A new simplified version of the Fuzzy Controller: The Natural Logic Controller

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Abstract: MIMO (multi inputs multi outputs) systems are difficult to be handled by simple Fuzzy Controllers (CF), because of number of rules to specified. The Natural Logic Controller (NLC) overcomes this difficulty by considering the connection of all necessary outputs as a fuzzy-logic combination using t-norms and a last adjustable parameter. This approach propose an important simplification of the FC that allow easy-design for SISO, SIMO and MIMO control problems. This approach takes in to account the constraints of actuator (saturation) and the supportable limits for certain measurable variables, as for example the output signal, to define the universes of discourse. The main features are, smooth input-output mapping, analytically well defined, and is linear around the state space origin. We show in this paper, that it is also possible to test stability and robustness using the Multivariable Circle Criterion.

Key-Words: Natural Logic Controller, Mixed fuzzy-logic connective, Stability, Robustness.

1 Introduction

All real system has constraints on some of its variables and all these constraints are due to physical reasons as well as technological or security reasons. It is therefore clear that any Control Approach must take it in account. Fuzzy Control takes advantage of this knowledge to define the universe of discourse of variables. We can always consider the utilization of a Fuzzy Controller (FC) when it is possible to describe the expert-knowledge under rules. Nevertheless the complexity of a FC increases exponentially with the number of variables to be considered. In recent years, the scientific community is interested in search techniques to reduce this complexity and several results are already gotten as [1]-[3], to mention only a few.

Our work is placed in this area. Two main ideas were motivated the conception of this new approach, initially introduced by Aguilar-Hernández [4]-[6] and Aceves-Aguilar [7]-[8]. The first is, to take into account explicitly system constraints. And the second is, the reduction at the maximum the complexity of fuzzy controller in order to control multivariable systems in a simple way. This approach has been labeled “Natural Logic Controller”, because it use mixed fuzzy-logic connective for combining all plant observations. And because its design depends only on the natural constraints either imposed by the physical control actions or defined with respect to the acceptable errors. In the next section, we present the definition of NLC from a fuzzy-control point of view. In the section three we present a method to test stability and robustness of this non-linear controller. And finally we discuss its performances on an example.

2 Definition of Natural Logic Controller

2.1 Control Problem

We consider the control problem illustrated on Figure 1. The vector e is composed of m variables $\varepsilon_1, \varepsilon_2 \dots \varepsilon_m \in \Re$ and constructed from the error between the *reference* and the output of system $G(s)$. The control action $u \in \Re$ belongs to the universe of discourse $U := [-U^{max}, U^{max}]$ and each variable ε_i belong respectively to the universe of discourse $E_i := [-E_i^{max}\ E_i^{max}]$. We are looking for a control function $\Psi : E_1 \times E_2 \times \dots \times E_m \rightarrow U$ that assign for each value of vector $e = [\varepsilon_1, \varepsilon_2 \dots \varepsilon_m]^T$ a control action $u \in U$. For this, we will consider the Fuzzy Control Approach.

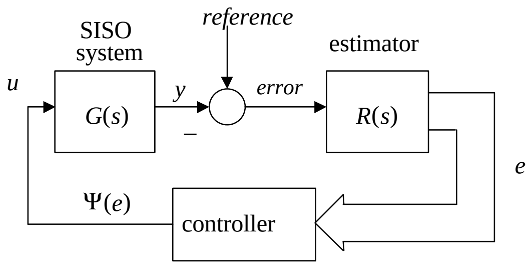

Figure 1.- Closed loop command system.

2.2 Definition of Fuzzy Semantic and Rule Base

In a fuzzy-control approach, it will be necessary to define a fuzzy partition for each variable. Knowing that we want to reduce at the maximum the complexity of the fuzzy controller, we will define the simplest fuzzy partition with two fuzzy subsets $\{N, P\}$, that represent the negative and positive values. Its membership functions $\mu_P(\varepsilon_i)$ and $\mu_N(\varepsilon_i)$ are shown in Figure 2.

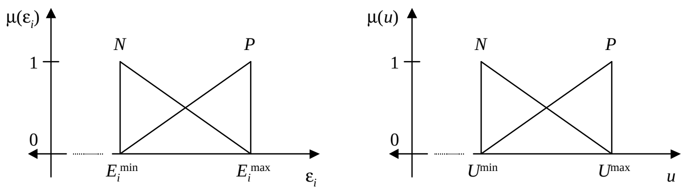

Figure 2.- Fuzzy partitions.

