Extraction of Distance From an Eddy-Current Sensor Response by a Source Separation Technique: Blind Context or A Priori Knowledge ?

MICHEL HARITOPOULOS, YANNICK NAUDET, PASCAL VASSEUR, ALAIN BILLAT Laboratoire d'Automatique et de Microélectronique Université de Reims Champagne-Ardenne Moulin de la Housse, BP 1039-51687 Reims cedex 2 FRANCE

Abstract: - Eddy-current displacement sensors are very sensitive to many environmental factors and especially to temperature variations. We deal here with a real world problem that is, distance measurement between an eddy-current sensor and a metallic target (an aluminun sheet) which temperature varies during displacement. We show, that source separation techniques applied to the experimental pre-processed data set issued from the sensor response permit to obtain the sensor-target distance as well as the target temperature profile, although disturbed by an additive noise due to the instrumentation. IMACS/IEEE CSCC'99 Proceedings, Pages:4441-4446

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1 Introduction

An eddy-current sensor is a contactless device which consists in a coil of wire excited by a high frequency current. The working principle can be resumed as follows: when the coil is excited by a sinusoidal current, a magnetic field is generated around it. The introduction of a nonmagnetic conducting object (target) into this field induces eddy currents on its surface, creating then another magnetic field which tends to oppose the coil's one (Lenz's law). It follows from there a reduction in the coil inductance, while its resistance increases. The mutual inductance between the coil and the target varies then with the gap probe-target, and this variation is exploited by a signal conditioner in order to provide a measurement of this distance. This can be done in different manners, and particularly, by introducing the coil in an oscillator, which central frequency changes with the mutual.

A real problem to deal with in the use of eddycurrent sensors is to obtain accurate measurements in spite of a varying environment. In particular, if they are very robust to fluids or dust contamination, it is not the same with temperature variations [1]. We are interested here in the effect that can have temperature changes on a distance measurement provided by an eddy-current sensor made in our laboratory.

First, without making any assumptions about the temperature and the distance measurement evolution, a Blind Source Separation (BSS) algorithm is used to extract the effect of the temperature from the sensor output. Secondly, use of a priori knowledge and its effects on separation performances are studied. This approach is justified by comparing results with a standard filtering.

2 Problem Formulation

The above problem will be formulated with an instantaneous source separation model, that can be stated as follows:

$$\mathbf{x}(t) = A(t)\mathbf{s}(t) + \nu(t) \tag{1}$$

where **x** is an $m \times 1$ vector of observations, **s** is an $n \times 1$ vector with statistically independent components corresponding to the different phenomena that are mixed together with A, an $m \times n$ invertible matrix called mixing matrix. Here, ν is an additive noise vector independent with **s**, often considered to be Gaussian. From this model, it is supposed that vector **s** has stationary or slowly varying over time components and that the



Fig.1 Source Separation scheme

mixing giving vector \mathbf{x} is linear.

Considering the two phenomena of interest, that are the distance d to be measured and the temperature variations θ , let $\{\theta(t), d(t)\}$ be the components of the two dimensional source vector $\mathbf{s}(t)$. It is obvious that because they have physically no common point, these two sources can be considered as independent.

2.1 A Brief Introduction to Source Separation

The source separation technique consists in retrieving the sources s(t) issued from real phenomena, considering only the response of some sensors which receive a mixing of these sources $\mathbf{x}(t)$. When there is really no a priori knowledge or assumption about the latter, the problem is known as Blind Source Separation (BSS) and was introduced by Hérault and Jutten [2]. But whether some a priori knowledge is used or not, the fundamental hypothesis is the *statistical independence* of the sources that is necessary for finding a solution.

So, extracting independent components from a random vector is now a well known technique called Independent Component Analysis (ICA). It can be considered as the underlying mathematical technique for solving a BSS problem, and was first developed in a rigorous manner by Pierre Comon [3] as an extension of the Principal Component Analysis (PCA) concept that provides only decorrelated components.

