Fuzzy Gain-Scheduling Control of Nonlinear Processes

R. E. King[1] & A. STATHAKI[2]
[1]Department of Electrical & Computer Engineering
University of Patras
Patras 26110, GREECE
[2]Computer Technology Institute
Athens 15773, GREECE
e-mail: rking@ee.upatras.gr | http//:www.lar.ee.upatras.gr

Abstract: - Fuzzy gain-scheduling is a special form of model-based fuzzy control that uses linguistic rules and fuzzy reasoning to determine the controller parameter transition policy for a dynamic plant subject to large changes in its operating state. Issues of stability and overall dynamic behavior are resolved using conventional modern control techniques. The design of a fuzzy gain-scheduling controller for a simple nonlinear process illustrates the technique.

Keywords: - Gain-scheduling, heuristic fuzzy control, model-based fuzzy control, fuzzy gain-scheduling.

1 Introduction

Two distinct trends in the design of fuzzy controllers have evolved in the past two decades or so. The first is based on *heuristic knowledge* of the control policy that is required to control a plant. This technique does not require deep knowledge of the controlled plant in order to be applied successfully [1,3,5-8]. This feature has led to extensive adoption of *heuristic fuzzy control* in industry and manufacturing where such knowledge is usually lacking In general, heuristic fuzzy control is possible only if the control policy is known *a priori*. Thus for new processes, where prior knowledge is unavailable, heuristic fuzzy control is not a suitable candidate. It is also clear that the heuristic approach can neither resolve the issue of stability of the closed system nor specify its dynamic performance [12]. This approach is clearly case-dependent and it is therefore impossible to generalize on the performance of this class of controllers from a knowledge of their behavior in other applications.

The limitations of the heuristic approach led to a search for more rigorous methods that could combine Zadeh's Fuzzy Logic [13,14] and Modern Control Theory. One such outcome has become known as *Model-based Fuzzy Control*, a technique that has been applied with success in a number of practical situations such as high-speed trains, helicopters, robotic arms etc. The technique assumes the existence of an explicit model of the controlled process of sufficient fidelity from which linearized models can be derived for every nominal operating state. The hybrid model-based approach is thus a fusion of *soft* control techniques based on *Computational Intelligence* [4,9,10] and *hard* control techniques based on Modern Control Theory and offers distinct advantages where closed system stability and dynamic characteristics must be assured.

Gain-scheduling is a well-known technique of industrial control [2] and is used when a plant is subject to large changes in its operating state, a situation that is typical in industry. Large changes in the operating state lead to corresponding variations in the parameters of the linearized models of the plant about these operating states. It is well known that it is not possible therefore to design a controller to operate satisfactorily at one operating state and expect it to perform equally well elsewhere without re-tuning it. Closed system performance is degraded since the controller cannot track the changes in the operating states.

Considerable effort has gone into developing controllers that can track the variations in plant parameters with a view to achieving invariant operation throughout the domain of operation of the plant. Adaptive controllers are one such approach, yet even these controllers do not always demonstrate satisfactory performance throughout the domain of operation of the plant and may, on occasion, lose control altogether. Robust controllers, another approach, also have their limitations since they must deal with system dynamics that vary over a wide range though using constant parameters only. Clearly this class of controllers can only operate satisfactorily over a limited domain.

This paper applies Fuzzy Gain-Scheduling to the problem of transfering the state of a nonlinear process from an initial state to a final state where the linearized process dynamics are very different from those at the initial state. Fuzzy Gain-Scheduling assures smooth transistions in the control law, yet maintains essentially invariant closed system characteristics.

2 Conventional Gain-Scheduling

In conventional gain-scheduling controllers the parameters (i.e. gains) of the controller are varied usually as a function of some exogenous variable in an attempt to compensate for the changes in the operating state of the plant through *stepwise* changes in the controller parameters. A typical example of conventional gain-scheduling in which the gains of the controller are changes step-wise can be found in aircraft surface control. Here, as the altitude of the aircraft increases, the sensitivity and consequently effectiveness of the control surfaces decreases because of the thinning air. This in turn requires a greater control action to achieve the same overall response. If altitude is the exogenous variable that adjusts the controller gains, then the controlling effect can scheduled to be independent of the altitude. The result of step-wise changes in the controller parameters may, however, result in bumpy motion evry time the controller parameters are adjusted.