From figure, the degree of truth of proposition $<\varepsilon_i$ is $P>$ is defined by :

$$\mu_P(\varepsilon_i) = \frac{\varepsilon_i - E_i^{\min}}{E_i^{\max} - E_i^{\min}} \tag{1}$$

Because N and P are a fuzzy partition, the degree of truth of proposition $<\varepsilon_i$ is $N>$ is :

$$\mu_N(\varepsilon_i) = 1 - \mu_P(\varepsilon_i) \tag{2}$$

With these fuzzy propositions we are able to characterize a specific situation of the system. This specific situation $\Im$ can be translated as a collection of fuzzy propositions $<\varepsilon_i$ is $X_i>$ where $X_i \in \{N, P\}$. Without loss of generality and with an adequate disposition of sign of signal form $R(s)$, we can consider a MacVicarWhelan-like base rule. The "complete" base has 2^m different rules, which the first and last are :

$$\text{First rule: If } \Im \text{ is } \{<\varepsilon_i \text{ is } N> \mid i = 1\ldots m\} \text{ then } <u \text{ is } N>$$
$$\vdots \qquad\qquad \vdots$$
$$\text{Last rule: If } \Im \text{ is } \{<\varepsilon_i \text{ is } P> \mid i = 1\ldots m\} \text{ then } <u \text{ is } P>$$

Each rule is a fuzzy implication $A \rightarrow B$, where A corresponds to the situation and B corresponds to control action. Here, we consider another simplification, perhaps too much abusive. Instead of to specify and make inference from all 2^m rules, we will only make inference from the first and the last rules. If no supplementary hypothesis is taken, two attitudes can be taken for the interpretation of antecedents. Either one takes the conjunction of the elementary fuzzy proposition, that corresponds to the strictest attitude, that is, it is necessary that all antecedents are true so that the situation is true. Either one takes the disjunction of elementary fuzzy proposition, that corresponds to the weakest attitude, that is, it is sufficient that one of antecedents is true so that the situation is true. Each of these two attitudes are coherent and it found its justification according to the degree of exigency wanted. Moreover, we can take all convex interpolation between these two extreme attitudes by a mixed fuzzy-logic connective operator, defined as : $f_\lambda(y,z) = (1-\lambda) \cdot t(y,z) + \lambda \cdot s(y,z)$, where the t-norm and s-norm are dual and $\lambda \in [0, 1]$. It was demonstrated in [9]-[11] that the exigency notion is ordered respect to λ. Therefore, this last parameter can be left to designer's choice according to performance wanted. In the following sections we will show the effect of this parameter in the control action. Finally, using Mamdani's inference and the associatively of t-norms, we can evaluate our rule base :

$$\mu_P(u) = (1-\lambda) \cdot t(\mu_P(e)) + \lambda \cdot s(\mu_P(e)) \tag{3}$$
$$\mu_N(u) = (1-\lambda) \cdot t(\mu_N(e)) + \lambda \cdot s(\mu_N(e)) \tag{4}$$

where : $\mu_P(e) = [\mu_P(\varepsilon_1), \mu_P(\varepsilon_2) \ldots \mu_P(\varepsilon_m)]^T$ and $\mu_N(e) = [\mu_N(\varepsilon_1), \mu_N(\varepsilon_2) \ldots \mu_N(\varepsilon_m)]^T$.

So, the NLC can be taken as a two-based rules CF. The final control action u is compute with a Center-of-Gravity-like defuzzification method :

$$u = \frac{\mu_P(u) U^{\max} + \mu_N(u) U^{\min}}{1 - \mu_P(u) + 1 - \mu_N(u)} \tag{5}$$

Thus, the command function Ψ is constructed from equation (1)-(5) and it depend on λ parameter. Let's note that our approach avoid the combinatory explosion involved in the Multivariable Fuzzy Approach. This is done thanks to the associatively of t-norms. In the next section we will show that, with some conditions, our approach achieves tangentially a linear control at the origin.

2.3 Scale Transformation

If we consider an application on systems with symmetric constraints, then NLC admit the transformation shows in Figure 3. Let's notice that this consideration is little restrictive because almost any dynamic system has symmetrical limits. In cases of systems whit asymmetric limits, a shift of variable can be applied.

