Robust Input-Output Decoupling via Static Measurement Output Feedback

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Abstract: - In this paper the problem of robust input-output decoupling for linear systems with nonlinear uncertain structure (NLUS), via an independent from the uncertainties static measurement output feedback law, is studied. The necessary and sufficient conditions for the problem to have a solution, are established. The general analytical expressions of the feedback matrices and the decoupled closed loop system are derived. The problem of robust input - output decoupling with simultaneous Hurwitz invariability, is also studied.

KeyWords:-I/O decoupling, robust control, uncertain systems, linear systems, output feedback, robust stability.

1 Introduction

The problem of input-output decoupling is one of the central control design problems having attracted considerable attention since the early 70s (see f.e. [1]-[3]). The case where the system is uncertain, i.e. the respective robust problem, has been solved in [4], using a static state feedback, while the robust input-output decoupling problem via performance output feedback has been solved in [5].

For the case of non uncertain systems the problem of input output decoupling via static measurement output feedback appears to be of limited interest. This holds since the general solution of the state feedback controller matrices solving the problem (see [2], [3]), can be equated to the measurement feedback, factor the measurement output matrix. The linear form of the free parameters in the state feedback general solution recasts the problem to that of solving a standard linear nonhomogeneous equation. For the case of uncertain systems the input output decoupling problem via measurement output feedback is not a trivial extension of the respective state feedback problem. The measurement output matrix is usually uncertain due to measurement devices errors. This problem has not as yet been solved. Motivated by the above observations, here, the problem of robust input-output decoupling is studied for the case of linear systems with NLUS via a static measurement output feedback, i.e. for systems described by $\dot{x}(t) = A(q)x(t) + B(q)u(t), y_M(t) = M(q)x(t),$

y(t) = C(q)x(t) (1.1) where $x \in \Re^n$, $u, y \in \Re^m$ and $y_M \in \Re^\mu$. The matrices A(q), B(q), C(q) and M(q) belong to $\wp(q)$ i.e. they are nonlinear function matrices depending upon the uncertainty vector $q \notin (q_1, ..., q_l) \in \mathcal{Q}$, where \mathcal{Q} is the uncertainty domain and $\wp(q)$ is the set of nonlinear functions of q. The uncertainty domain can be any set, while the values of the functions of $\wp(q)$ are considered to be real. The uncertainties $q_1, ..., q_l$ do not depend upon time. The vector $y_m(t)$ denotes the measurable part of the state vector and y(t) is the output vector. Note that the uncertainties of M(q)(sensoring errors) may be different than those of the system data. However all types of uncertainties have been grouped to the vector $q \notin (q_1, ..., q_l)$.

With regard to the problem of robust input output decoupling via measurement output feedback, the following results are derived: The necessary and sufficient conditions, the general analytical expression of the independent from the uncertainties static measurement output feedback matrices and the general form of the decoupled closed loop system, sufficient conditions for the solvability of robust decoupling with simultaneous robust stabilized ity.

2 Transformation of the problem

To system (1.1) apply the static measurement output feedback law

 $u(t) = Fy_M(t) + G\omega(t) = FM(q)x(t) + G\omega(t) \quad (2.1)$

where $\omega(t)$ is the external input vector. The robust decoupling via static measurement output feedback problem is stated as follows:

Definition 2.1. The problem of robust input output decoupling via static measurement output feedback is solvable if there exist independent from the uncertainties matrices F and G, such that

 $C(q)[sI - A(q) - B(q)FM(q)]^{-1}B(q)G = \operatorname{diag}\{h_i(s, q)\}$ $\forall q \in \mathcal{Q} (2.2)$

For (2.2) to be satisfied it is necessary for the matrix *G* to be invertible. *G* and *F* are independent from *q* if and only if Γ and Φ , defined by

$$\Gamma = \begin{bmatrix} \gamma_1 \\ \vdots \\ \gamma_m \end{bmatrix} = G^{-1} , \Phi = \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_m \end{bmatrix} = G^{-1}F$$
 (2.3)

are independent from q. According to (2.3) equation (2.2) can be rewritten equivalently as follows

diag{ $s^{d_i(q)+1}$ } $P(s,q)C(q)[sI_n - A(q)]^{-1}B(q) =$ = $\Gamma - \Phi M(q)[sI_n - A(q)]^{-1}B(q)$ (2.4) where

$$d_i(q) = \begin{cases} \min\{j \mid c_i(q) [A(q)]^j B(q) \neq 0\} & \text{for } j = 0, ..., n-1 \\ n-1 & \text{if } c_i(q) [A(q)]^j B(q) = 0 & \text{for } j \le n \end{cases}$$

