# **Fuzzy and Neuro-Fuzzy Control of A Fluid Mixer**

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Abstract: - This paper presents the use of both conventional fuzzy and neuro-fuzzy structures in a qualitative control of a fluid mixer, which is a multivariable and intrinsically non-linear plant. The mixer has as inputs two fluids of different colours and, as its output, the colour of the resulting mix. The actual control system consists of two independent fuzzy controllers which are responsible for maintaining the water level at a given height and for adjusting the colour of the fluid in the mixing tank. Initially, the set of rules is established based upon operator's experience. Since, in general, one of the main difficulties in the design of fuzzy control systems, especially when the plant is a complex one, is the definition of an optimal or near-optimal rule-base, a neuro-fuzzy controller is also implemented. This offers the possibility of creating the rule-base automatically, through the constant evaluation of the system error during a learning phase. Simulation results show that responses can be improved by the use of neuro-fuzzy controllers.

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### **1** Introduction

Ordinary fuzzy controllers have been succesfully applied to a variety of plants since the pioneering works of the mid-1970s. In the case of multivariable processes, the natural approach would be to consider as rules antecedents all the controller inputs, which grow in number as the number of desired outputs, or reference inputs, grows. This would certainly make the process of designing the control strategy, or rulebase, a more complex one. On the other hand, if independent controllers are used for each reference input, rule-base design becomes simpler and the control strategy becomes potentially more reliable.

In the case of robot control through a learning fuzzy controller [1], it has been shown that the use of independent controllers for each link can give good results. That is, the controllers, by adjusting their sets of rules, cope very well with the coupling between variables.

In this work the approach of using separate fuzzy controllers is also employed. The plant, described in the next section, is a fluid mixer, which, by presenting non-linear characteristics, provides an additional complexity and constitutes a good test for the designed fuzzy and neuro-fuzzy control systems. These are employed in the control of the coloration of the resulting mix of two different fluids, while avoiding overflow in the mixing tank. The introduction of a new variable allows the reasoning process to occur in a decoupled fashion, and, in consequence, two independent controllers can be used. A similar strategy has been employed before [2] and results have been encouraging.

The aim of the current work is to compare process responses under different strategies, one based on operator's experience and the other on automatic rule-base generation. In this, a neurofuzzy approach is used to create an appropriate rulebase through training. A linguistic description with fuzzy rules is used for the error definition in the rule-base learning process. The replacement of both fuzzy controllers by two neuro-fuzzy ones is expected to give better responses. The size and shape of the fuzzy sets are kept the same in both cases, so that results obtained with the fuzzy and neuro-fuzzy structures can be compared.

### **2** Description of the Plant

The plant, shown in Fig. 1, consists of a mixing and two auxiliary tanks. The first auxiliary tank contains coloured water  $c_1$ , while the second one contains clear water  $c_2$ . The input flow **q** to the mixing tank is controlled by two valves, which regulate the output flows  $q_1$  and  $q_2$  from the auxiliary tanks. The output flow  $q_0$ , taken as a disturbance, has the coloration c of the resulting mix and is a function of the output pipe cross-section ab, of the liquid level h in the mixing tank, and of a constant  $C_d$  related to the shape and material of the output pipe. In order to simplify the simulation, it has been assumed that the auxiliary tanks always contain sufficient liquid for the process to keep on running. The mixing tank dimension is 20x10x80 cm and the output crosssection ab can be set to values between 0 and 0.4  $cm^2$ . The output flow  $q_0$  lies between 0 and 60 cm<sup>3</sup>/sec. Colorations  $c_1$  and  $c_2$  are set to 1 and 0, respectively.