Automatic transmission in autos is another example. The step-wise changes in the gear ratios, however are often causes of uneven and jerky motion. To avoid these sudden changes and to provide smooth acceleration, a number of auto manufacturers today offer infinitely variable ratio transmissions. Yet another example is a robotic arm whose dynamics change as it is extended.

3 Fuzzy Gain-Scheduling

Fuzzy gain-scheduling, which is a derivative of *Model-Based Fuzzy Control*, offers a simple yet robust solution to the problem of controlling a non-linear plant subject to large changes in its operating state. In this technique, fuzzy linguistic rules and fuzzy inference mechanisms are used to establish the required control policy. Given explicit models of the plant and the corresponding control laws at a finite set of states, the technique infers the control laws at all states in between. The resultant state-weighted control policy leads to smooth state transition.

4 Model-based control

In the mid-1980s Takagi and Sugeno [11] proposed using fuzzy reasoning to *specify* the control law of a state feedback controller so that the overall system had guaranteed properties. The controller that Takagi and Sugeno proposed was characterized by a set of fuzzy rules that related the *current state* of the process to its *process model* and the corresponding *control law*. These composite rules have the generic form:

R: IF (*current state*)
THEN (*fuzzy process model*)
AND (*fuzzy control policy*).

If, now, it is possible to generate a set D of known nominal states x^i, for which a corresponding set S of linearized models of the controlled plant is determined through first principles or identification, then fuzzy gain-scheduling can offer distinct advantages. In this scheme, the transition from one nominal state to another is smooth since the system parameters can be made to vary smoothly along the trajectory. These two sets are clearly invariant with changes in the nominal states. For any nominal state that does not belong to the set D, an approximating description can be derived from models belonging to the set without recourse to further linearization.

For each nominal state x^d the locally linearized model is stored in the *model base* S while the corresponding control policy is stored in the *control policy base* U. The nominal states x^i in the *nominal states base* D can be conveniently chosen to be the centers of the fuzzy regions ΦX, i.e., the states $x^i \in \Phi X^i$ at which the values of the fuzzy membership values are unity, i.e.

$$\mu_{IX^i}(x^i) = \min(1,1,\ldots,1) = 1.$$

Consider the *crisp process state vector* x define on a closed, real set X. The *fuzzy state variables* x_i are fuzzy sets defined on X. The values of these variables are termed the *fuzzy values* of the fuzzy state variable and are written as the fuzzy set

$$\Phi X_{ij} = \int_X \mu_{\Phi X_{ij}}(x)/x$$

For every fuzzy value of x_i there exists a corresponding membership function $\mu_{\Phi X_{ij}}(x)$, which specifies the membership value of the crisp value x_i^* of this variable. The *universe of discourse* is defined as the set:

$$TX_i = \{\Phi X_{i1}, \Phi X_{i2} \ldots \Phi X_{ik_i}\}$$

where k_i is the number of fuzzy values x_i. In order to simplify the analysis that follows, it will be assumed that:

- the shapes of the fuzzy sets of ΦX_i are identical for all i,
- the number of fuzzy numbers $k_1 = k_2 = \ldots = k_n$ and that
- $\Phi X_{1i} = \Phi X_{2i} \ldots = \Phi X_{ij} = \Phi X_{nj}$

Examples of such triangular fuzzy sets are shown in Figure 1.

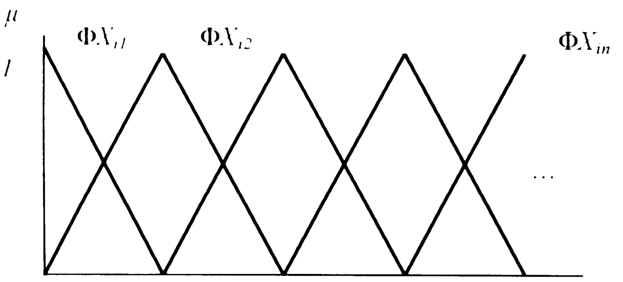

Figure 1. Fuzzy sets of the state x_i

The *state vector* x of the controlled plant is defined over some space. Every *crisp* value x^* of the state vector corresponds to a specific state in state space. In the case of fuzzy model-based controllers, the states take on fuzzy values and consequently the concept of state space must be modified to account for the fuzzy values of the state vector. Knowing that every fuzzy variable has a finite number of fuzzy values, then a finite number of fuzzy vectors that result from the combinations of the fuzzy values can be generated.

