MRAC using a Variable Structure-based Neuron-like model

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Abstract: -In this work it is proposed a model reference adaptive control (MRAC) law for a class of nonlinear systems. It is used a virtual (neuron-like) time variant linear model, that updates its parameters using a variable structure-based algorithm. It is proved that the virtual model signals will converge to the nonlinear systems, and using the linear control law, it yields to a accurate tracking for the reference model. It is presented an illustrative example using a nonlinear system.
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1 Introduction

Since the creation of the sliding regime theory [1], there have been many publications that uses these approach for creating different kinds of control laws [2,3,4]. One of the most recent paper can be found in [5]. The use of Variable Structure Control (VSC) for updating the parameters in dynamical weights neural networks [6,7,8] have been proposed together with some identification and control applications. The authors of these paper have also proposed an adaptive control law using a state feedback control which parameters were adapted using a VSC-based algorithm, but this approach is only for regulation in time variant linear systems [9]. In this paper, it is proposed a Model Reference Adaptive Control law, as the one presented by Slotine and Li [10], but for a class of nonlinear systems, using a VSC-based virtual linear model for obtaining the parameters to be used in the control law.

The paper is organized as follows: Section 2 presents the formulation of the problem. In Section 3 is presented using a theorem the reference model adaptive control proposed and the parameters adaptation law. Section 4 show an example for the control scheme proposed. Section 5 gives some conclusions and some further works in this direction.

2 Problem Formulation

Let consider the nonlinear system given by:

$$\overset{(n)}{y} = f(y, \dot{y}, \dots, \overset{(n-1)}{y}, t) + g(y, \dot{y}, \dots, \overset{(n-1)}{y}, t)u(t) \qquad (1)$$

with g(.)≠0 and where the components $y, \dot{y}, \dots, \overset{(n-1)}{y}$ are measurable.

Defining the following time variant linear neuron-like model:

$$\begin{aligned}\overset{(n)}{y_v}(t) = &-a_{n-1}(t)\overset{(n-1)}{y_v}(t) - \dots - a_1(t)\dot{y}_v(t) \\ &- a_0(t)y_v(t) + u(t) + \gamma(t)m\end{aligned} \qquad (2)$$

where $a_i(t)$ and $\gamma(t)$ are adjustable parameters.

The objective is to use the parameters found using a vsc-based algorithm for the virtual model (2), in order to give an accurate track of the nonlinear system (1) to the reference model given by:

$$\begin{aligned}\alpha_m \overset{(n)}{y_m}(t) = &-\alpha_{n-1}\overset{(n-1)}{y_m}(t) - \dots - \alpha_1(t)\dot{y}_m(t) \\ &- \alpha_0(t)y_m(t) + r(t)\end{aligned} \qquad (3)$$

3 MRAC using a Variable Structure-based Neuron-like model

In this section it is presented the theorem used for fill the requirements given in the previous section in order to provide an accurate track to a reference model for a given nonlinear system.

Theorem 1:
If the parameters of the equation (2) are adjusted according to:

$$\begin{bmatrix} a_0(t) \\ a_1(t) \\ \vdots \\ a_{n-1}(t) \\ -\gamma(t) \end{bmatrix} = \frac{\text{NUM}}{y_v^2 + \dot{y}_v^2 + \ldots + \overset{(n-1)}{y}_v{}^2 + m^2} \begin{bmatrix} y_v \\ \dot{y}_v \\ \vdots \\ \overset{(n-1)}{y}_v \\ m \end{bmatrix}$$

$$\text{NUM} = -\overset{(n)}{y} + u(t) - \lambda_{n-1}e_n(t) - \ldots - \lambda_1 e_2(t) - W\,\text{Sign}(e(t)) \qquad (4)$$

where the virtual model error signals are defined by:

$$e_1(t) = y(t) - y_v(t)$$

$$e_2(t) = \dot{y}(t) - \dot{y}_v(t) = \dot{e}_1(t)$$

$$\vdots$$

$$e_n(t) = \overset{(n-1)}{y} - \overset{(n-1)}{y_v} = \dot{e}_{n-1}(t)$$

and the sliding surface is given by:

$$e(t) = e_n(t) + \lambda_{n-1}e_{n-1}(t) + \ldots + \lambda_1 e_1(t)$$

(where the λ_i are selected in order to guarantee that e(t) is a Hurwitz polynomial).

Then, it can be used the following control law for making that the non linear system (1) tracks the reference model (3) accurately:

$$u(t) = \overset{(n)}{y}_m - \beta_{n-1}\overset{(n-1)}{e_T}(t) - \ldots - \beta_0 e_T(t) + a_{n-1}\overset{(n-1)}{y_v}(t) + \ldots + a_0 y_v(t) - \gamma(t)m \qquad (5)$$

Where the tracking error signal is defined as:

$$e_T(t) = y_v(t) - y_m(t) \qquad (6)$$

and the parameters β_i are selected such that $s^n + \beta_{n-1}s^{n-1} + \ldots + \beta_0$ is a Hurwitz polynomial.