 $(c_i(q) \text{ denotes the } i\text{-th row of } C(q)) \text{ and }$

 $P(s,q) = P_0(q)s^0 + P_1(q)s^{-1} + \cdots =$

diag{ $s^{-d_i(q)-1}$ }diag{ $[h_i(q)]^{-1}$ }, $P_0(q) \neq 0$ Equation (2.4) is equivalent to (2.2) if the following condition is satisfied

$$\det\{\Gamma\} \neq 0 \tag{2.5}$$

3 Necessary and Sufficient Conditions

Before establishing our main results some definitions are presented. Let

$$C^{*}(q) = \begin{bmatrix} c_{1}^{*}(q) \\ \vdots \\ c_{m}^{*}(q) \end{bmatrix}; \quad c_{i}^{*}(q) = c_{i}(q)A(q)^{d_{i}(q)} \quad (3.1)$$

$$j^{*}(i,q) = \begin{cases} \min \left[j \in \{1, ..., m\} / c_{i}^{*}(q)b_{j}(q) \neq 0 \right] \\ 0 \text{ if } c_{i}^{*}(q)b_{j}(q) = 0 \\ ; b_{j}(q) : j - \text{th column of } B(q) (3.2a) \end{cases}$$

$$v_{i}(q) = \begin{cases} c_{i}^{*}(q)b_{j^{*}(i,q)} , & \text{if} \quad j^{*}(i,q) \neq 0\\ 1 & , & \text{if} \quad j^{*}(i,q) = 0\\ (v_{i}(q) \neq 0, \forall q \in \mathcal{Q}) & (3.2b) \end{cases}$$

Taking into account the above definitions and the definition of the operator $\operatorname{rank}_{\Re}[\cdot]$ denoting the rank of an uncertain matrix on the field of real numbers [4] the main theorem is established.

Theorem 3.1. The necessary and sufficient conditions for the robust input output decoupling of the system (1.1), via the independent from q static measurement output feedback law (2.1), are

$$\det\{C^*(q)B(q)\} \neq 0 \quad , \quad \forall q \in \mathcal{Q}$$
 (3.3)

$$\operatorname{rank}_{\mathfrak{R}} \begin{bmatrix} c_i^*(q)B(q) \end{bmatrix}^T = 1 , \quad i = 1, ..., m \quad (3.4)$$
$$\operatorname{rank}_{\mathfrak{R}} \begin{bmatrix} M(q)L_i(q) \\ [v_i(q)]^{-1}c_i^*(q)A(q)L_i(q) \end{bmatrix} =$$
$$=\operatorname{rank}_{\mathfrak{R}} \begin{bmatrix} M(q)L_i(q) \end{bmatrix}, \quad i = 1, ..., m \quad (3.5)$$

where

$$\Delta(q) = B(q) \begin{bmatrix} C^*(q)B(q) \end{bmatrix}^{-1}$$

$$A_c(q) = A(q) - \Delta(q)C^*(q)A(q)$$

$$L_i(q) = \begin{bmatrix} \Delta_i(q) \cdots \begin{bmatrix} A_c(q) \end{bmatrix}^{2n-1}\Delta_i(q) \end{bmatrix}$$

 $\Delta_i(q) = \begin{bmatrix} \delta_1(q) & \delta_2(q) & \cdots & \delta_{i-1}(q) & 0 & \delta_{i+1}(q) & \cdots & \delta_m(q) \end{bmatrix}$ and where $\delta_i(q)$ is the *i*-th column of Δ .