Fig. 1 - The Fluid Mixer

To derive a practical mathematical model so that simulated experiments can be performed, the time needed to obtain a uniform mixture, time delays related to flows in the pipes, and the dynamics of input and output valves have been neglected [3]. The following equations model the plant dynamics:

$$q - q_0 = \frac{dV}{dt} = S.\frac{dh}{dt}$$
(1)

 $\mathbf{q} = \mathbf{q}_1 + \mathbf{q}_2 \tag{2}$ 

$$q_0 = Cd.ab. \sqrt{2gh}$$
(3)

where g is the gravity acceleration, V is the volume of liquid and S is the area of the liquid surface in the mixing tank. By using (3) in (1):

$$\frac{dh}{dt} = -\frac{Cd.ab.\sqrt{2gh}}{S} + \frac{q}{S}$$
(4)

The mixing process is modelled by:

$$\frac{dh}{dt} = \frac{1}{S}(q_1 + q_2 - q_0)$$
(5)

$$c_1.q_1 - c_0.q_0 = \frac{d(c_0.S.h)}{dt} = S.(c_0.\frac{dh}{dt} + h.\frac{dc_0}{dt})$$
 (6)

By combining (5) and (6):

$$\frac{dc_0}{dt} = \frac{1}{Sh} (c_1 \cdot q_1 - c_0 (q_1 + q_2))$$
(7)

Equations (4) and (7) describe the system's dynamics; **h** and  $c_0$  are the variables to be manipulated by the controlling system.

### **3** Fuzzy Control Strategy

The control strategy used is described by a set of linguistic statements, or rules. Consider, for example, the case where each control rule relates two input variables e and ce to the controller output u, and a control algorithm consisting of a set of rules  $R^1$ ,  $R^2$ ,...,  $R^n$ , of the IF (E is  $E^j$ ) AND (CE is  $CE^j$ ) THEN (U is  $U^j$ ) form, connected by a ELSE connective. The combination of those rules can be expressed mathematically (by its membership function) as:

$$\mu_{\mathbb{R}^{N}}(e,ce,u) = f_{1}[\mu_{\mathbb{R}^{1}}(e,ce,u), ..., \mu_{\mathbb{R}^{n}}(e,ce,u)]$$
(8)

where  $f_1$  expresses the ELSE connective. In (8), each control rule j can be expressed as:

$$\mu_{R_{i}}(e,ce,u) = f_{2}[f_{3}(\mu_{E_{i}}(e), \mu_{CE_{i}}(ce)), \mu_{U_{i}}(u)]$$
(9)

In (9),  $E = \{e\}$ ,  $CE = \{ce\}$ ,  $U = \{u\}$  are finite universes and  $E^{j}$ ,  $CE^{j}$  and  $U^{j}$  are fuzzy subsets of those universes. The operator  $f_{2}$  stands for implication [4], and  $f_{3}$  is the interpretation of the connective AND, which is usually taken as *min* ( $\wedge$ ). The controller decides which action to take through a compositional rule of inference. As is generally the case in control, the controller inputs are real measured values given by singletons, and called here  $e_{s}$  and  $ce_{s}$ . The controller output fuzzy set Us will thus be given by:

$$\mu_{Us}(u) = f_1[f_2(\mu_{E^1}(e_s) \land \mu_{CE^1}(ce_s), \mu_{U^1}(u)), .... ...., f_2(\mu_{E^n}(e_s) \land \mu_{CE^n}(ce_s), \mu_{U^n}(u))]$$
(10)

Since the process requires at its input non-fuzzy values, the controller output fuzzy set must be defuzzified, the result being a value  $\mathbf{u}_{s}$ .

### 4 Neuro-Fuzzy System

The neuro-fuzzy controllers have been implemented through the use of the NEFCON system [5] which is able to learn and to optimize online the rule-base of a Mamdani-like fuzzy controller by a reinforcement learning algorithm that uses a fuzzy error measure. The NEFCON model is based on a backpropagation network, with one hidden layer. An example of a neuro-fuzzy controller structure with 6 rules, 2 inputs and one output is shown in Fig. 2.