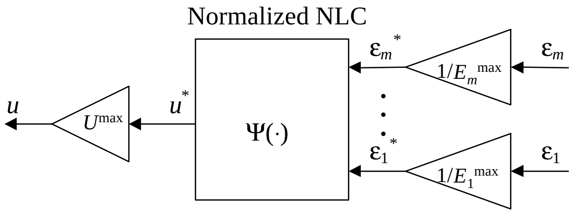

Figure 3.- Normalized NLC.

Definition 1 : Let to be $\varepsilon_i^* \in [-1, \; 1]$ for $\forall$i = 1…*m*, $u^* \in [-1, \; 1]$ and $\lambda \in [0, 1]$ then the ***normalized NLC*** is a non linear control function $\Psi_\lambda : [-1, 1]_1 \times \ldots \times [-1, 1]_m \rightarrow [-1, 1]$ defined by :

$$u^* = \frac{t(\mu) + s(\mu) - 1}{(2\lambda - 1)[t(\mu) - s(\mu)] + 1} \qquad (6)$$

where $t(\cdot)$ and $s(\cdot)$ are dual triangular norms, and :

$$\mu = \begin{bmatrix} \frac{\varepsilon_1^* + 1}{2} & \cdots & \frac{\varepsilon_m^* + 1}{2} \end{bmatrix}^T .$$

□

2.4 Equivalence with Linear Control

An important characteristic of NLC is that it is equivalent, under some considerations, to linear control.

Propriety 1 : The control action constructed by (6) is *exactly equal* to $u^* = ½(\varepsilon_1^* + \varepsilon_2^*)$, for any t-norm and s-norm dual, if m=2 and λ=½.

□

Demonstration : Knowing form Fuzzy Logic that $t(\mu_1, \mu_2) + s(\mu_1, \mu_2) \equiv \mu_1 + \mu_2$ for any t–norm and s–norm dual, equation (6) is equal to :

$$u^* = \left[\frac{t(\mu_1, \mu_2) + s(\mu_1, \mu_2)}{2} \right] - 1 = \frac{1}{2}\left(\varepsilon_1^* + \varepsilon_2^*\right).$$

□

This propriety is verified only for m=2. Nevertheless, it is possible to demonstrate that the control action u^*, choosing the probabilistic t-norms, is linear at the origin. In other words, if universes of discourses are very large (E_i^{max}, $U^{max} \rightarrow \infty$), then the NLC perform a linear control action.

Propriety 2 : Using probabilistic t-norms, the control action u^* obtained from (6) is linear, if $e^* = [\varepsilon_1^*, \varepsilon_2^* \ldots \varepsilon_m^*]^T$ is near of zero, then :

$$u^* \cong \frac{1}{(1-\lambda)(2^m - 1) + \lambda} \sum_{i=1}^{m} \varepsilon_i^* \qquad (7)$$

with $e^* \rightarrow [0 \quad 0 \quad \ldots \quad 0]^T$.

□

Demonstration : The general expression of probabilistic t-norms is $t(\mu) = \prod_{i=1}^{m} \mu_i$ and $s(\mu) = 1 - \prod_{i=1}^{m} (1 - \mu_i)$, so equation (6) is :

$$u^* = \frac{\prod (1 + \varepsilon_i^*) - \prod (1 - \varepsilon_i^*)}{(2l - 1)\left(\prod (1 + \varepsilon_i^*) + \prod (1 - \varepsilon_i^*) - 2^m\right) + 2^m} \qquad (8)$$

Knowing that $\prod_{i=1}^{m} (\varepsilon_i^* + 1) \cong 1 + \sum_{i=1}^{m} \varepsilon_i^*$, and $\prod_{i=1}^{m} (1 - \varepsilon_i^*) \cong 1 - \sum_{i=1}^{m} \varepsilon_i^*$, then (8) is approximately equal to:

$$u^* \cong \frac{1}{(1-\lambda)(2^m - 1) + \lambda} \sum_{i=1}^{m} \varepsilon_i^*$$

□

We tested successfully the NLC in some systems, showing interesting temporal features obtained with the adjustable parameter λ. Nevertheless, it is important to specify under which conditions the system remains stable. Then a serious study of stability of the NLC has been developed and it is presented here next.

3 Stability Analysis of NLC

3.1 The Circle Criterion

There are many results in Nonlinear Stability Theory, like the Nyquist Criterion, Circle Criterion, Popov Criterion, or Small-Gain Criterion, see [12]-[13] for details. In this paper we will work with the Multivariable Circle Criterion, which we recall in the next paragraph.