Proof : According to the reformulation of the problem (at the end of Section 2), the robust decoupling problem is solvable if and only if (2.5) is satisfied and there exist (independent of the uncertainties) Γ and Φ that satisfy (2.4). Eq. (2.4) after some algebraic manipulation may be broken down to the following set of algebraic equations

$$\gamma_{i} = p_{i,0}(q)c_{i}^{*}(q)B(q) \qquad (3.6a)$$

$$\begin{cases} p_{i,0}(q)c_{i}^{*}(q)A(q) + \phi_{i}M(q) \\ p_{i,1}(q) p_{i,2}(q) \cdots p_{i,2n}(q) \end{bmatrix} = -\begin{cases} p_{i,0}(q)c_{i}^{*}(q)A(q) + \phi_{i}M(q) \\ \delta_{i}(q) A_{c}(q)\delta_{i}(q) \cdots [A_{c}(q)]^{2n-1}\delta_{i}(q) \end{bmatrix} (3.7)$$

where $p_i(s,q)$ is the *i*-th row of the matrix P(s,q) and $p_{i,0}(q), p_{i,1}(q), \ldots$ are the coefficients of negative powers of *s* of $p_i(s,q)$. According to (3.6a)condition (2.5) is satisfied if and only if (3.3) is satisfied and $p_{i,0}(q) \neq 0$, $\forall q \in Q$. The vector $\gamma_i = p_{i,0}(q)c_i^*(q)B(q)$ is independent from q, with $p_{i,0}(q) \neq 0$, $\forall q \in Q$, if and only if there exist a function, let $v_i(q) \in \wp(q)$ and a vector $b_i^* \in \Re^{1 \times m}$, satisfying the condition $c_i^*(q)B(q) = v_i(q)b_i^*$ where $b_i^* = \begin{bmatrix} b_{i,1}^* \cdots b_{i,m}^* \end{bmatrix}$ is independent from q. According to the definition of rank_{\Re}[·] [4] the above condition is equivalent to (3.4). From (3.5) it holds that $p_{i,0}(q) = p_i^* [v_i(q)]^{-1}$ where $p_i^* \in \Re - \{0\}$ is an arbitrary parameter.

The vectors ϕ_i solving (3.6b), may or may not be independent from q. Here, we are interested for all the solutions that are independent from q. Substitution $p_{i,0}(q) = p_i^* [v_i(q)]^{-1}$ in (3.6b) yields $\tilde{\phi}_i M(q) L_i(q) = [v_i(q)]^{-1} c_i^*(q) A(q) L_i(q)$;

$$\tilde{\phi}_i = -(p_i^*)^{-1} \phi_i$$
 (3.8)

The above equation is a non homogeneous uncertain equation. According to [4] equation (3.8) is solvable for $\tilde{\phi}_i$ independent from *q* iff (3.5) is satisfied.

The symbol "Rank" denotes the rank of rational matrices over the field of rational functions of *s* ($\Re(s)$). The normal rank of a matrix with real or complex entries is denoted by "rank". An algorithm for the computation of rank_{\Re}[·] may be found in [6].

4 The Controller Matrices

Define

$$\eta_{i} = \left\langle \begin{bmatrix} v_{i}(q) \end{bmatrix}^{-1} c_{i}^{*}(q) A(q) L_{i}(q) \ \left\langle M(q) L_{i}(q) \right\rangle_{\Re} \\ B^{*} = \begin{bmatrix} b_{1}^{*} \\ \vdots \\ b_{m}^{*} \end{bmatrix}, H = -\begin{bmatrix} \eta_{1} \\ \vdots \\ \eta_{m} \end{bmatrix}, H^{*} = -\begin{bmatrix} \begin{bmatrix} M(q) L_{1}(q) \end{bmatrix}_{\mathbb{R}}^{\perp} \\ \vdots \\ \begin{bmatrix} M(q) L_{m}(q) \end{bmatrix}_{\mathbb{R}}^{\perp} \end{bmatrix}$$

In order to derive the general solution of the controller matrices the operators $[\cdot]_{\mathbb{R}}^{\perp}$ and $\langle \cdot \setminus \cdot \rangle_{\mathbb{R}}$ as well as the above definitions will be used. In particular, $[\cdot]_{\mathbb{R}}^{\perp}$ denotes an independent from q matrix which is orthogonal to the argument matrix, and $\langle \cdot \setminus \cdot \rangle_{\mathbb{R}}$ denotes the projection (in the field of real numbers) of an uncertain vector to the subspace defined by the rows of the uncertain matrix ([4]).