Hence, recall that sources must be independent, it is also often assumed that the model is non-degenerate (the number of sensors is at least equal to the number of sources, that is $m \ge n$), and that at most one source can be Gaussian. Without prior knowledge about their distributions, source signals can be recovered at best up to a permutation, scale and sign. Finally, the source separation problem in the case of two sources and two observations, can be resumed as illustrated in Fig.1, in the estimation of a demixing matrix W(t), that tends to invert A, in order to provide source estimates $e_i(t)$ fulfilling the conditions stated before.



Fig.2 Model for linear filtering problem

2.2 BSS, the Blind Context

We can consider that an eddy-current sensor provides, without post-processing, a response which contains both distance and temperature information. By modifying it in order to obtain two or more responses for a same distance, and if we consider that the sources d(t) and $\theta(t)$ are perceived by the sensor in the same time intervals (no delay), the model (1) holds and we are able to apply a BSS technique: taking only into account the available responses as vector **x**, we are in blind context.

2.3 Non-Blind Source Separation

Now, because we focus on the distance measurement, one can suppose that prior information concerning the evolution of the plate temperature $\theta(t)$ is available. One or more components of the observation vector **x** given by (1), can then be transformed according to:

$$x_i(t) \mapsto x_i(t) \pm \alpha \theta(t).$$
 (2)

In this way, the mixing is somehow enriched by increasing the contribution of one of the sources that was poor before. The choice for coefficient α is arbitrary. Yet, it has to be chosen not too large, in order to avoid the other contributions to become negligible because this would have no sense. Furthermore, it is evident that it must minimize the error between the true distance probe-target and the estimated one. As Fig.3 shows, an optimal value for α can be:

$$\alpha^* = \max_{\alpha} \frac{E\{\left|\widehat{\theta_{\alpha}} - \theta\right|\}}{E\{\left|\widehat{d_{\alpha}} - d\right|\}}$$
(3)

where $\widehat{\theta_{\alpha}}$ and $\widehat{d_{\alpha}}$ designate respectively the estimates of θ and d for a certain value of α , after application of a source separation algorithm.

2.4 Noise Cancellation

If the plate true temperature $\theta_c(t)$ is known, the model of Fig.1 can be transformed in that of Fig.2. Such a fil-



Fig.3 Error of the estimated sources for a non-blind source separation scheme

tering scheme, known as Wiener filtering, is optimal in the sense of minimum mean-square error (MMSE) and could permit to denoise the observations. Here, to be coherent with the instantaneous BSS model, filtering resumes to the estimation of one coefficient h, relative to the temperature. Hence, we can write:

$$x_i(t) = h\theta_c(t) + d(t).$$
(4)

Given the observations x and the true temperature $\theta_c(t)$, a denoised version of distance d can be computed after optimal estimation of h.

3 Experimental Context

The sensitive element of the sensor developed for our experiments is a flat spiral coil engraved on the two faces of a printed circuit board [1]. The coil is the principal element of two Clapp oscillators with different central frequency which values in the air (without target) are respectively $f_{01} = 497KHz$ and $f_{02} = 989KHz$. Hence, the modification of the coil-target mutual inductance due to the presence of the target in the magnetic field created by the coil, modifies both of the oscillator frequencies f_1 and f_2 by 6% to 13% in our displacement range. The so-called target is here an aluminum sheet, chosen so as to have dimensions much greater than these of the coil, in order to prevent some edge effects.

Recall now that the "standard depth of penetration", where the eddy-currents density is 1/e of its surface value, can be expressed as

$$\delta = \frac{500}{\sqrt{\mu_r \sigma f}} \tag{5}$$



Fig.4 Experimental set-up.

where σ denotes the material conductivity and the relative permeability μ_r equals 1 for a non-ferromagnetic material as the aluminum is. Frequency f is expressed in Hz while δ is in m. Because δ depends on the frequencies f_1 or f_2 , the penetration in the metal is different and thus the temperature effect is not in the same proportions.

Our purpose being mainly to focus on the temperature variations inside the target, a heating resistor is placed on the back side of the aluminum sheet. The whole device is also placed in an oven, regulated in temperature and humidity, so as to minimize the contribution of the other environmental changes. A stepping motor permits to move the object, within a $[0, 5000]\mu m$ range, displacement resolution $(1\mu m)$ being of the same order than that required in industrial applications. It is controlled by a PC giving us also measurements of the two oscillator frequencies (Fig.4). The air and plate temperature, the humidity degree and the real probetarget distance are obtained from external sensors.