The error between the virtual model and the nonlinear system will converge to zero in finite time and a sliding mode will maintain the error trajectories in bounded neighborhood of the surface e(t)=0.

Proof:

Compute the time derivative of the sliding surface as:

$$\dot{e}(t) = \overset{(n)}{y}(t) - \overset{(n)}{y_v}(t) + \lambda_{n-1}e_n(t) + \ldots + \lambda_1 e_2(t) \qquad (7)$$

By substituting the proposed parameter adaptation laws (4) in (7), the following expression is obtained:

$$\dot{e}(t) = -w\,\text{sign}(e(t)) \qquad (8)$$

It can be seen that for all e(t)≠0, $e(t)\dot{e}(t) = -w|e(t)| < 0$ for all w>0 (Noting that e(t)sign(e(t)= |e(t)|)

Thus, the sliding surface e(t), satisfies a differential equation with discontinuous right hand side whose solution exhibits a sliding regime in a finite time tr (Utkin,1992).

Notice that equation (8) may be solved explicitly as:

$$\int_0^{tr} \dot{e}(t)dt = -w\int_0^{tr} \text{sign}(e(t))dt \Rightarrow$$

$$\Rightarrow e(tr) - e(0) \le -w\,tr\,\text{sign}(e(0))$$

At time tr the value of e(t) is zero and therefore:

$$e(0) \ge w\;tr\;\text{sign}(w(0)) \qquad (9)$$

Multiplying both sides of inequality (9) by «sign(e(0))» yields:

$|e(0)| \ge w\,tr$

and therefore

$$tr \le \frac{|e(0)|}{w} \qquad (10)$$

Substituting the control law (5) in the virtual model expression(2) it is obtained:

$$\overset{(n)}{y_v} = \overset{(n)}{y_m} - \beta_{n-1}\overset{(n-1)}{e_T} - \ldots - \beta_0 e_T \qquad (12)$$

but, $\overset{(n)}{y_v} - \overset{(n)}{y_m} = \overset{(n)}{e_T}$, so (12) can be rewritten as:

$$\overset{(n)}{e_T} + \beta_{n-1}\overset{(n-1)}{e_T} + \ldots + \beta_0 e_T = 0 \qquad (13)$$

Considering that the β_i were selected according to a Hurwitz polynomial, it can be seen that $e_T \to 0$, and consequently $y_v \to y_m$. It has been previously shown that selecting the virtual model parameters using the VSC-based algorithm it will occur that $y_v \to y$. So it yields that $y \to y_m$.

...

It is important to point out that if « $\overset{(n)}{y}(t)$ » is not measurable, the updating law for the parameters in (3.4) may be rewritten in the following terms:

$$\begin{bmatrix} a_0(t) \\ a_1(t) \\ \vdots \\ a_{n-1}(t) \\ -\;((t \end{bmatrix} = \frac{\text{num}}{y_v^2 + \dot{y}_v^2 + \ldots + \overset{(n-1)}{y}_v{}^2 + m^2} \begin{bmatrix} y_v \\ \dot{y}_v \\ \vdots \\ \overset{(n-1)}{y}_v \\ m \end{bmatrix}$$

$$\text{num} = u(t) - \lambda_{n-1}e_n(t) - \ldots - \lambda_1 e_2(t) - W\,\text{Sign}(e(t)) \qquad (11)$$

with W being a positive design constant satisfying $w > B_{\overset{(n)}{y}}$, where $B_{\overset{(n)}{y}}$ is a bound for $\overset{(n)}{y}$.

4 Illustrative Example

Let consider the nonlinear system:

$$\ddot{y}(t) = f(y(t), \dot{y}(t), t) + g(y(t), \dot{y}(t), t) * u(t)$$

where:

$$f(y(t), \dot{y}(t), t) = 4 * \sin(3t) * y(t) * \dot{y}^2(t) * \cos(\dot{y}(t))$$

$$g(y(t), \dot{y}(t), t) = [1 + \sin(t) * \cos(y(t) * \dot{y}(t))]$$

It is defined the virtual model as:

$$\ddot{y}_v(t) = -a_1(t)\dot{y}_v(t) - a_0(t)y_v(t) + u(t) + \gamma(t)m$$

Our goal is to reach an accurate tracking to the following reference model:

$$\ddot{y}_m = -y_m - 0.25\dot{y}_m + ref$$

where

$$ref = \begin{cases} 1 + \sin(4t) & \text{if } t \le 30\,\text{s.} \\ 0.5 & \text{if } t > 30\,\text{s.} \end{cases}$$

In Figure 1 it is depicted the tracking error (difference between the reference signal and the virtual model output), and the sliding surface created in such a way that the difference between the virtual model output and the nonlinear system output converge to zero.

