Position Control for Constrained Robots

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Abstract: - The problem of position control, for robot manipulators constrained to carry a load, is studied. Using a nonlinear P-D feedback law the design requirements of command matching and command following are proved to be always satisfied for constrained robots. Particular feedback laws solving the above problems are determined in the form of analytic expressions of the mechanical characteristics of the manipulator and the load.

Keywords: - Input-Output Decoupling, Multivariable Control, Robotic Systems, Position Control, Command Following, Nonlinear Control, PD controllers. IMACS/IEEE CSCC'99 Proceedings, Pages:6131-6134

1 Introduction

The case of robot systems with constrained kinematics appears to attract increasing interest during the last years [1]-[12]. It is plausible that many manipulator operations, such as the carrying of a load or working on a surface, are modelled as constrained robot systems. Constrained robot systems are described by differential and algebraic equations [4]. The most often met tasks for this robot category are position control and force control [7]-[9]. Both tasks are difficult to be satisfied due to the impulsive characteristics of the constrained robot models.

In this paper the attention is focused on the case of manipulators carrying a load (rigid body). The load is considered to be rigidly gripped by the manipulator’s end effector. The distribution of mass of the load is not considered to be significant, thus only the equations of forces are assumed to govern the motion of the load. The forces applied to the load are equal to the forces applied to the gripper and consequently to the manipulator. The equations of the manipulator are the well known Euler-Lagrange equations [13], modified to involve the forces applied to the end effector by the load. The interaction between the load and the manipulator is modelled as a holonomic constraint. It is important to mention that such a robot configuration (manipulator + load) can be met in a variety of practical applications such as to warehouse goods or to manipulate heavy tools (manufacturing).

Here, position control is achieved for the model of the constraint manipulator carrying a load. With the term position control we mean the placement and orientation of the load. The feedback law feeds back the displacements, velocities and accelerations of the joints together with the position and the velocity of the load as well as the force applied to the load. The motion variables of the joints (displacement and velocity) can easily be measured using sensors embedded into the joints. The motion variables of the load (position and orientation) can be computed solving direct kinematic problem for the end effector and using measurements of the joint displacements. Finally, the forces applied to the load are considered to be measured via force sensors on the gripper. It is important to mention that torque actuators in the joints are assumed to follow the respective commands (voltage signal) in full accuracy.

Using a P-D (Proportional plus Derivative) state feedback law the position control problem is proved to be always satisfied in the sense of command matching. This way, the trajectory of the load, as well as some of the joint displacements are proved to be mathematically equal to the respective external commands. For stability purposes the requirements of command matching are relaxed to those of command following. The latter problem is proved to be always solvable.

It is important to mention that the problem of command matching, via P-D feedback, for general descriptor nonlinear (NL) systems has been studied in [14]. Furthermore, it is mentioned that the case of command matching with simultaneous asymptotic stability for a robot gripping a load has been studied in [12], for the special case of linearized models, with static state linear feedback.

2 Robot Model

Consider a robot with k links. The dynamics of the robot are expressed by the Euler-Lagrange equations. For the case of robot involving contact forces applied to the gripper, the robot dynamics are described by the following NL set of differential equations [13,4]

$$M(q)\ddot{q}+R(q,\dot{q})\dot{q}+N(q)=\tau-J^{T}(q)f \qquad (2.1)$$

the dynamic equations of the load are

$$M_c(p)\ddot{p}+R_c(p,\dot{p})\dot{p}+N_c(p)=L^{T}(p)f \qquad (2.2)$$

The constraints between the load and the manipulator are expressed by the relation

$$\pi(p)=h(q) \qquad (2.3)$$

where $q\in\mathbb{R}^k$ is the vector of joint coordinates of the robot; $p\in\mathbb{R}^6$ is the vector of coordinates of the load (placement and orientation); $M\in\mathbb{R}^k$, $M_c\in\mathbb{R}^6$ are the positive definite inertia matrices of the robot and the load, respectively; $R(q,\dot{q})\dot{q}\in\mathbb{R}^k$, $R_c(p,\dot{p})\dot{p}\in\mathbb{R}^6$ are the vectors of Coriolis and centrifugal forces of the robot and the load, respectively; $N\in\mathbb{R}^k$, $N_c\in\mathbb{R}^6$ are the vectors of gravitational forces of the robot and the load, respectively; $\tau\in\mathbb{R}^k$ is the vector of joint generalized driving forces of the robot; $f\in\mathbb{R}^6$ is the vector of generalized forces (forces and torques) exerted at the end effector of the robot; $h(\cdot):\mathbb{R}^k\to\mathbb{R}^6$ is a function which characterizes the forward kinematics of the robot; $\pi(p)$ denotes the function characterizing the coordinate transformation from the object frame to the robots end effector frame; $J(\cdot):\mathbb{R}^k\to\mathbb{R}^6\times\mathbb{R}^k$ is the manipulator Jacobian of the robot defined by