4 Signal Processing

The true measured distance obtained from an external sensor permits to build reference curves (RC) providing the real frequencies obtained on the distance interval we use at a given reference temperature $T_0 = 18^{\circ}C$. These RC's permit to do a parametric estimation of the theoretical model describing the frequency response of the target. Thus, for each measurement we do next, while the temperature is changing, we compute the distance deduced from the two frequencies by using directly the previous model. This is the preprocessing step of the experimental data set. The main step is to separate the temperature from the distance



Fig.5 Experimental data set.

measurement by application of source separation. Then, a post-processing step will be necessary to remove the indeterminacies of source estimates amplitude and sign inherent to any non-blind or BSS problem.

4.1 Parametric Estimation of the Theoretical RC Model

Earlier work in this domain [4] proved that the frequency response of the eddy-current sensor under the same experimental conditions can be matched by the following mathematical model:

$$f_{1,2}(i) \cong f_{01,02}(1-k)^{-\frac{1}{2}},$$

$$k^2 = \beta \exp(-\alpha_1 d_i - \alpha_2 d_i^2)$$
(6)

where *i* denotes the discrete time and $w = \{\beta, \alpha_1, \alpha_2\}$ is a set of parameters to be estimated by, for example, a gradient descent method. The theoretical RC computed using the Levenberg-Marquardt algorithm, leads to a non unique estimated parameters set \hat{w} achieving the minimum residual sum of squares. This algorithm is a combination of the Newton method which converges fast but requires a good initial guess, and the steepest descent method which converges slowly but does not require a good initial guess. The global error ε_t corresponding to the parametric estimation is computed following

$$\varepsilon_t = \sqrt{\sum_{i=1}^N \varepsilon_i^2} \text{ and } \varepsilon_i = \frac{|f_{1,2}(i) - \widehat{f_{1,2}(i)}|}{f_{1,2}(i)} \quad (7)$$

where N designates the number of samples. It takes the value 2.57×10^{-3} for the higher frequency corresponding RC and 7.12×10^{-4} for the lower one.



Fig.6 (a) Estimated (dotted) and exact (plain) values of the sheet temperature and (b) estimated (point) and exact (square) values of the probe-target distance for fifty points.

4.2 From the Frequency Response to the Distance Measurement

Let $\hat{w} = \left\{ \hat{\beta}, \hat{\alpha}_1, \hat{\alpha}_2 \right\}$ be the estimated parameters set obtained as described previously. By using the model of equation (6), the frequency response of the sensor can be converted directly in distance measurement according to:

$$d_i \cong \left(\sqrt{\Delta} - \hat{\alpha}_1\right) / 2\hat{\alpha}_2,$$

$$\Delta = \hat{\alpha}_1^2 - 4\hat{\alpha}_2 \left\{ 2\log\left[1 - \left(\frac{f_{01,02}}{f_{1,2}(i)}\right)^2\right] - \log(\hat{\beta}) \right\}. (8)$$

So, when the aluminum sheet is moving while its surface temperature is varying, we obtain a series of distance measurements, corrupted by the temperature effect, for the used theoretical model (6) is valid only at a constant temperature. Thus, these series will be the components of the input observation vector \mathbf{x} to a BSS algorithm. Fig.5 represents the experimental data set of the sources and the observation vectors corresponding to the two operating frequencies f_1 and f_2 . In order to treat the entire displacement range, choice of distance in time is arbitrary within it and its distribution is also almost uniform. Thus, mixtures 1 and 2 are also uniform processes.

5 Experimental Results

In this section, we will show that BSS can enhance the distance measurement in the case of a sheet temperature signal comprising three peaks at about 56, 76 and $70^{\circ}C$. Another case is also examined, for which poor results of BSS can be circumvented by taking into ac-

count the a priori knowledge of temperature. We will show that this technique can greatly improve performances. Comparison with linear filtering is resumed in Table 1, which results justify the use of the non-blind source separation approach.