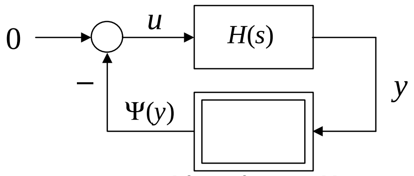

Figure 4.- Canonical form of Lure problem.

Theorem 1 : *The Multivariable Circle Criterion*. For the closed loop system of Figure 4, let us consider :

- $H(s)$ be a linear system,
- $\Psi(\cdot)$ be a static memory-less no-linearity satisfying the sector condition :
  $$[\Psi(e)-K_{\min}e]^T[\Psi(e)-K_{\max}e]\le 0 \text{ for } \forall e\in \Pi \subset \Re^m$$
  for some real matrices $K_{\min}$ and $K_{\max}$, and Π is a convex compact set that includes the origin,
- and let us define :
  $$H_T(s)=H(s)\left[I+K_{\min}H(s)\right]^{-1}=C(sI-A)^{-1}B+D$$
  where (A, B, C, D) is minimal realization,

then the closed loop system is absolutely stable if :

- A is Hurwitz,
- $W^TW=2I+KD+D^TK$ is a definite positive matrix,
- and there exist a symmetric positive definite matrix P solution of the Riccati equation:
  $$PX_A+X_A^TP+PB\left(W^TW\right)^{-1}B^TP+C^TK\left(W^TW\right)^{-1}KC=0 \quad (9)$$
  where $X_A=A-B\left(W^TW\right)^{-1}KC+\frac{\varepsilon}{2}I$, $K=K_{\max}-K_{\min}$ and ε is a small-enough positive constant.

□

In the case that the stability is guarantee in a finite domain (that's mean $\Pi \subset \Re^m$), then we have to compute an estimation of this domain solving the nonlinear optimization problem:

$$\mu=\max_x\left(x^TPx\right) \text{ under } K_{\min}\left(Cx+DU^{\max}\right)=U^{\max}$$

The solution a this problem is given by :

$$\begin{bmatrix}2P & C^TK_{\min}^T\\ K_{\min}C & 0\end{bmatrix}\begin{bmatrix}\bar{x}\\ \phi\end{bmatrix}=\begin{bmatrix}0\\ (I-K_{\min}D)U^{\max}\end{bmatrix} \quad (10)$$
$$\mu=\bar{x}^TP\bar{x}$$

where ϕ is the Lagrange's variable. So, for all initial condition verifying $x(0)^TPx(0)\le\mu$ the behavior of system tends asymptotically to origin.

3.2 Application to NLC

We are interested to study the stability of a linear system with the NLC under the control configuration of Figure 5. This configuration is already under the canonical shape.

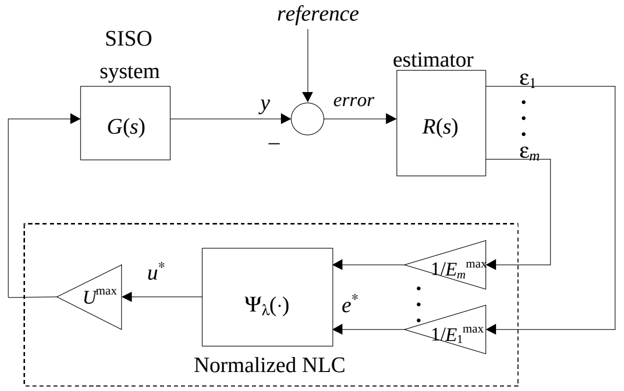

Figure 5.- Command configuration using NLC.

The linear part is :

$$H(s)=\left[diag\left(E_i^{\max}\right)\right]^{-1}R(s)G(s)U_{\max} \quad (11)$$

and the sector condition is :

$$\left[\Psi_\lambda\left(e^*\right)-K_{\min}e^*\right]^T\left[\Psi_\lambda\left(e^*\right)-K_{\max}e^*\right]\le 0 \quad (12)$$

We are looking for $K_{\min}$ and $K_{\max}$ verifying inequality (12). For instant, let us consider a NLC with two inputs $e^*=\left[\varepsilon_1^* \quad \varepsilon_2^*\right]^T$ and let us consider that all normalized variables are in the interval [−1, 1].