In the model-based technique, each element of the crisp state variable x is fuzzified as in the heuristic fuzzy control case. In each *fuzzy region* of fuzzy state space, a rule uniquely defines the *local plant model* in that region, e.g.,

$$R_S^i: \text{IF } x = \Phi x^i \text{ THEN } \dot{x}^i = f_i(x^i, u^i)$$

The symbolism $x = \Phi X^i$ implies that the state of the process x belongs to the fuzzy region ΦX^i. The consequent of each rule describes an explicit local model of the process in the corresponding fuzzy region ΦX^i.

Thus at the centers of each fuzzy region, the linearized model defined in the consequents of the fuzzy rules with x^i in place of x^d. The set of rules which describe the fuzzy plant model reduce to:

$$R_S^j: \text{IF } x^d = \Phi X^i \text{ THEN } \dot{x}^i = A^i(x - x^d) + B^i(u - u^d)$$

where $x \in \Re^n$ and $u \in \Re^m$, while the plant matrix pair (A, B) uniquely specifies the linearized plant. Likewise the control policy, or *process rules*, is given by the linear state feedback law:

$$R_C^j: \text{IF } x^d = \Phi X^j \text{ THEN } u = K^j(x - x^d) + u^d$$

where the gain matrix $K(x^j, u^j)$ is computed so as to ensure stability and specified closed system transient performance through suitable pole-placement on the basis of the linearized closed system defined in the corresponding fuzzy region LX^j. The fuzzy model of the plant is now specified in terms of the deviations of the state and the corresponding control actions from their nominal values, i.e.:

$$\dot{x} = \sum_i w^i(x^d) \cdot \left[A^i(x - x^d) + B^i(u - u^d)\right]$$

The *overall control law* is given by:

$$u = \sum_j w^j(x^d) \cdot K^*(x - x^d) + u^d$$

The closed system model and the overall control law are both seen to be *linear* since the normalized membership functions $w^i(x^d)$ and $w^j(x^d)$ are constants. The closed system is therefore given by:

$$\dot{x} = \sum_i \sum_j w^i(x^d) w^j(x^d) \left[A^i + B^i K^j\right](x - x^d)$$

It follows that

$$\sum_i w^i(x^d) = \sum_j w^j(x^d) = \sum_i \sum_j w^i(x^d) w^j(x^d) = 1$$

Here

$$A^d = \sum_i w^i(x^d) A^i ; \quad B^d = \sum_i w^i(x^d) B^i$$

and

$$K^d = \sum_j w^j(x^d) K^j$$

In an analogous way

$$\dot{x} = A^d(x - x^d) + B^d(u - u^d)$$

and

$$u = K^d(x - x^d) + u^d$$

which are linear state equations involving the state and control input *deviations* from their nominal states x_d and u_d respectively, while

$$A^d = A(x^d, u^d) = \left.\frac{\partial f_i(x,u)}{\partial x}\right|_{x=x^d, u=u^d}$$

and

$$B^d = B(x^d, u^d) = \left.\frac{\partial f_i(x,u)}{\partial u}\right|_{x=x^d, u=u^d}$$

It is noted that the overall closed system is *stable* at the nominal state x^d if and only if $\mathrm{Re}\{\lambda_m(A^*)\} < 0 \;\; m = 1, 2, \ldots, n$, where λ_m are the eigenvalues of the matrix $A^* = A^d + B^d K^d$.

3 The Fuzzy Process Model

It is observed that whereas the antecedent of the fuzzy process rules are similar to those used in the heuristic fuzzy control case, the consequents are analytical expressions that describe a process model. The *process rules* can be expressed in terms of the elements of the crisp process state as:

R_S^i: IF $x_1 = \Phi x^i_1$ AND $x_2 = \Phi x^i_2$ AND … $x_n = \Phi x^i_n$
THEN $\dot{x}^i = f_i(x,u)$

In any fuzzy region ΦX^i the process can thus be specified by:

$$\dot{x}^i = \mu_S^i(x) \cdot f_i(x,u)$$

where

$$\mu_S^i(x) = \mu_{\Phi X_1^i}(x_1) \wedge \mu_{\Phi X_2^i}(x_2) \wedge \ldots \wedge \mu_{\Phi X_n^i}(x_n)$$
$$= \min(\mu_{\Phi X_1^i}(x_1), \mu_{\Phi X_2^i}(x_2), \ldots, \mu_{\Phi X_n^i}(x_n)).$$

are the degrees of fulfillment of the local linearized models of the plant using *Mamdani's* fuzzy compositional rule [7].