Theorem 4.1. Assume that system (1.1) satisfies the conditions of Theorem 3.1. Then, the general explicit expression of the independent from the uncertainties controller matrices G and F are

$$G = (B^*)^{-1} (P^*)^{-1}; P^* = \text{diag}\{p_i^*\}, i = 1, ..., m(4.1)$$
$$F = (B^*)^{-1} H + (B^*)^{-1} T H^*$$
(4.2)

where

 $T = \operatorname{diag}_{i=1,\dots,m} \{\tau_i\} \ (\tau_i \in \mathbb{R}^{1 \times \pi_i}, \pi_i = \mu - \operatorname{rank}_{\mathbb{R}}[M(q)[L_i(q)]_{\mathbb{R}}^{\perp}]$

is an arbitrary and independent of q diagonal matrix. *Proof*: According to (2.3, 3.6a) and $p_{i,0}(q) = p_i^* [v_i(q)]^{-1}$, *G* is given by (4.1). According to (3.8) and [4] the general solution of $\tilde{\phi}_i$ is

$$\begin{split} \tilde{\phi}_i &= \tau_i \big[M(q) L_i(q) \big]_{\Re}^{\perp} + \\ &+ \left\langle \big[\upsilon_i(q) \big]^{-1} c_i^*(q) A(q) L_i(q) \ \left\langle M(q) L_i(q) \right\rangle_{\Re} \right. \tag{4.3} \end{split}$$

Substitution of $\phi_i = -(p_i^*)^{-1}\phi$ in (4.3) yields $\phi_i = -p_i^* \tau_i [M(q)L_i(q)]_{\Re}^{\perp} - p_i^* \eta_i$ (4.4) From (4.4, 2.3), the general solution of F, is obtained to be as in (4.2).

5 The decoupled closed loop system 5.1 Closed loop transfer function

Define

$$R_{i}(q) = [M(q)L_{i}(q)]_{\Re}^{\perp}M(q) [\delta_{i}(q)\cdots[A_{c}(q)]^{2n-1}\delta_{i}(q)] (5.1a)$$

$$\zeta_{i}(q) = \left[\eta_{i}M(q) - \left[\upsilon_{i}(q)\right]^{-1}c_{i}^{*}(q)A(q)\right] \times \left[\delta_{i}(q)\cdots[A_{c}(q)]^{2n-1}\delta_{i}(q)\right] (5.1b)$$

 $\begin{bmatrix} \delta_i(q) \cdots [A_c(q)] & \delta_i(q) \end{bmatrix} (5.1b)$ Let $r_{i,j}(s,q)$ and $\eta_i(s,q)$ are the rational vector functions corresponding to the vectors $v_i(q)[R_i(q)]_j$ and $v_i(q)\zeta_i(q)$ respectively, where $[R_i(q)]_j$ is the *j*-th row of $R_i(q)$. The rational vector functions $r_{i,j}(s,q)$ and $\eta_i(s,q)$ can be expressed as the ratio of two polynomials as $r_{i,j}(s,q) = \mu_{i,j}(s,q)/a_i(s,q)$ $(j = 1, ..., \pi_i)$ and $\eta_i(s,q) = \mu_{i,0}(s,q)/a_i(s,q)$ where $\mu_{i,j}(s,q)$ and $a_i(s,q)$ are prime between themselves. For each $q \in \mathcal{Q}$ $a_i(s,q) = s^{\sigma_i} + a_{i,\sigma_i-1}s^{\sigma_i-1} + \cdots + a_{i,0}$ is the least common multiplier of the denominators of $r_{i,j}(s,q)$ and $\eta_i(s,q)$ $(j = 1, ..., \pi_i)$. The polynomials $\mu_{i,j}(s,q)$ $(j = 0, ..., \pi_i$ are of the form $\mu_{i,j}(s, q) = (\mu_{i,j})_{\sigma_i-1}s^{\sigma_i-1} + \cdots + (\mu_{i,j})_0$ where $(\mu_{i,j})_{\zeta} = [(\mu_{i,j})_{\zeta,1}, ..., (\mu_{i,j})_{\zeta,i}](\zeta = 0, ..., \sigma_i - 1)$. Note that $(a_i)_j$ $(\mu_{i,j})_{\zeta,j}$ ($\zeta = 0, ..., \sigma_i - 1$) and σ_i are functions of q. Based upon the above definitions the following theorem is established.