Proposition 1 : Let to be $e^*=\left[\varepsilon_1^* \quad \varepsilon_2^*\right]^T$ with $\varepsilon_1^*, \varepsilon_2^* \in [-1\ 1]$, let $u^*=\Psi_\lambda(e^*)$ defined by (6) and let $K_{\min}=\frac{1}{2}[1 \quad 1]$ and $K_{\max}=\frac{1}{4(1-\lambda)}[1 \quad 1]$; then the sector condition is globally verified for any t-norms dual and any λ chosen.

□

Demonstration.- From (6) we can write :

$$\Psi_\lambda\left(e^*\right)=\frac{\varepsilon_1^*+\varepsilon_2^*}{2(2\lambda-1)\{t(\mu_1,\mu_2)-s(\mu_1,\mu_2)\}+2}$$

Using its expression defined for $K_{\min}$ and $K_{\max}$ in equation (12), then :

$$\frac{\left(\varepsilon_1^*+\varepsilon_2^*\right)^2(2\lambda-1)^2}{2(1-\lambda)}\left(s(\mu_1,\mu_2)-t(\mu_1,\mu_2)\right)\left(s(\mu_1,\mu_2)-t(\mu_1,\mu_2)-1\right)\le 0 \quad (13)$$

The first factor is always positive for any value of ε_1^*, ε_2^* and λ, so the negative definition of (13) depend only on last factor involving the t-norms. From Fuzzy Logic we know that $0 \le s(\mu_1, \mu_2) - t(\mu_1, \mu_2) \le 1$ for any t-norms dual, then it follows easily that (13) is negative semi-definite.

□

Now, let us consider a two-inputs-NLC with saturating control action, then matrixes $K_{\min}$ and $K_{\max}$ are:

$$K_{\min} = \frac{\gamma}{4\max\left(\frac{1}{2}, 1-\lambda\right)}[1 \quad 1]; K_{\max} = \frac{1}{4\min\left(\frac{1}{2}, 1-\lambda\right)}[1 \quad 1]$$

for all $\lambda \in [0, 1]$ and a scalar small-enough $\gamma \ge 0$.

Now, we have all elements to study stability. Notice that we have to fix a priori λ value in order to apply the Multivariable Circle Criterion. But we are looking for all values of λ that gives a stable behavior, so we propose the next search algorithm :

Algorithm 1 :

Step 1.-To obtain a minimal realization (A,B,C,D) of (11). If $(A–½BC)$ is not Hurwitz then STOP because there does not exit any value of λ that gives stable behavior, CONTINUE if otherwise.

Step 2.-To search the smallest $\lambda=\lambda_{\inf} \ge 0$ verifying theorem 1 with

$$K_{\min} = \frac{1}{2}[1 \quad 1] \text{ and } K_{\max} = \frac{1}{4(1-\lambda)}[1 \quad 1]$$

Step 3.-To search the biggest $\lambda=\lambda_{\sup} \le 1$ verifying theorem 1.

Step 4.-To search the smallest $\gamma=\gamma_{\lim} \ge 0$ verifying theorem 1 with

$$K_{\min} = \frac{\gamma}{4\max\left(\frac{1}{2}, 1-\lambda_{\inf}\right)}[1 \quad 1] \text{ and}$$

$$K_{\max} = \frac{1}{4\min\left(\frac{1}{2}, 1-\lambda_{\sup}\right)}[1 \quad 1]$$

Step 5.- To compute μ solution of (10) with

$$K_{\min} = \frac{\gamma_{\lim}}{4\max\left(\frac{1}{2}, 1-\lambda_{\inf}\right)}[1 \quad 1], \text{ and } U^{\max} = 1.$$

Step 6.-The closed loop system is absolutely stable for $\lambda \in [\lambda_{\inf}, \lambda_{\sup}]$ and all initial condition verifying : $\mu \ge x(0)^T P x(0)$.