For each nominal state of the process, the state equations of the closed system are determined. Using the set of fuzzy process rules the *fuzzy plant model* of the overall process is the weighted sum of the local linearized plant models $f_i(x,u)$ i.e.

$$\dot{x} = \sum_i w_S^i(x) \cdot f_i(x,u)$$

where

$$w_S^i(x) = \frac{\mu_S^i(x)}{\sum_i \mu_S^i(x)} \in [0, 1]$$

are the *normalized degrees of fulfillment* or *process function weights*. Clearly the sum of the process function weights is unity, i.e.

$$\sum_i w_S^i(x) = 1$$

4 The Fuzzy Gain-Scheduling Law

For every antecedent there exist *two* consequents, the second one of which specifies the *state feedback control law* that must be applied. This, of course, assumes that *all* states are measurable, often a fundamental restriction of the technique in practice. Where it is not possible to measure all the state variables then observers or estimators may be used to provide the missing states.

The second consequent of every rule in the fuzzy region ΦX^j specifies the control law and has the generic form:

R_S^j: IF $x = \Phi x^j$ THEN $u = g_j(x)$

while the *control law* is given by:

$$u^j = \mu_C^j(x) \cdot g_j(x)$$

where

$$\mu_c{}^j(x) = \wedge_k (\mu_{\Phi X_k{}^j}(x_k))$$
$$= \min(\mu_{\Phi X_1{}^j}(x_1), \mu_{\Phi X_2{}^j}(x_2), \dots, \mu_{\Phi X_n{}^j}(x_n)).$$

The *overall control law* is the weighted sum:

$$u = \sum_j w_c{}^j(x) \cdot g_j(x)$$

where $w_c{}^j$ are the corresponding *control weights*.

5 Application: Fuzzy gain-scheduling control of a simple nonlinear process

Many industrial processes exhibit dynamic behavior that depends strongly on their nominal operating state. Beacause of their nonlinear nature the performance of any controller that is tuned at any nominal state would clearly be degraded at any other state not in the proximity of the initial nominal state.

Fuzzy gain-scheduling is an effective way to adapt the parameters of the controller as a function of the state so that the performance of the process be essentially invariant in state space.

In order to describe the procedure in simple terms, it will be assumed that the process can be modeled by a scalar state equation. Clearly the technique can be extended to the multivariable case.

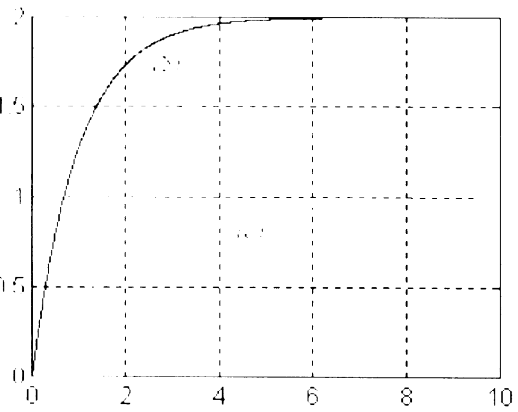

Figure 3. Step responses of the process at the end states

The objective of the controller is to force the process to move from some initial state to some terminal state in a smooth manner while simultaneously satisfying given performance specifications. It will be assumed furthermore that the process model can be linearized at (a) some initial (normalized) state $x^d = 0$ and (b) at a second (terminal) state $x^d = 1$. In case (a) the *process rule* (i.e. linearized model) of the process is taken to be:

R^a: IF $x^d = 0$ THEN
$\dot{x} = f_1(x,u) = -0.5x + 0.5u$

while in case (b) the model of the plant differs radically and the *process rule* is taken to be:

R^b: IF $x^d = 1$ THEN
$\dot{x} = f_2(x,u) = -x + 2u$

The step responses of the plant for the two cases are shown in Figure 3. In case (a), the dynamics of the plant are evidently slow with a normalized time constant of $T=2$ whereas in case (b), the response is much faster, with a time constant $T = 0.5$ and the sensitivity to any control action is increased.

