PLENARY LECTURE
dedicated to Professor L.ZADEH

Fuzziness in Multidimensional Systems
by

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Abstract: In this paper, the concept of deterministic multidimensional systems with fuzzy variables, the concept of stochastic multidimensional systems with fuzzy variables, and the concept of fuzzy multidimensional systems with fuzzy variables are introduced and briefly studied. We mainly examine 2-D Roesser models and 2-D Fornasini-Marchesini models, though the analysis can be extended in m-D Roesser models and m-D Fornasini-Marchesini models (with m>2).

I. Introduction

Fuzzy Sets, Fuzzy Logic and Fuzzy Systems theory was founded by L.A. Zadeh by his historical paper [3] in 1965, with the purpose of formalizing the representation and management of imprecise or approximate knowledge. Sets, Logic and Systems contain or manage in many cases imprecise (approximate) information, specifically when we deal with very complex systems for which the complete mathematical description is impossible or non-necessary. Such Sets, such Logic or such Systems are called Fuzzy.

It is also well known, that events, situations, activities, etc of the human kind or the nature are not always described by algebraic or differential equations due to their imperfect knowledge.

So, for example we have precise methods for studying a classical electric circuit, an electron in a voltage well, a wave in a wave-guide, but we have lack for methods that deal with systems which are too complex or too ill-defined or contain uncertain or imprecise or incomplete information, for examples in fields like life science, philosophy, economics, phychology, ethics, sociology, religion etc.

Multidimensional Systems are systems that process information (or signal) in two or more dimensions. In the case of precise or non-fuzzy multidimensional systems there exists already an extended mathematical theory accompanied by many algorithms, computational techniques and practical applications [1,2]. Multidimensional Systems describe dynamical phenomena parametrized by many independent variables (e.g. time and space coordinates) and are mainly used in Image Processing (image deblurring, X-ray enhacement, seismology, computerised tomography and other biomedical two-dimensional signal processing). Radar and Sonar Technology, Large Scale Systems Analysis, Multivariable Analysis, Distributed System Analysis and Analysis of many problems of Mathematical Physics (Laplace equation, Helmholtz equation, Wave equation, Elasticity equation, Diffusion equation, Absorption equation etc in various fields with various boundaries). Multidimensional systems can be linear or non-linear, shift-invariant or shift-variant, causal or non-causal, discrete variables or continuous variables or mixed (hybrid) variables, single-input-single-output or multiple-input-multiple-output. Detailed Reviews can be found in [1] and [2].

However, in many cases multidimensional systems can also include fuzziness in their variables or/and in their structure. The investigation of such multidimensional systems, i.e. with fuzziness, is the scope of this paper. The fuzziness of a multidimensional system is also due to the ill-defined or complex or imperfect (uncertain or imprecise or incomplete) information both for the system itself and its input and its output.

The present paper is organized as follows:
In Section 2, an introductury presentation of multidimensional systems with fuzziness is given. In Section 3, the analysis is restricted (without loss of generality) in fuzzy 2-D systems described by the Roesser model or by Fornasini-Marchesini model.

II. Fuzzy Multidimensional Systems

The main models for an *m*-dimensional systems are the Roesser (or Givone-Roesser) model and the Fornasini-Marchesini model [1,2].

a) Roesser model:

$$x(n+1) = f\left(x(n),u(n);n\right), \quad y(n) = g\left(x(n),u(n);n\right)$$

where $x(n),x(n+1),y(n),u(n)$ are vectors of appropriate dimensions where by n is denoted the *m*-tuple $(n_1,\ldots,n_m)$ and $x(n+1)$ has a special meaning where

$$u(n) = \begin{bmatrix} u_1(n_1,\ldots,n_m) \\ \vdots \\ u_{m_1}(n_1,\ldots,n_m) \end{bmatrix}$$

$$x(n)=\begin{bmatrix} x_1(n_1,\ldots,n_m) \\ \vdots \\ x_m(n_1,\ldots,n_m) \end{bmatrix}$$

$$y(n)=\begin{bmatrix} y_1(n_1,\ldots,n_m) \\ \vdots \\ y_{m_2}(n_1,\ldots,n_m) \end{bmatrix}$$