END

4 Illustratif Example

Let us consider a SISO linear system to control with a NLC as is shows in Figure 5. The system linear model and the pseudo-estimator are :

$$\tilde{G}(s) = \frac{s+2}{s^2 + \alpha s + 2} \quad \text{and} \quad R(s) = \begin{bmatrix} 1, & \frac{1}{s} \end{bmatrix}^T$$

with : $\alpha = \alpha_o + \Delta\alpha$, $\alpha_o = -2$ and $\Delta\alpha \in [-1.1 \ \ 1.1]$. So, the nominal model is :

$$G(s) = \frac{s+2}{s^2 - 2s + 2}$$

Let us define its minimal realizations as: $G(s) \leftrightarrow [A_G, B_G, C_G, D_G]$, $R(s) \leftrightarrow [A_R, B_R, C_R, D_R]$ with initial conditions $x_G(0)$ and $x_R(0)$ equals to zero. The *reference* is a step function with amplitude r. Let us suppose that universe of discourses of NLC are $E_1 \in [-1, 1]$ and $E_2 \in [-0.5, 0.5]$. The first variable corresponds to error and the second one corresponds to its integral. Let the command limits $U \in [-30, 30]$. And let us choose probabilistic t-norms.

4.1 Temporal responses

In Figure 6 we show the temporal responses of $y(t)$ controlled by the NLC specified above. The amplitude of *reference* was fixed to 1. We note that the temporal response with $\lambda=1$ is faster than this with $\lambda=0$. Also we note that overshoot and steady time are more important with $\lambda=0$ that $\lambda=1$. These results confirm that the exigency notion is ordered respect to λ, as we discuss in section 2.2.

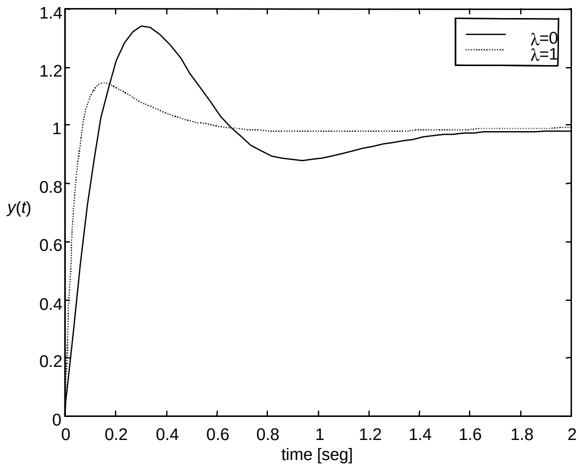

Figure 6.- Temporal response of closed loop system.

4.2 Stability analysis of nominal system

Now we will study stability of steady state $x_H(\infty) = [x_G(\infty), x_R(\infty)]^T$. Buts $x_H(\infty) \neq 0$, so we have to make the next change of variable $x(t) = x_H(t) - x_H(\infty)$, therefore the new variable has equilibrium point at origin $x(\infty)=0$. By definition $error(\infty)=r-y(\infty)=0$, then :

$$\begin{bmatrix} x_G(\infty) \\ u(\infty) \end{bmatrix} = \begin{bmatrix} A_G & B_G \\ C_G & D_G \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ y(\infty) \end{bmatrix}$$

$$\begin{aligned} u(\infty) &= U^{\max} \Psi_\lambda^{\text{Prob}} \left(error(\infty) E_1^{\max}, x_R(\infty) E_2^{\max} \right) \\ &= x_R(\infty) \frac{E_2^{\max} U^{\max}}{3-2\lambda} \end{aligned}$$

A minimal state space realization of $H(s)$ is :

$$\dot{x} = \begin{bmatrix} 2 & -2 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & 0 \end{bmatrix} x + \begin{bmatrix} 30 \\ 0 \\ 0 \end{bmatrix} u$$

$$e = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$

with initial condition :

$$x(0) = -\begin{bmatrix} x_G(\infty) \\ x_H(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ -0.5 \\ \frac{3-2\lambda}{60} \end{bmatrix} r$$

Applying algorithm 1, we found that the system is absolutely stable for any $\lambda \in [0, 0.9999]$ and $r \le 5.01$, $\gamma \ge 0.3$. By temporal simulation we could found a limit cycle at $r = 5.56$ and $\lambda = 0$. This shows that the stability method is not very conservative.

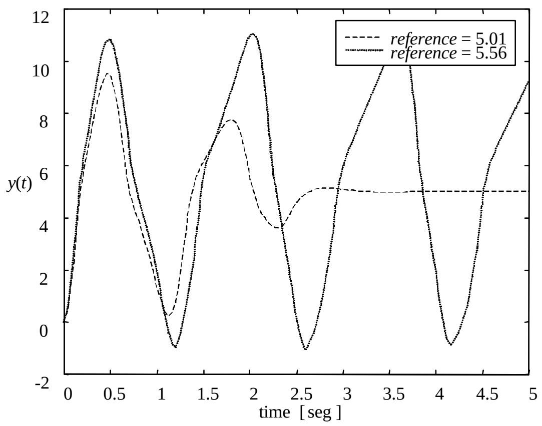

Figure 7.- Temporal response with a probabilistic-NLC and $\lambda = 0$.

