases very fast within the first iteration steps. The reason is, that at the beginning the compensation of the low frequency linear coefficients takes place dominantly. Past 40 iterations the output cycle is almost exactly conform to the setpoint cycle (figure 8), even for higher frequency coefficients.

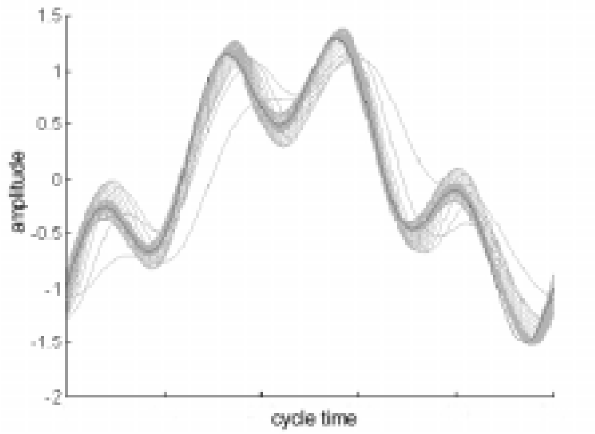

Figure 8: Output value cycles.

The iterational process finally results in an optimal feedforward trace (figure 9). This indicates the main difference to classical control algorithms:
We do obtain a time displacement toward prior time, the control algorithm leads to a predetermination. Furthermore, high frequent errors due to amplitude depending boundary are compensated.

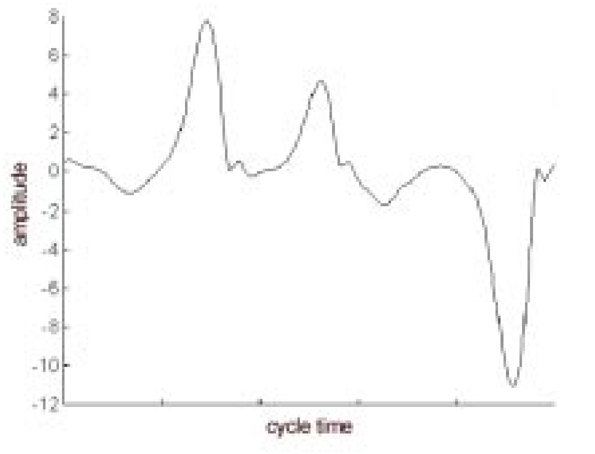

Figure 9: Correcting value cycle.

A Fourier transform of the setpoint value cycle and the correcting value cycle demonstrates the changes within Fourier domain (figure 10). Because of the nonlinearity it is necessary to generate much higher frequencies to finally receive the desired output. For adequate reduction of the control error the frequencies within the correcting value cycle have to be calculated up to ten times the maximum frequency of the setpoint value cycle.

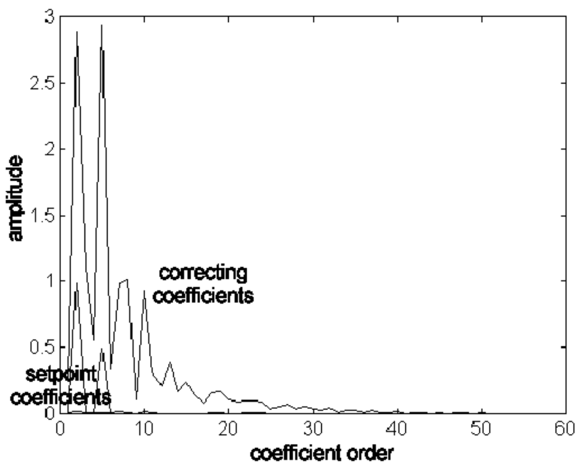

Figure 10: Setpoint value and correcting value coefficients.

5 Summary

Besides linear compensation within the first iteration steps the decrease of the maximum control error mainly depends on compensation of higher order frequencies due to nonlinearities. Additive correction will be sufficient in many cases, but iteration speed is inevitably limited by low frequency modifications effecting high frequency coefficients. Therefore, iteration speed depends on degree and characteristic of the nonlinearity and on the number of coefficients. Therefore, iteration can take quite a time.
To get a universal solution to speed up iteration, it would be necessary to evaluate the relation between any input frequency coefficient and all output coefficients. Although it is fixed relation, generally it will be very difficult to detect it by use of classical methods [4].
The combination of proportional and additive correction is offering a reasonable solution to retrieve both, fast iteration time and high iteration accuracy. Iteration will usually converge for low damping multipliers.

6 Conclusion

Within this paper we proposed an enhanced control structure for nonlinear control that takes advantage of the ability of cyclic processes. It is applied to prediction tasks by means of input and output Fourier coefficients. This basic approach can be used to achieve better accuracy than common control systems, i.e. for example in use for fatigue lifetime stability research or for enhancement of synchronous run of rotating systems.

Actually, the presented control method is not the optimal solution at all. There are too many restrictions now. But these first results prove that control within Fourier domain is a very promising approach for control of a wide range of cyclic processes. It represents a good starting point for further research activities.

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