It is worth obtaining that for this model

$$x(n+1)=\begin{bmatrix} x_1(n_1+1,n_2,\ldots,n_m) \\ x_2(n_1,n_2+1,\ldots,n_m) \\ \vdots \\ x_m(n_1,n_2,\ldots,n_m+1) \end{bmatrix}$$

If *f,g* are linear functions of their arguments then the above system is said to be linear. A great number of results has been produced during the last decades [1], [2], [8]÷[12]. If we omit the part "*;n*" in the expression of *f,g*, the system is shift-invariant, otherwise is shift variant.

b) Fornasini-Marchesini model:

$$x(n_1+1,n_2+1,\ldots,n_m+1)= f\left(x(n_1,n_2,\ldots,n_m),\ x(n_1+1,n_2,\ldots,n_m),\right.$$
$$\left. x(n_1,n_2+1,\ldots,n_m),\ldots,\ x(n_1,n_2,\ldots,n_m+1)\ ,u(n_1,n_2,\ldots,n_m);(n_1,n_2,\ldots,n_m)\right)$$
$$y(n_1,\ldots,n_m) = g\left(x(n_1,\ldots,n_m),u(n_1,\ldots,n_m);(n_1,n_2,\ldots,n_m)\right)$$

where u,y are defined as previously. $x(n)$ is defined as $x(n)=\begin{bmatrix} x_1(n_1,\ldots,n_m) \\ \vdots \\ x_M(n_1,\ldots,n_m) \end{bmatrix}$ (where in general $M \neq m$). The previous notation for $x(n+1)$ in the Roesser model does not hold here. If *f,g* are linear functions of their arguments then the above system is said to be linear. If we omit the part "$;(n_1,n_2,\ldots,n_m)$" in the expression of *f,g* the system is shift-invariant, otherwise is shift variant.

In the sequel, we will specialize our analysis for $m=2$ without loss of generality. The results can easily be extended for a fuzzy *m*-D model *m*>2.

a) Fuziness in Roesser 2-D model

Let us consider the 2-D model (for simplicity we consider it as shift-invariant)

$$x_1(n_1+1,n_2)=f_1(x(n_1,n_2),\, u(n_1,n_2))$$

$$x_2(n_1,n_2+1)=f_2(x(n_1,n_2),\, u(n_1,n_2))$$

$$y(n_1,n_2)=\mathrm{g}(x(n_1,n_2),\, u(n_1,n_2))$$

where u,y

In many applications x (i.e. x_1, x_2), u, y must be considered as fuzzy variables X (i.e. X_1, X_2), U, Y.

This model is a deterministic 2-D model with fuzzy variables. If instead of the previous equations we have the conditional probability vectorial functions

$$p_1\left(X_1(n_1+1,n_2)\,\middle|\,x(n_1,n_2),u(n_1,n_2)\right)\in[0,1]$$

$$p_2\left(X_2(n_1,n_2+1)\,\middle|\,x(n_1,n_2),u(n_1,n_2)\right)\in[0,1]$$

$$p_g\left(y(n_1,n_2)\,\middle|\,x(n_1,n_2),u(n_1,n_2)\right)\in[0,1]$$

where x(i.e. x_1, x_2), u, y are considered as values of the fuzzy variables X (i.e. X_1, X_2), U, Y, then we have a stochastic 2-D model with fuzzy variables.

Finally, if we have

$$X_1(n_1+1,n_2)=F_1(X(n_1,n_2),\, U(n_1,n_2))$$

$$X_2(n_1+1,n_2)=F_2(X(n_1,n_2),\, U(n_1,n_2))$$

$$Y(n_1,n_2)=G(X(n_1,n_2),\, U(n_1,n_2))$$

where F_1, F_2, G are fuzzy mappings of the fuzzy variables X (i.e. X_1, X_2), U, Y, then we have a fuzzy 2-D model with fuzzy variables. Fuzzy m-D models with fuzzy variables will be simply called fuzzy m-D models. This fuzzy 2-D model (i.e. fuzzy 2-D system) is also described by the conditional membership vectorial functions

$$\mu_1\left(x_1(n_1+1,n_2)\,\middle|\,x(n_1,n_2),u(n_1,n_2)\right)\in[0,1]$$

$$\mu_2\left(x_2(n_1,n_2+1)\,\middle|\,x(n_1,n_2),u(n_1,n_2)\right)\in[0,1]$$

$$\mu_g\left(y(n_1,n_2)\,\middle|\,x(n_1,n_2),u(n_1,n_2)\right)\in[0,1]$$

All the previous models were considered as 2-D shift invariants models that is f_1, f_2, g (deterministic); p_1, p_2, p_g (stochastic), μ_1, μ_2, μ_g (fuzzy) are not dependent (directly) on n_1 and n_2.