The hidden nodes represent the rules  $R_1$ ,  $R_2$ ,  $R_3$ ,... $R_6$ ; the input layer nodes ( $\xi_1$ ,  $\xi_2$ ) represent the input values; and the output node ( $\eta$ ) corresponds to the controller output. The weights  $\mu_r^{(i)}$  represent the antecedents  $A_r$  and the weights  $v_r^{(i)}$  represent the consequent  $B_r$ . For example, rule 1 ( $R_1$ ) is translated as:

 $R_1 \Rightarrow$  IF  $\xi_1$  is  $A_1$  and  $\xi_2$  is  $A_2$  THEN  $\eta$  is  $B_1$ 



Fig. 2 - Neuro-Fuzzy System

Rules with the same antecedents  $(A_r)$  have the same weights, thus ensuring the integrity of the rule-base.

The learning process of the neuro-fuzzy system is accomplished in two steps, which are (i) weight initialisation and supervised learning process and (ii) rule-base optimisation. Since the learning process is supervised, the generation of a suitable set of rules is highly associated to the error definition supplied to the neuro-fuzzy system during the learning phase. The NEFCON system has several options for that error definition and in this work a linguistic error description with fuzzy rules is used. These rules describe, in an intuitive way, the system should behave when it is driven towards its optimal state. As the starting rule-base is an empty one, initial fuzzy partitions of the input and output universes are supplied to the neuro-fuzzy system, which, in turn, derives the rules based upon the fuzzy error information. Rule optimisation is performed by shifting rules consequents and changing the support of antecedents. Since the main objective of this work is to compare rule-bases designed either by an operator or in an automatic way, and then evaluate the responses in each case, it was deemed more sensible to keep fuzzy sets definitions and positions the same in both cases and, therefore, skip optimisation.

In the fluid mixer case, where variables can be decoupled, two separate neuro-fuzzy structures are used for the control of coloration and level (height) of fluid in the mixing tank. The decision must be taken upon information of height and coloration errors, as well as of the output flow. The fuzzy sets and universes used for the error definition for rule learning are shown in Figs. 3 and 4 while some typical rules, out of 25 possible ones, of the linguistic error description for height control are:

- If height error is nb an output flow is z then error is n
- If height error is nm and output flow is z then error is n
- If height error is z and output flow is z then error is nz
- If height error is pm and output flow is s then error is pz
- If height error is pb and output flow is z then error is pz

The rules for coloration control are similar, with the obvious adaptation of the fuzzy variables. The rules consequents provide the error information the neuro-fuzzy system needs for rule learning.



Fig. 3 - Membership functions for *height error* for rule learning



Fig. 4 - Membership functions for *output flow* and for *error* for rule learning

#### **Control System** 5

In the fluid mixer under consideration, the control system has to be designed to keep both the output coloration  $\mathbf{c}_0$  and the liquid height **h** in the mixing at desired setpoints. The quantitative tank information needed by the control system in order to attain these goals is given by the coloration error  $e_c$ and change in coloration error  $\Delta e_c$ , the height error  $\mathbf{e}_{\mathbf{h}}$ , and the output flow  $\mathbf{q}_{0}$ . Since the chosen strategy makes use of two independents fuzzy controllers, one for height control and another for coloration control, it is more convenient to choose as output variables the total flow q (eq. 2) and the proportion  $\mathbf{q}_{\mathbf{r}}$  of coloured water in the total flow, defined as:

$$q_{r} = \frac{q_{1}}{q_{1} + q_{2}} \tag{11}$$

The height controller has two quantitative inputs,  $e_h$ and  $q_0$ , and one output, q. The coloration controller has as quantitative inputs  $\mathbf{e}_{c}$  and  $\Delta \mathbf{e}_{c}$ , and  $\Delta \mathbf{q}_{r}$  as its output. There is a slight difference between those controllers: while in the former precision is not a fundamental factor, in the latter the goal is to have a fine control, with null steady-state error if possible; thus the use of a structure with PI characteristics.