Theorem 5.1. Assume that the conditions of Theorem 3.1 are satisfied and the feedback matrices are those given in Theorem 4.1 The i-th diagonal element of the resulting robustly decoupled closed loop transfer function is

$$\begin{split} h_{i}(s,q) &= (p_{i}^{*})^{-1} \upsilon_{i}(q) \frac{a_{i}(s,q)}{\beta_{i}(s,q)s^{d_{i}(q)+1}} = \\ &= (p_{i}^{*})^{-1} \upsilon_{i}(q) \frac{s^{\sigma_{i}+a_{i,\sigma_{i}^{-1}}(q)s^{\sigma_{i}^{-1}+\cdots+a_{i,0}(q)}}{\left[s^{\sigma_{i}+\beta_{i,\sigma_{i}^{-1}}(q)s^{\sigma_{i}^{-1}+\cdots+\beta_{i,0}(q)}\right]s^{d_{i}(q)+1}} \quad , \\ &\forall q \in \mathcal{Q} \quad (i = 1, ..., m) \ (5.2) \end{split}$$

where $\beta_{i,k} = a_{i,k} + (\mu_{i,0})_k + \sum_{j=1}^{\pi_i} \tau_{i,j}(\mu_{i,j})_k \ (k = 0, ..., \sigma_i - 1)$

and $\tau_{i,j}$ is the *j*-th element of τ_i .

Proof. Substitute (4.4) and the relation $p_{i,0}(q) = p_i^* [v_i(q)]^{-1}$ in (3.7) to yield $\begin{bmatrix} p_{i,1}(q) \cdots p_{i,2n}(q) \end{bmatrix} = p_i^* \tau_i R_i(q) + p_i^* \zeta_i(q)$ (5.3)

 $\begin{bmatrix} p_{i,1}(q) \cdots p_{i,2n}(q) \end{bmatrix} = p_i^* \tau_i R_i(q) + p_i^* \zeta_i(q) \quad (5.3)$ To derive the general form of the transfer function of the decoupled closed-loop system, (5.3) is written as

$$\left[p_{i,1}(q)\cdots p_{i,2n}(q)\right] = p_i^* \sum_{j=1}^{\pi_i} \tau_{i,j} [R_i(q)]_j + p_i^* \zeta_i(q) \quad (5.4)$$

Using the unique bilateral correspondence, between a strictly proper rational function (with order less than n) and the vector involving the first 2n coefficients of its negative power series expansion the relation (5.4) can be expressed equivalently as follows:

$$p_i(s,q) = p_{i,0}(s,q) + p_i^* \sum_{j=1}^{\pi_i} \tau_{i,j} r_{i,j}(s,q) + p_i^* \eta_i(s,q) \quad (5.5)$$

Using (5.1,5.5), relation $p_{i,0}(q) = p_i^* [v_i(q)]^{-1}$ and the comments after (5.1) the theorem is proved.

5.2 Cancelled out polynomial

In this subsection the poles which are cancelled out in the general form of the diagonally decoupled closed loop system will be studied. To study the problem first define $a_i^*(s,q) = a_i(s,q)s^{\kappa_i^*(q)}$ where $\kappa_i^*(q)$ is the multiplicity of the root at the origin of $a_i(s,q)$. Define $\kappa_i(q) = \min\{d_i(q) + 1, \kappa_i^*(q)\} (q \in Q)$. According to this definition the *i*-th element of the transfer function of the closed loop system is $h(q,q) = (n^*)^{-1}h(q) = \frac{a_i^*(s,q)}{q}$

$$\begin{aligned} h_i(s,q) &= (p_i^*)^{-1} v_i(q) \frac{a_i(s,q)}{\beta_i(s,q) s^{d_i(q) - \kappa_i(q) + 1}} , \\ &\forall q \in \mathcal{Q} \ (i = 1, ..., m) \end{aligned}$$
 (5.6)

The polynomials $a_i(s, q)$ and $\beta_i(s, q)$ are prime between themselves. This holds since $a_i(s, q)$ and $\mu_{ij}(s, q)(j = 0, ..., \pi_i)$ are prime between themselves, at any $q \in Q$, by construction. Hence, the polynomials $a_i^*(s, q)$ and $\beta_i(s, q)s^{\mu_i(q)-\kappa_i(q)+1}$ are also prime between themselves, at any $q \in Q$. The polynomial involving the poles which are cancelled in the general form of the transfer function of the robustly decoupled closed loop system are the roots of the polynomial $p_u^*(s,q) = \frac{\det[sI-A(q)-B(q)FM(q)]}{\cosh(s,q)\}} = \frac{\det[sI-A(q)-B(q)FM(q)]}{\prod_{i=1}^m \{\beta_i(s,q)s^{d_i(q)-\kappa_i(q)+1}\}}$. The following theorem regarding the determination of the polynomial involving the poles which are cancelled out in the general form of the closed loop system transfer function is proven in [4].