$$J(q)=\frac{\partial h(q)}{\partial q} \qquad (2.5a)$$

$L(p):\mathbb{R}^6\to\mathbb{R}^6\times\mathbb{R}^6$ is the Jacobian of the load defined by

$$L(p)=\frac{\partial \pi(p)}{\partial p} \qquad (2.5b)$$

The set of the above equations can be grouped into nonlinear state space form as follows:

$$\begin{bmatrix} I_k & 0 & 0 & 0 & 0\\ 0 & I_6 & 0 & 0 & 0\\ 0 & 0 & M(q) & 0 & 0\\ 0 & 0 & 0 & M_c(p) & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}\left(\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} q\\ p\\ a\\ v\\ f\end{bmatrix}\right)=$$

$$\begin{bmatrix} a\\ v\\ -R(q,a)a+-N(q)+J^{\top}(q)f\\ -R_c(p,v)v+N_c(p)-L^{\top}(p)f\\ h(q)-\pi(p)\end{bmatrix}+\begin{bmatrix}0\\0\\I_k\\0\\0\end{bmatrix}\tau \qquad (2.6)$$

where I_k is the k-th dimension unity matrix and where $a=\dot{q}$ is the rate of change of the joint displacement, $v=\dot{p}$ is the velocity of the position variable. According to (2.6) the state vector is

$$x(t)=\begin{bmatrix} q(t) & p(t) & a(t) & v(t) & f(t)\end{bmatrix}^{\top}$$

while the input (command) vector is the vector $\tau(t)$. The commands driving the joint actuator are assumed to be equal to the generated torques. The model in (2.6) is clearly a nonlinear singular model [4], [7]. The rank of the coefficient matrix of the derivative of the state vector is clearly less than the number of its rows or the number of its columns.

In order to complete the model it suffices to determine the output (performance) vector. The performance vector, let y, is chosen to have k entries (equal to the number of inputs). Since, the task is position control, the first 6 elements of the output vector are the elements of the vector p. The rest of the output variables, namely the rest $(k-6)$ variables, are selected to be elements of the vector q, i.e. to be appropriate joint displacements. There is a selection matrix, let J_{qs}, resulting to the vector q_s of selected joint variables $(q_s=J_{qs}q)$ it is obvious that the matrix J_{qs} is of full row rank

$$\mathrm{rank}\left[J_{qs}\right]=k-6 \qquad (2.7)$$

Furthermore, it is noted that in order to involve joint displacements in the performance vector it is necessary that $k>6$. In the case where $k=6$ the matrix J_{qs} is considered to be of zero dimension. According to the above it is clear that the manipulator must be redundant, i.e. the number of joints must be greater than or equal to the number of elements of the position vector. In concluding, the output vector is defined to be

$$y=\begin{bmatrix} p\\ q_s\end{bmatrix}\begin{matrix}\}6\\ \}k-6\end{matrix} \qquad (2.8)$$

or equivalently as a map of the state vector

$$y=\begin{bmatrix} p\\ q_s\end{bmatrix}=\begin{bmatrix} 0 & I_6 & 0 & 0 & 0\\ J_{qs} & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix} q\\ p\\ a\\ v\\ f\end{bmatrix}=$$

$$=C(q,p,a,v,f) \qquad (2.9)$$

In the case where the number of the joints is less than 6, the elements of the performance vector can be appropriately selected elements of the vector p (f.e. the performance is required to be only the placement coordinates). Alternatively, there are many applications where the load is described by less than 6 coordinates (f.e. point mass load). Indeed, if the load is considered to be of point mass and the number of manipulator joints is 3 the dimension of the performance vector is still equal to the number of joints.

3 Control Objective

To achieve position control, apply to the system (2.6) the P-D feedback law

$$\tau(t) = a(q, p, \alpha, v, f, \dot{f}, \dot{\alpha}, \dot{v}) + b(q, p, \alpha, v, f)\omega(t) \quad (3.1)$$

where $\omega(t) \in \mathbb{R}^k$ is the vector of external inputs. The controller is of P-D type since the first feedback term $a(q, p, \alpha, v, f, \dot{f}, \dot{\alpha}, \dot{v})$ depends not only upon the state vector but also upon the derivatives of the state vector. It is mentioned that the derivatives of the forces have not been fed back since it is difficult to be measured and usually sensitive to be computed. The design goal is to determine, if there exist, functions a and b such that the performance vector y follows precisely the external command, i.e.

$$y = \omega \quad (3.2)$$

The above problem is known in the literature as the command matching problem [12], [14]. In order to have response free of impulses, the restriction of command matching is relaxed to that of command following. It is important to guarantee stability together with the command following. The type of stability considered here is that of strong stability. Strong stability requires that for every bounded input and any initial condition the state of the system remains bounded.

It is important to mention that for the treatment of the problem it has been assumed that the functions $M,\ M_c, R, R_c, N, N_c, J, L$ are precisely known to the designer, i.e. that the kinematics of the robot have been computed in details. Furthermore, it is assumed that the function matrix $b \in \mathbb{R}^{k \times k}$ is invertible for every q, p, α, v and f. This holds in order to guarantee the independence of the outputs.