4.3 Robustness analysis

Now, let us consider the uncertainty model, which is also equal to:

$$\tilde{G}(s) = \frac{G(s)}{1+\Delta_g G(s)} \qquad \text{with} \qquad \Delta_g = \Delta\alpha \frac{s}{s+2}$$

Then the command configuration is transformed into Figure 8.

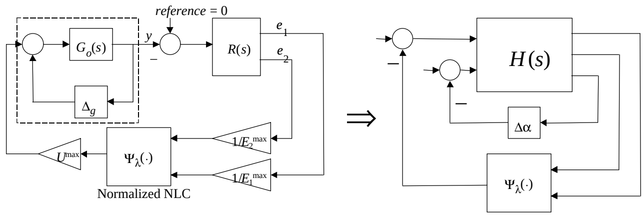

Figure 8.- Equivalent Schema.

A minimal state realization of the new $H(s)$ is :

$$\dot{x} = \begin{bmatrix} 2 & -2 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & 0 \end{bmatrix} x + \begin{bmatrix} 30 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} u$$

$$e = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 2 \\ 1 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} u$$

with :

$$x(0) = \begin{bmatrix} 0 \\ -0.5 \\ \frac{3-2\lambda}{60} \end{bmatrix} r$$

and the new matrixes $K_{\min}$, $K_{\max}$ are :

$$K_{\min} = \begin{bmatrix} \frac{\gamma}{4\max\left(\frac{1}{2},1-\lambda\right)} & \frac{\gamma}{4\max\left(\frac{1}{2},1-\lambda\right)} & 0 \\ 0 & 0 & -\Delta\alpha \end{bmatrix};$$

$$K_{\max} = \begin{bmatrix} \frac{1}{4\min\left(\frac{1}{2},1-\lambda\right)} & \frac{1}{4\min\left(\frac{1}{2},1-\lambda\right)} & 0 \\ 0 & 0 & \Delta\alpha \end{bmatrix}$$

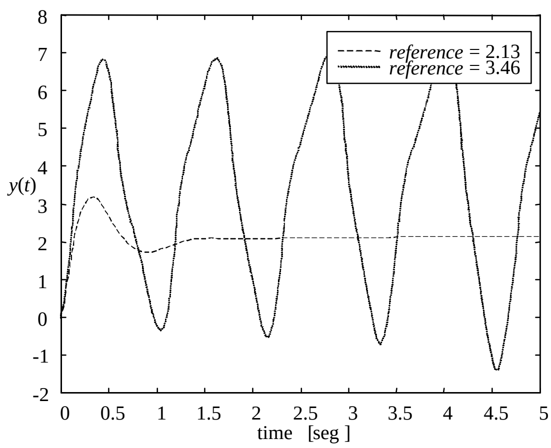

Figure 9.- Temporal response of NLC and uncertainty model.

Applying Circle Criterion we found that the system is robust for any $\lambda \in [0, 0.9]$, $\gamma \geq 0.31$ and all $\Delta\alpha \in [-1.1\ \ 1.1]$. The maximal amplitude estimated for *reference* is $r \leq 2.13$. By temporal simulation we found a limit cycle with $\lambda = 0$, $\Delta\alpha = -1.1$ and $r = 3.46$, showing again that our test is not very conservative.

5 Conclusions

The NLC approach proposes an important simplification of the FC that allows easy-design for SISO, SIMO and MIMO control problems. It can be noticed that only the universes of discourse have to be defined.

We think that the interest of NLC is in three facts :

a) When linear control was applied, our approach could give interesting results.
b) If absolute stability is guarantee for an interval $\lambda \in [\lambda_{inférieur}\ \lambda_{supérieur}]$, then an operator (or supervisor) can choose freely a specific value of λ depending on desired performances.
c) Because the NLC has only a few parameters, an automatic tuning could be developed.

With Circle Criterion we are able to test stability and robustness of a two-inputs NLC and for any t-norm chosen. Practice show that our method is not very conservative. The principal disadvantage is that the stability is guarantee for *any* control actions satisfying the sector condition (linear control included). This test takes the NLC as a source of uncertainty.

As future works we propose the application of NLC to a real multivariable process.

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