These systems are called shift invariant deterministic (with fuzzy variables), stochastic (with fuzzy variables), fuzzy 2-D systems respectively. However we can have shift variant (analogously to time-variant of 1-D systems) deterministic (with fuzzy variables), stochastic (with fuzzy variables), fuzzy 2-D systems. In this case the equations are:

a) Shift variant deterministic 2-D system with fuzzy variables

$$x_1((n_1+1,n_2)=f_1(x(n_1,n_2),u(n_1,n_2);n_1,n_2))$$

$$x_2((n_1,n_2+1) = f_2(x(n_1,n_2),u(n_1,n_2);n_1,n_2))$$

$$y = g(x(n_1,n_2),u(n_1,n_2),n_1,n_2)$$

b) Shift variant stochastic 2-D system with fuzzy variables

$$p_1(x_1(n_1+1,n_2)|x(n_1,n_2),u(n_1,n_2);n_1,n_2) \in [0,1]$$

$$p_2(x_2(n_1,n_2+1)|x(n_1,n_2),u(n_1,n_2);n_1,n_2) \in [0,1]$$

$$p_g(y(n_1,n_2)|x(n_1,n_2),u(n_1,n_2);n_1,n_2) \in [0,1]$$

c) Shift variant fuzzy 2-D system

$$\mu_1(x_1(n_1+1,n_2)|x(n_1,n_2),u(n_1,n_2);n_1,n_2) \in [0,1]$$

$$\mu_2(x_2(n_1,n_2+1)|x(n_1,n_2),u(n_1,n_2);n_1,n_2) \in [0,1]$$

$$\mu_g(y(n_1,n_2)|x(n_1,n_2),u(n_1,n_2);n_1,n_2) \in [0,1]$$

b) Fuzziness in Fornasini-Marchesini 2-D model

Quite analogously we have (in the most general case) the equations:

$$X(n_1+1,n_2+1) = f(X(n_1,n_2),X(n_1+1,n_2),X(n_1,n_2+1),u(n_1,n_2);n_1,n_2)$$

$$y(n_1,n_2) = g(x(n_1,n_2),u(n_1,n_2))$$

$$p(x(n_1+1,n_2+1)|x(n_1,n_2),x(n_1+1,n_2),x(n_1,n_2+1),u(n_1,n_2);n_1,n_2)$$

$$p_g(y(n_1,n_2)|x(n_1,n_2),x(n_1+1,n_2),x(n_1,n_2+1),u(n_1,n_2);n_1,n_2)$$

$$\mu(x(n_1+1,n_2+1)|x(n_1,n_2),x(n_1+1,n_2),x(n_1,n_2+1),u(n_1,n_2);n_1,n_2)$$

$$\mu_g(y(n_1,n_2)|x(n_1,n_2),x(n_1+1,n_2),x(n_1,n_2+1),u(n_1,n_2);n_1,n_2)$$

for deterministic (with fuzzy variables), stochastic (with fuzzy variables), fuzzy 2-D systems correspondingly. It is noted that for shift-invariant models the part "$;n_1,n_2$" in the above equations must be omitted.

The analysis of deterministic or stochastic systems with fuzzy variables is the same to the analysis of deterministic or stochastic systems with non-fuzzy variables, where by the term analysis we mean the evolution of the system under known initial conditions and input. So, in the sequel, we restrict our analysis to fuzzy 2-D systems with fuzzy variables which will be simply called fuzzy 2-D systems.

III Fuzzy 2-D Systems

We start our analysis from 2-D Systems described by the Roesser model. The system is supposed to be shift invariant.