4 Solvability of Command Matching

The possibility of achieving command matching, for the constrained robot system at hand, is examined in the following theorem.

Theorem 4.1: The problem of command matching, for a robot manipulator constrained to grip a load, is always solvable.

Proof: Consider the P-D feedback law (3.1) with

$$a(q, p, \alpha, v, f, \dot{f}, \dot{\alpha}, \dot{v}) = M(q)\dot{\alpha} + R(q, \alpha)\alpha + \\ + N(q) - J^{\top}(q)f - C(q, p, \alpha, v, f),\ b(q, p, \alpha, v) = I_k \quad (4.1)$$

Clearly, the invertibility of b is guaranteed. Applying the feedback law (4.1) to the system (2.6), the resulting closed loop system is derived to be

$$\left.\begin{array}{c} \alpha = \dot{q} \\ v = \dot{p} \\ \pi(p) = h(q) \\ M_c(p)\dot{v} + R_c(p, v)v + N_c(p) = L^T(p)f \\ y = \omega \end{array}\right\} \quad (4.2)$$

According to (4.2) the design requirement has been satisfied and thus the theorem has been proved. ■

On the basis of the proof of Theorem 4.1, the following corollary is established.

Corollary 4.1: A P-D feedback law solving the command matching problem is

$$\tau(t) = M(q)\dot{\alpha} + R(q, \alpha)\alpha + N(q) - J^{\top}(q)f + \\ -C(q, p, \alpha, v, f) + \omega(t) \quad (4.3)$$

■

It is important to mention that the control law in (4.3) appears to have similarities to the well known *inverse dynamic control* (or *torque control*) (see f.e.[13]). However, it appears to have the distinct advantage of perfect command following independently of the singular points of the inertia matrix $M(q)$.

5 Command Following

As it can readily be observed from the closed loop system (4.2), the obvious advantages of the feedback law (4.3) are obscured by the disadvantage of generating impulses (Dirac signals) for every discontinuity of the (possibly bounded) external command. To overcome this characteristic an additional feedback law is proposed. This way the external command $\omega(t)$ is controlled to be

$$\omega(t) = -K\dot{y}(t) + \Gamma w(t) \quad (5.1)$$

where $w(t)$ is the new external input (command). The feedback law is decoupled, i.e. the feedback matrices are chosen to be

$$K = \underset{1,\dots,k}{\mathrm{diag}}\{\kappa_i\}\ ,\quad \Gamma = \underset{1,\dots,k}{\mathrm{diag}}\{\gamma_i\} \quad (5.2)$$

where $\kappa_i, \gamma_i \in \mathbb{R}$ and where $\underset{1,\dots,k}{\mathrm{diag}}\{\cdot\}$ is a diagonal matrix of dimension $k \times k$. Combining (5.1) and (4.3) the overall feedback law can be computed to be

$$\tau(t) = M(q)\dot{\alpha} + R(q, \alpha)\alpha + N(q) - J^{\top}(q)f + \\ -C(q, p, \alpha, v, f) - K\frac{\mathrm{d}\left[C(q, p, \alpha, v, f)\right]}{\mathrm{d}t} + \Gamma w(t) \quad (5.3)$$

Applying the feedback law (5.3) to the system (2.6), or equivalently the feedback law (5.1) to the system (4.2), the resulting closed loop system is derived to be

$$\left.\begin{array}{c} \alpha = \dot{q} \\ v = \dot{p} \\ \pi(p) = h(q) \\ M_c(p)\dot{v} + R_c(p, v)v + N_c(p) = L^T(p)f \\ K\dot{y} + y = \Gamma w \end{array}\right\} \quad (5.4)$$

From the last equation in (5.4) it is observed that the performance variables are governed by the equations

$$\dot{y}_i(t) + \kappa_i^{-1} y_i(t) = \kappa_i^{-1} \gamma_i w_i(t) \ ; \qquad i = 1, \ldots, k \quad (5.5)$$

where y_i is the i-th element of the performance vector y. According to (5.5) and after appropriate choice of κ_i^{-1}, γ_i, the performance variable $y_i(t)$ can follow sufficiently close the command $w_i(t)$. From (5.5) it is further observed that for bounded $w_i(t)$ $(i = 1, ..., k)$ the variables p, v, $\dot{v}$, $q_s, \dot{q}_s$ are impulse free and consequently that q, a, v are also impulse free. Furthermore, the selected joint coordinates q_s as well as the load position coordinates p are stable.

Alternative results guaranteeing stability can also be found in [16].

7 Conclusions

The problem of position control, for robot manipulators constrained to carry a load, has been extensively studied. Using a nonlinear P-D feedback law the design requirements of command matching and command following have been proved to be always satisfied for constrained robots (Theorems 4.1 and 5.1, respectively). Particular feedback laws solving the above problems have been determined (Corollary 4.1, 5.1). The controllers are analytic expressions of the mechanical characteristics of the manipulator and the load. The extension of the present results for uncertain mechanical characteristics of the manipulator is currently under investigation.

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