#### 5.1 **Height Control**

In the design of the height controller, fuzzy sets NB, NM, Z, M, B, PB are assigned to  $E_h = \{e_h\}$ , and Z, S, M, B, VB, to  $Q = \{q\}$  and  $Q_0 = \{q_0\}$ , as specified by their membership functions shown in Figs. 5 and 6. The universes for each variable are shown in those figures. The non-fuzzy actual values of the measured input variables are mapped to the chosen universes of discourse through scaling factors  $GE_h$  and  $GQ_0$ , which are part of blocks S<sub>0</sub> and S<sub>1</sub> in the diagram of Fig. 9. The resulting values are called  $e_{hs}$  and  $q_{0s}$ . The defuzzified controller out-put  $\mathbf{q}_s$  is mapped to the process input actual values through a scaling factor **GQ**, so that  $q = q_s \times GQ$ . In this controller,  $f_1$ is implemented by max and f2 by min [6]; Mean of Maxima (MOM) is used for defuzzification.



Fig. 5 - Membership functions for  $E_h$ 



Fig. 6 - Membership functions for Q and  $Q_0$ 

The rule-base for height control for the ordinary fuzzy controller (in italics) and the resulting rulebase for the neuro-fuzzy controller (after learning) are shown in Table 1. The entries correspond to the values of the total flow **Q**.

	Q	Ζ	S	Μ	B	VB
e <sub>h</sub>	PB	М	В	VB	VB	VB
		Μ	VB	VB	VB	VB
	PM	S	М	В	VB	VB
		Μ	Μ	Μ	Μ	VB
	Z	Ζ	S	М	В	VB
		Ζ	Μ	Μ	Μ	VB
	NM	Ζ	Ζ	S	М	В
		Μ	Ζ	Ζ	Μ	Μ
	NB	Ζ	Ζ	Ζ	М	В
		Μ	Μ	Ζ	Ζ	Ζ
	→ q₀					

Table 1 - Fuzzy (italics) and Neuro-Fuzzy rulebases for height control

#### 5.2 **Coloration Control**

In the coloration controller, fuzzy sets NB, NB, Z, PM and PB are used for  $E_c = \{e_c\}$  and  $\Delta E_c = \{\Delta e_c\}$ , while NVB, NB, NM, NS, Z, PS, PM, PB and PVB are used for the controller output  $\Delta Q_r = \{q_r\}$ , as specified by their membership functions shown in Figs. 7 and 8. The universes are as shown in those figures. The actual measured values of the controller inputs are mapped to the universes of discourse through scaling factors  $GE_c$  and  $G\Delta E_c$  (contained in blocks  $S_2$  and  $S_3$  in the diagram of Fig. 9), the result being variables  $e_{cs}$  and  $\Delta e_{cs}$ . The defuzzified controller output  $\Delta q_{rs}$  is mapped to the process input actual values through a scaling factor  $G\Delta Q_r$ . The actual input flows  $q_1$  and  $q_2$  to the fluid mixer are given by  $q_1=q_r \times q$  and  $q_2=q_r - q_1$ . In this controller,  $f_1$ is implemented by *max*, and  $f_2$  by *product* [6]; defuzzification is performed through Center of Gravity (COG).



Fig. 8 - Membership functions for  $\Delta Q_r$ 

The larger number of fuzzy sets used for the controller output in coloration control, as compared to height control, is due to the requirement of high accuracy in the coloration of the resulting mix. The reason for keeping  $\mathbf{h}$  at a desired setpoint is mainly to prevent an undesirable overflow in that tank.

The rule-bases for coloration control for the ordinary fuzzy controller (in italics) and for the neuro-fuzzy one (after learning) are shown in Table 2, where the entries correspond to the values of  $\Delta Q_r$ .