Fuzzy gain-scheduling permits adaptation of the gain of the controller as a function of the state smoothly instead of in a step-wise manner as in conventional gain-scheduling. To achieve this, consider the *fuzzy generic control rules* given below for the two nominal states:

R^a: IF $x^d = 0$ THEN $u_1 = g_1(x,x^d) = k_1(x-x^d)$
R^b: IF $x^d = 1$ THEN $u_2 = g_2(x,x^d) = k_2(x-x^d)$

In order to obtain the desired closed system response at *both* state extremes, state feedback gains $k_1=0.6$, $k_2=0.4$ are selected so that the eigenvalues of the closed system are at (-0.2, 0) in *both* cases. This implies that in the vicinity of both nominal states, at least, the closed system behavior will be identical. Ideally it is desirable to maintain the same closed system behavior in *all* states in between and ideally forcing the closed system behavior to be independent of the state.

To simplify the analysis, let the simple fuzzy membership functions shown in Figure 4 indicate *how* the transition of the dynamics of the plant must follow the state. It is evident that for states close to the initial nominal state, the model of the plant is predominantly of type (a) but as the norm of the state increases the type (a) model fades gadually into case (b) until $x = x^d = 1$ at which the model is entirely case (b).

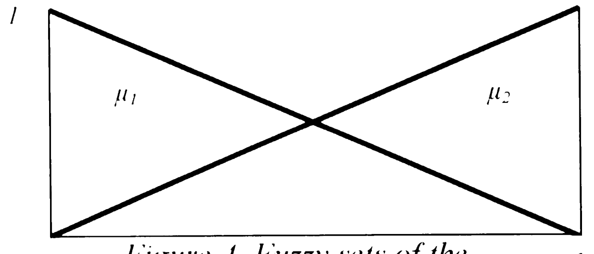

Figure 4. Fuzzy sets of the fuzzy gain-scheduling controller

The gain-scheduling fuzzy sets for two nominal states are shown in Figure 4. These can be described analytically by $\mu_1(x) = 1-x$ and $\mu_2(x) = x$. It is clear that $\mu_1 + \mu_2 = 1 \ \forall x$. The *overall fuzzy process model* is thus given by the weighted sum:

$$\dot{x} = w_1(-0.5(x - x^d) + 0.5u) + w_2(-(x - x^d) + 2u)$$

where w_1 and w_2 are the normalized membership functions:

$$w_i = \frac{\mu_i}{\mu_1 + \mu_2} = \mu_i$$

The *overall fuzzy control law* is therefore the weighted sum:

$$u = 0.6w_1(x - x^d) + 0.4w_2(x - x^d)$$

The fuzzy gain-scheduling controller provides the control actions to force the closed system to follow the desired policy for all values of the state. The response of the closed system to a large step demand in the nominal state from $x = 0$ to $x = x^d = 1$ is shown as trajectory (b) in Figure 5 while for comparison trajectory (a) shows the response of an *invariant* system governed by:

$$\dot{x} = -0.2x + 0.2u$$

As a final note, it is evident that since the resultant closed system is nonlinear, it is unreasonable to expect the step responses in Figure 5 to be identical. It is noted, however, that in the proximity of $x = 0$ and $x = x^d$ the response of the closed system approaches the response of the invariant system.

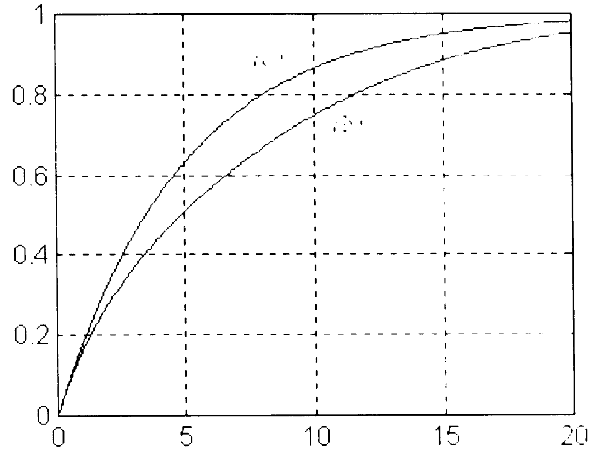

Figure 5 Responses of (a) the invariant system with the ideal dynamic characteristics and (b) the fuzzy gain-scheduling controller

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