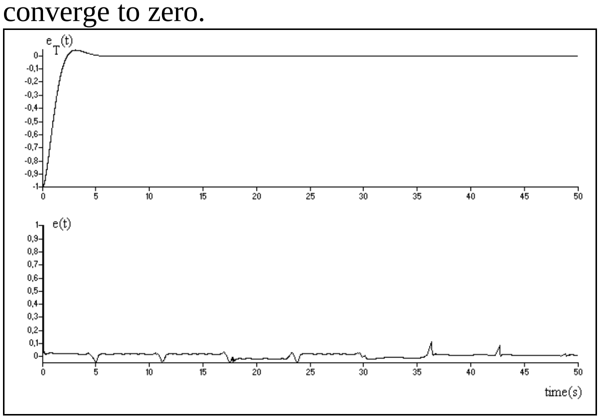

Fig.1. Tracking Error and Sliding surface for the example

Figure 2 illustrates the reference model signal, the virtual model output and the nonlinear system output with different initial conditions. Note that it doesn't takes much time for reaching the reference signal, and after that moment, it follows accurately the reference model including when it has a change on the reference signal (30 sec.)

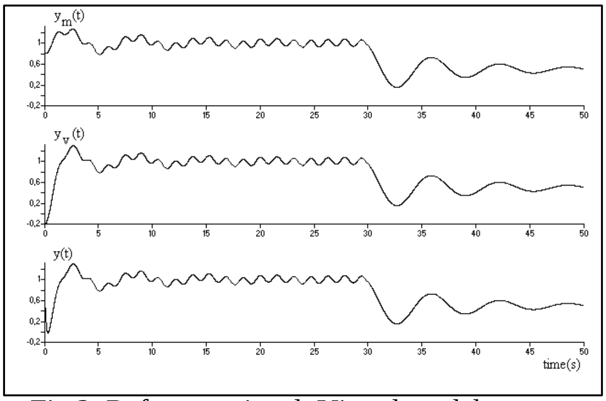

Fig.2. Reference signal, Virtual model output, Nonlinear system output

In Figure 3 can be seen the virtual model parameters and in Figure 4 it is shown the control signal generated using the proposed adaptive law.

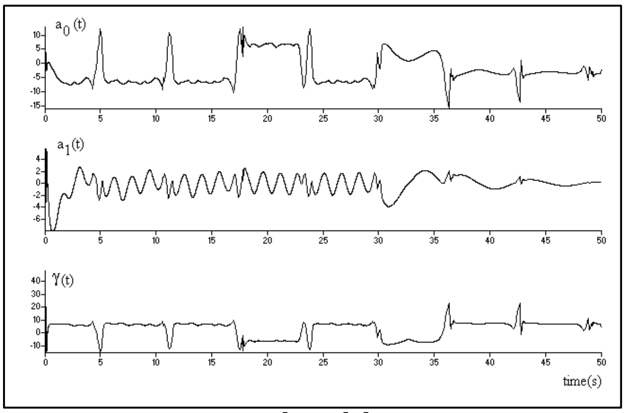

Fig.3. Virtual model parameters

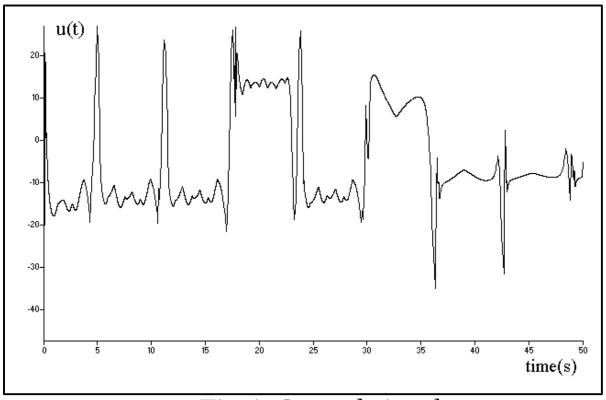

Fig.4. Control signal

5 Conclusions

In this paper have been proposed an adaptive control law for a class of nonlinear systems for an accurate tracking of a reference model. It was proposed a linear time variant virtual model for obtaining the parameters to be used in the control law. The virtual model parameters were adjusted using a Variable Structure-based algorithm.

It was shown that the control scheme proposed works was very easy to implant and the parameters such as the convergence time and the tracking error can be defined according to the performance desired for the system.

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