Theorem 5.2. Assume that the conditions of Theorem 3.1 are satisfied. The poles which are cancelled out from the diagonally decoupled closed loop transfer function, resulting after the application of the class of controllers given in Theorem 4.1, depend entirely upon the open loop system and they are the roots of the polynomial $p_u(s,q) = \frac{\det\{C(q)|sI-A(q)|^{-1}B(q)\}\det[sI-A(q)]}{a(s,q)}$; $a(s,q) = \prod_{i=1}^m a_i^*(s,q)$

6 Robust decoupling with simultaneous Hurwitz invariability

The stabilizability of the transmission poles of the closed loop system is equivalent to

$$\mu_i(q) - \kappa_i(q) + 1 = 0$$

$$\beta_i(s,q) : is \text{ Hurwitz } \forall q \in \mathcal{Q} \quad (i = 1, ..., m) \text{ (6.1b)}$$

The condition (6.1a) is necessary in order to avoid $\mu_i(q) - \kappa_i(q) + 1$ poles at the origin. The second condition, i.e. the Hurwitz invariability of $\beta_i(s,q)$, depends upon the degrees of freedom $\tau_{i,j}$. The degree σ_i of $\beta_i(s,q,\tau_{i,j})$ is a nonlinear map of qwhile the coefficients of $\beta_i(s,q,\tau_{i,j})$ are piecewise continuous functions of q [4]. The degree of $\beta_i(s,q,\tau_{i,j})$ is depending from q. The uncertain domain Q is divided into finite sets, let Q_k (k = 1, ..., v) in which the degrees $\sigma_i(q) = \sigma_{i,k}$ of $\beta_i(s,q,\tau_{i,j})$ remain independent of q. The number of these sets is finite. The sets Q_k (k = 1, ..., v) are considered to be compact. Define

 $A_{i,k}^{**}(q) = \begin{bmatrix} \bar{a}_{i,k}(q) & \bar{A}_{i,k}(q) \end{bmatrix}, \ k = 1, ..., v \quad (6.2a)$ where

$$\bar{A}_{i,k}(q) = \begin{bmatrix} 0 & \cdots & 0 \\ (\mu_{i,1})_{\sigma_{i,k}-1} & \cdots & (\mu_{i,\pi_i})_{\sigma_{i,k}-1} \\ \vdots & \vdots & \vdots \\ (\mu_{i,1})_0 & \cdots & (\mu_{i,\pi_i})_0 \end{bmatrix}$$

$$\bar{a}_{i,k}(q) = \begin{bmatrix} 1 \\ (\mu_{i,0})_{\sigma_{i,k}-1} + a_{i,\sigma_{i,k}-1} \\ \vdots \\ (\mu_{i,0})_0 + a_{i,0} \end{bmatrix}$$
(6.2b)

Based upon the above definitions and the respective results in [4, 7], the following theorem is established. **Theorem 6.1.** Robust decoupling with simultaneous transmission pole robust stabilizability, via an independent from q static measurement output feedback law, can be accomplished if: The conditions

of Theorem 3.1 together with condition (6.1a), are satisfied. The elements of $A_{i,k}^{**}(q)$ (k = 1, ..., v) are continuous functions of q for all $q \in \mathcal{Q}_k$ (k = 1, ..., v). There exist $\sigma_{i,k}$ -row submatrices of $A_{i,k}^{**}(q)$ (k = 1, ..., v), let $A_{i,k}^*(q)$ (containing columns of $A_{i,k}^{**}(q)$ appearing in like positions (k = 1, ..., v)) which are simultaneous positive antisymetric [4].

With regard to the robust stabilizability of the closed loop system poles, combining Theorems 5.2 and 6.1 the following theorem is established.

Theorem 6.2. Robust decoupling via static measurement output feedback with simultaneous robust stabilizability can be achieved if the conditions of Theorem 6.1 are satisfied and $p_u(s,q)$, is Hurwitz invariant for every $q \in Q$.

7 Conclusions

The problem of robust input output decoupling via static measurement output feedback, has extensively been solved. The necessary and sufficient conditions for the problem to have a solution, have been established, the general analytical expressions of the independent from the uncertainties static feedback and the general form of the decoupled closed loop system, have been derived. The polynomial of the cancelled out poles is determined. Finally, sufficient conditions for robust decoupling with simultaneous robust stabilizability, have been derived.

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