Suppose that our initial conditions are

$$X_1(0,n_2) \text{ and } X_2(n_1,0)$$

Consider first the special case where $u(n_1,n_2)$ is a non-fuzzy vector. Then, the evolution of our fuzzy 2-D system is described by the recursive equations

$$\mu_1(x_1(n_1+1,n_2)) = \bigvee_{x(n_1,n_2)} \left(\mu_1(x_1(n_1+1,n_2) \mid x(n_1,n_2),u(n_1,n_2)) \wedge \mu(x(n_1,n_2))\right)$$

$$\mu_2(x_2(n_1,n_2+1)) = \bigvee_{x(n_1,n_2)} \left(\mu_2(x_2(n_1,n_2+1)) \mid x(n_1,n_2),u(n_1,n_2) \wedge \mu(x(n_1,n_2))\right)$$

$$\mu_g(y(n_1,n_2+1)) = \bigvee_{x(n_1,n_2)} \left(\mu_g(y(n_1,n_2+1)) \mid x(n_1,n_2),u(n_1,n_2) \wedge \mu(x(n_1,n_2))\right)$$

where

$$\mu(x(n_1,n_2)) = \begin{bmatrix} \mu_1(x_1(n_1,n_2)) \\ \mu_2(x_2(n_1,n_2)) \end{bmatrix}$$

where the following notation is used: A$\wedge$B=*min*(A,B), A$\vee$B=*max*(A,B), $\underset{i}{\wedge}$A=*min*($A_1,A_2,\ldots$), $\underset{i}{\vee}$A=*max*($A_1,A_2,\ldots$). If *i* takes infinite values, *min* becomes *inf* (infimum) while *max* becomes *sup* (supremum).

In the general case where $u(n_1,n_2)$ is also a fuzzy vector and given the same initial conditions the evolution is described by

$$\mu_1(x_1(n_1+1,n_2)) = \bigvee_{x(n_1,n_2)} \bigvee_{u(n_1,n_2)} \left(\mu_1(x_1(n_1+1,n_2) \mid x(n_1,n_2),u(n_1,n_2)) \wedge \mu(x(n_1,n_2)) \wedge \mu(u(n_1,n_2))\right)$$

$$\mu_2(x_2(n_1,n_2+1)) = \bigvee_{x(n_1,n_2)} \bigvee_{u(n_1,n_2)} \left(\mu_2(x_2(n_1,n_2+1)) \mid x(n_1,n_2),u(n_1,n_2) \wedge \mu(x(n_1,n_2))\right) \wedge \mu(u(n_1,n_2))$$

$$\mu_g(y(n_1,n_2)) = \bigvee_{x(n_1,n_2)} \bigvee_{u(n_1,n_2)} \left(\mu_g(y(n_1,n_2)) \mid x(n_1,n_2),u(n_1,n_2) \wedge \mu(x(n_1,n_2)) \wedge \mu(u(n_1,n_2))\right)$$

In the most general case where $x(n_1,n_2)$ and $u(n_1,n_2)$ are fuzzy vector and interacting with the same initial conditions, we have the equations:

$$\mu_1(x_1(n_1+1,n_2)) = \bigvee_{x(n_1,n_2)} \bigvee_{u(n_1,n_2)} \left(\mu_1(x_1(n_1+1,n_2) \mid x(n_1,n_2),u(n_1,n_2)) \wedge \mu(x(n_1,n_2),u(n_1,n_2))\right)$$

$$\mu_2(x_2(n_1,n_2+1)) = \bigvee_{x(n_1,n_2)} \bigvee_{u(n_1,n_2)} \left(\mu_2(x_2(n_1,n_2+1)) \mid x(n_1,n_2),u(n_1,n_2) \wedge \mu(x(n_1,n_2),u(n_1,n_2))\right)$$

$$\mu_g(y(n_1,n_2)) = \bigvee_{x(n_1,n_2)} \bigvee_{u(n_1,n_2)} \left(\mu_g(y(n_1,n_2)) \mid x(n_1,n_2),u(n_1,n_2) \wedge \mu(x(n_1,n_2),u(n_1,n_2))\right)$$