5.1 Case of Three Temperature Peaks

We applied a BSS method to the observations shown in Fig.5. The aluminum sheet temperature is stabilized around $20^{\circ}C$ (inside the oven) throughout the acquisition of the sensor response, but in the meantime, we modified it on three several occasions and for short time delays. After the pre-processing step described in sections 4.1 and 4.2, we applied JADE [5], a batch BSS algorithm, to the whole observation set.

After centering and rescaling the amplitude levels of the sources and their estimates, we computed the estimation errors by simply considering the mean of the absolute values of the difference between exact values and estimated ones provided by the BSS separation algorithm. The global error was $3.7^{\circ}C$ for the plate temperature and $7.6\mu m$ for the distance. As the least perturbed observation gives a total error of about $50\mu m$ that is not too accurate, the above results are quite good.

We compared then the performances of JADE with a non-blind source separation method. By adding prior information concerning the target temperature according equation (2), to the first, to the second and to both components of the observation vector \mathbf{x} , for the optimal value of α , the errors on the distance estimate are $9.4\mu m$, $7.2\mu m$ and $6.3\mu m$. These errors are of the same order that those provided by JADE. In such context, the interest is also that some information about the sheet temperature can be retrieved in addiction to distance measurement (see Fig.6).

One can also remark that, even after the source separation, there still remains an important level of additive noise into the estimates, that is due to the instrumentation (quality of the electronic components, vibrations, thermal noise ...).

5.2 Case of One Peak and Four Steps at Different Constant Temperatures

This experience took place outside the oven, so we have to take into consideration the variations of the environmental temperature that can be considered as



Fig.7 Experimental data set outside the oven.

a noise. Fig.7 shows the sources. We applied a BSS method and we compared it with non-blind approach and linear filtering. Table 1 shows the errors associated to the above three methods, computed as described in the previous section.

Error	Temperature ($^{\circ}C$)	Distance (μm)
BSS	9.6	319.2
Filtering	-	92.6
	-	82.667
Non-Blind	0.6	6.7
Source	0.7	8.9
Separation	0.7	7.9

Table 1 Comparative results.

BSS algorithm based on model (1), obviously, can not deal with such complicated temperature signals that influence separation quality. Indeed, after BSS, the stair profile is embedded in noise, and thus we can not cope with this kind of temperature variations as long as the adopted BSS model does not permit to minimize each effect that contribute to this noise. Simple linear filtering as described in section 2.4 improves the results accuracy; but an important error on the distance estimate remains. Hence, we supposed that the target temperature profile is known and we coupled this prior knowledge with a source separation method. In this non-blind context, distance estimation is more accurate than that provided by BSS or filtering. Temperature injection to the first, second and to both components of \mathbf{x} (Table 1) is made as in section 5.1 and the obtained error becomes acceptable, as with the three pics profile.

6 Conclusion and Discussion

Results presented in this paper show that source separation can be a suitable way to gain some robustness against temperature effect on an eddy-current sensor. When considering a distance measurement between the probe and a metallic target, BSS is able to improve the sensor response in the case of short plate temperature variations, such as peaks. When there is too much fast changes in the latter, in particular in case of temperature varying by steps, BSS is no more accurate owing to an important remaining noise in the source estimate signals. We showed that if a priori knowledge of the temperature variation is used, source separation in non-blind context can also provide good results. As we could have thought in a first approach, linear filtering is not sufficient.

The interest of BSS in our problem is that it can provide not only distance estimate, but also temperature profile. Nevertheless, the importance of the remaining noise after separation lead us to reconsider the validity of model (1). Indeed, parts of this noise can be due to external environment variations. Because even if they are regulated by the oven, the sensor remains sensible through the plate, to air fluctuations and even to small variations of the ambient temperature. This can cause small non-stationarities or non-linearities in the sources or in their mixing, especially in the temperature value, when they are perceived by the sensor. These assumptions explain poor performances of linear filtering compared to non-blind separation, because in this method, we can consider that the linear part in the mixing is reinforced by temperature addiction, while non-linear parts contribution is reduced.

Some further researches are in progress, in order to take under consideration the above problems. In particular, a non-linear and a convolutive BSS model will be studied. Finally, a mathematical model describing temperature propagation inside the plate, issued from heat conduction theory, will also be taken into account in the future.

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