### **6** Simulated Experiments

The process response will depend on the scaling factors, which can be empirically set beforehand and then tuned in order to improve the controller performance. For example, in the case of the fluid mixer, the range established for  $\mathbf{q}$  and  $\mathbf{q}_0$  was from 0 to 60 cm<sup>3</sup>. Since the corresponding universes are [0,4], **GQ** and **GQ**<sub>0</sub> were initially set at 15. In the

case of the conventional fuzzy controller, this value resulted in satisfactory responses and did not need to be tuned. The maximum possible height error  $\mathbf{e}_{\mathbf{h}}$  was assumed to be  $\pm$  7 cm, which, by considering the corresponding universe as [-2,2], gives  $\mathbf{GE}_{\mathbf{h}}=0.3$ . In order to achieve good accuracy in coloration, and considering that the universe is [-2,2] for that variable, the scaling factor  $\mathbf{GE}_{\mathbf{c}}$  was set at 200;  $\mathbf{G}\Delta\mathbf{E}_{\mathbf{c}}$  was set at 1500 and  $\mathbf{G}\Delta\mathbf{Q}_{\mathbf{r}}$ , to 1/15.

	∆Qr	NB	NM	Ζ	PM	PB
e <sub>c</sub> ▲	PB	NVB	NB	NM	NS	Ζ
		NV	NV	NV	NV	NV
	PM	NB	NM	NS	Ζ	PS
		NV	NV	Ζ	Ζ	NV
	Z	NM	NS	Ζ	PS	РМ
		NV	Ζ	Ζ	Ζ	NV
	NM	NS	Ζ	PS	РМ	PB
		PV	Ζ	Ζ	Ζ	Ζ
	NB	Z	PS	PM	PB	PVB
		PV	PV	PV	PV	PV
	→ <sub>∆e</sub>					

Table 2: Fuzzy (italics) and Neuro-Fuzzy rule-bases for coloration control

For simulation, MATLAB<sup> $\degree$ </sup>, its associated tool SIMULINK and the FuzzyToolbox have been used. The simulation diagram is shown in Fig. 9.

*Experiment 1*: The resulting mix coloration behaviour is shown in Fig. 10, when a step from 0 to 0.3 is applied to the reference input; the level of liquid in the mixing tank should be kept constant at 30 cm. The response obtained with the neuro-fuzzy controller (NF) reaches the setpoint faster than that obtained with the ordinary fuzzy controller (F).



Fig. 9 - Simulation Diagram



Fig. 10 - Experiment 1: coloration control

*Experiment 2:* The level (height) of liquid in the mixing tank is shown in Fig. 11, when steps from 0 to 40 and then to 25 cm are applied to the height reference input. The final coloration should be kept unchanged at 0.5. It can be observed that in both cases the reference input is reached, but the process response obtained with the the neuro-fuzzy controller (NF) is once again faster than that obtained with the simple fuzzy controller (F).



Fig. 11 - Experiment 2: height control

*Experiment 3*: Coloration and height variations are shown in Fig. 12, when steps of 0 to 0.3 and then to 0.15 are applied to the coloration reference input and steps of 0 to 40 and then to 25 cm. are applied at the same time to the height reference input. As can be observed, the response provided by the neuro-fuzzy controller for coloration is faster than in experiment 1. This is caused by a large input flow; the height controller is trying to reach the setpoint and the input flow is at its maximumat the same time.

## 7 Conclusions

Experiments have shown that the approach of using two independent neuro-fuzzy controllers in the control of a MIMO system can give good results. The rule-base generated through the learning process, although different from that defined by an expert, works perfectly well and the main objective of reaching a specified coloration for the liquid in the mixing tank was achieved with good precision. The neuro-fuzzy system is able to create an appropriate set of rules, thus overcoming the difficulty of defining the rule-base. The neuro-fuzzy and fuzzy rule-bases are similar in some cases, and in general they do not conflict, being different only in terms of intensity of the fuzzy variables (ex. NVB  $\Leftrightarrow$  NM) in the region near to the setpoint.



Fig. 12 - Experiment 3: Neuro-Fuzzy simultaneous coloration (a) and height control (b)

Some care must be taken when defining the error to be supplied to the neuro-fuzzy system during the learning process. The neuro-fuzzy system is highly sensitive to this information, which plays a crucial part in the effectiveness of the learning process.

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