We continue our analysis for fuzzy 2-D Systems described by the Fornasini-Marchesini model. The system is supposed to be shift invariant. Suppose that our initial conditions are $X(0,0),\ \ X(1,0),\ \ X(0,1)$

For Fuzzy 2-D systems described by the Fornasini-Marchesini model the corresponding equations are $\left(u(n_1,n_2)\, non-\, fuzzy\, vector\right)$.

$$\mu\left(x(n_1+1,n_2+1)\right)=\bigvee_{x(n_1,n_2)}\left(\mu\left(x(n_1+1,n_2+1)\,\middle|\,x(n_1,n_2),x(n_1+1,n_2),x(n_1,n_2+1),u(n_1,n_2)\right)\right.$$
$$\left.\wedge\mu\left(x(n_1,n_2)\right)\wedge\mu\left(x(n_1+1,n_2)\right)\wedge\mu\left(x(n_1,n_2+1)\right)\right)$$

$$\mu_g\left(y(n_1,n_2)\right)=\bigvee_{x(n_1,n_2)}\left(\mu_g\left(y(n_1,n_2)\,\middle|\,x(n_1,n_2),x(n_1+1,n_2),x(n_1,n_2+1),u(n_1,n_2)\right)\right.$$
$$\left.\wedge\mu\left(x(n_1,n_2)\right)\wedge\mu\left(x(n_1+1,n_2)\right)\wedge\mu\left(x(n_1,n_2+1)\right)\wedge\mu\left(u(n_1,n_2)\right)\right)$$

If $u(n_1,n_2)$ is a fuzzy vector we have:

$$\mu\left(x(n_1+1,n_2+1)\right)=\bigvee_{x(n_1,n_2)}\bigvee_{u(n_1,n_2)}\left(\mu\left(x(n_1+1,n_2+1)\,\middle|\,x(n_1,n_2),x(n_1+1,n_2),x(n_1,n_2+1),u(n_1,n_2)\right)\right.$$
$$\left.\wedge\mu\left(x(n_1,n_2)\right)\wedge\mu\left(x(n_1+1,n_2)\right)\wedge\mu\left(x(n_1,n_2+1)\right)\wedge\mu\left(u(n_1,n_2)\right)\right)$$

$$\mu_g\left(y(n_1,n_2)\right)=\bigvee_{x(n_1,n_2)}\bigvee_{u(n_1,n_2)}\left(\mu_g\left(y(n_1,n_2)\,\middle|\,x(n_1,n_2),x(n_1,n_2),x(n_1+1,n_2),x(n_1,n_2+1),u(n_1,n_2)\right)\right.$$
$$\left.\wedge\mu\left(x(n_1,n_2)\right)\wedge\mu\left(x(n_1+1,n_2)\right)\wedge\mu\left(x(n_1,n_2+1)\right)\wedge\mu\left(u(n_1,n_2)\right)\right)$$

In the most general case where $x(n_1,n_2)$ and $u(n_1,n_2)$ are fuzzy vector and interacting with the same initial conditions, we have the equations:

$$\mu\left(x(n_1+1,n_2+1)\right)=\bigvee_{x(n_1,n_2)}\bigvee_{u(n_1,n_2)}\left(\mu\left(x(n_1+1,n_2+1)\,\middle|\,x(n_1,n_2),x(n_1+1,n_2),x(n_1,n_2+1),u(n_1,n_2)\right)\right.$$
$$\left.\wedge\mu\left(x(n_1,n_2),x(n_1+1,n_2),x(n_1,n_2+1),u(n_1,n_2)\right)\right)$$

$$\mu_g\left(y(n_1,n_2)\right)=\bigvee_{x(n_1,n_2)}\bigvee_{u(n_1,n_2)}\left(\mu_g\left(y(n_1,n_2)\,\middle|\,x(n_1,n_2),x(n_1,n_2),x(n_1+1,n_2),x(n_1,n_2+1),u(n_1,n_2)\right)\right.$$
$$\left.\wedge\mu\left(x(n_1,n_2),x(n_1+1,n_2),x(n_1,n_2+1),u(n_1,n_2)\right)\right)$$

All the previous results (of this section, Section III) refer to 2-D systems described by the Roesser or Fornasini-Marchesini shift-invariant models. It is straitforward to re-write all the equations for the shift-variant case, if we add the part "$;n_1,n_2$" in the all the μ's of the right-hand side of all the above equations!

The previous analysis refers to the fuzzy 2-D systems which use the so-called Zadeh's t-norm and s-norm. Roughly speaking, a t-norm (s-norm) refers to the membership function of a fuzzy set which is the intersection (union) of two other fuzzy sets and is

$$t(\mu_1,\mu_2) = \mu_1 \wedge \mu_2 = min(\mu_1,\mu_2)$$

$$s(\mu_1,\mu_2) = \mu_1 \vee \mu_2 = max(\mu_1,\mu_2)$$

These, t-norm and s-norm, are the most utilized norms in fuzzy Systems Theory, because they are quite well founded, intuitively appealing and have proved useful in many applications. However, in the literature, some other *t*-norms and *s*-norms are defined.

They may be used in some specific applications. Under these *t*-norms and *s*-norms our definitions for deterministic 2-D systems with fuzzy variables, stochastic 2-D systems with fuzzy variables, fuzzy 2-D systems must be re-formulated. These *t*-norms and *s*-norms are given as follows (Note that a *t*-norm is dual to an *s*-norm in the sense: $\text{t}(\mu_1,\mu_2)$=1-s$(1-\mu_1,1-\mu_2)$)

	t-norm	s-norm
Propabilistic	$\text{t}(\mu_1,\mu_2) = \mu_1\mu_2$	$s(\mu_1,\mu_2) = \mu_1 + \mu_2 - \mu_1\mu_2$
Lukasiewicz	$t(\mu_1,\mu_2) = max(0,\mu_1 + \mu_2 - 1)$	$s(\mu_1,\mu_2) = min(1,\mu_1 + \mu_2)$
Weber family $\lambda \in (-1,\infty)$	$t_\lambda(\mu_1,\mu_2) = max\left(0,\frac{\mu_1 + \mu_2 - 1 + \lambda\mu_1\mu_2}{1+\lambda}\right)$	$s_\lambda(\mu_1,\mu_2) = min\left(1,\mu_1 + \mu_2 - \frac{\lambda\mu_1\mu_2}{\mu_1 + \mu_2}\right)$
Hamacher family $\gamma > 0$	$t_\gamma(\mu_1,\mu_2) = \frac{\mu_1\mu_2}{\gamma + (1-\gamma)(\mu_1 + \mu_2 - \mu_1\mu_2)}$	$s_\gamma(\mu_1,\mu_2) = \frac{\mu_1 + \mu_2 - \mu_1\mu_2 - (1-\gamma)\mu_1\mu_2}{1-(1-\gamma)\mu_1\mu_2}$
Yager family p>0	$t_p(\mu_1,\mu_2) = 1 - min\left\{\left((1-\mu_1)^p + (1-\mu_2)^p\right)^{1/p},1\right\}$	$s_p(\mu_1,\mu_2) = min\left\{\left(\mu_1^p + \mu_2^p\right)^{1/p},1\right\}$

with λ,γ,p are appropriate parameters correspondingly for Weber's, Hamacher's and Yager's families.

IV. Conclusion

In this paper, we introduced the concept of deterministic multidimensional systems with fuzzy variables, the concept of stochastic multidimensional systems with fuzzy variables, and the concept of fuzzy multidimensional systems with fuzzy variables. Without loss of generality, we studied 2-D systems that are described by the Roesser model or Fornasini-Marchesini model. These systems may be or not shift-invariant. Furthermore, the analysis was focused in fuzzy 2-D systems described by the Roesser model and Fornasini-Marchesini model (shift invariant or non shift-invariant). All, these models considered in the non-linear case. Fuzzy linear 2-D or *m*-D systems is therefore an interesting subcase. Also, all the previous models include discrete variables. Similar analysis can take place in the corresponding models with continuous variables [10]. On the other hand, all the previous models were considered as causal models. Similar analysis can also take place for non-causal multidimensional systems [11].

There exist many problems in the theory of these systems that must be studied while the field of the applications seems to be very extended. To the best of the authors' knowledge, other relevant studies concerning fuzziness in 2-D systems have not been published. The problem of control of such 2-D systems in left for further study.

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