Variable Structure Control Approach for a Photovoltaic Generator Coupled to a DC Water Pump

A. SELLAMI[1], R. ANDOULSI[1], R. M'HIRI[2], G. DAUPHIN TANGUY[3], and M. KSOURI[2]

1- Laboratoire LAS / INRST B.P. 95, 2050 Hammam-Lif, TUNISIA
Fax: 216(1)430 934
2- Laboratoire ACS / ENIT B.P. 37 1002 Tunis Belvedere, TUNISIA
3- LAIL/E.C.de Lille B.P.48, 59651 Villeneuve d'ascq. Cedex, FRANCE

Abstract: This paper proposes a new robust control strategy for the maximum power-point tracking (MPPT) of a photovoltaic (PV) generator coupled to a DC water pump. The proposed strategy is based on the sliding mode theory and allows a direct non linear control of the power converter. Stability and robustness issues are addressed, and a control design scheme is carried out. This work is motivated by the need to track the maximum power from the PV generator which is a non-linear device having insulation-dependent volt-ampere characteristic. Because of its relatively high cost, the system designer is interested in optimum matching of the motor and its mechanical load to the PV source so that maximum power is obtained during the entire operating period. The advantages of the proposed control scheme are indicated in comparison of the conventional pulse switch modulation (PWM) approach. Reliable results are presented and discussed in the last part of the paper.

Keywords: Photovoltaic,DC Water Pump,MPPT, Variable Structure Control, Sliding Mode,Robustness

1 Introduction

The application of photovoltaics has been increasingly popular, especially in remote areas, where power from utility is not available or is too costly to install. PV powered water pumping is frequently used for agriculture and in households.

However the major disadvantages of using PV arrays are their associated energy conversion efficiency, which is quite low. Therefore, in order to enhance the cost effectiveness of PV array-powered systems, the electric power generated by the PV arrays should be efficiently utilised, and any improvement in efficiency is considered to be precious.

In a direct coupled (with no battery storage) PV system, the solar cell array is directly connected to the motor load couple. These systems are relatively simple and inexpensive to operate. A direct coupled system may include a maximum power point tracker (MPPT) to improve its performance at starting and at steady state operation whenever it is needed [2,7].

The DC motor is supplied from the PV generator whose volt-ampere characteristics depend non-linearly on the solar insulation and temperatures variations and on the current drawn by the DC motor. To match the point at which the PV generator power is maximum, the system designer use an electronic device (converters) known as MPPT, which continuously matches the output characteristics of the PV generator to the input characteristics of the DC motor [1]. The MPPT consists, generally, of a power processing circuit, as buck or boost converters. The classical control approach of the system is made through the duty cycle of the converter. However, in this case the PV generator is characterised by a strongly non-linear current-voltage characteristics giving a non-linear state equation and a linearised model around a given steady-state condition of the I-V characteristic.(peak power point).

This paper presents a new control approach for MPPT of a PV generator coupled to a DC water pump during the entire operating period, and using only the linear part of the system. The proposed strategy is based on the sliding mode control theory and allows a direct non linear control of the power converter. The optimum matching is achieved by imposing the PV current as a reference value which is proportional to insolation level.

2 PV Generator Characteristics

A PV generator consists of an array of photovoltaic cell modules connected in series-parallel combination to provide the desired dc voltage and current. The corresponding I-V characteristic, strongly non-linear, can be represented by the following equation [6]:

$$I_p = I_{ph} - I_s(\exp[\frac{q(V_p + I_p R_s)}{AKT}] - 1) - \frac{V_p + I_p R_s}{R_{sh}} \qquad (1)$$

where I_p is the current, is the photo current, I_s is the reverse saturation current, q is the electron charge, V_p is the terminal voltage , R_s is the serial resistance , A is the ideality factor , k is the Boltzman constant, T is the absolute temperature and R_{sh} is the shunt resistance. Fig.1 and Fig.2 show I_V and P_V characteristics respectively for two values of solar radiation and two values of panel temperature.

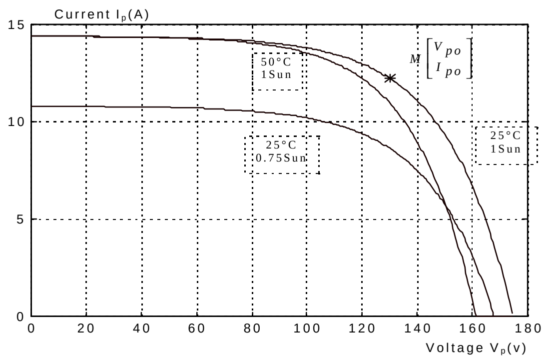

Fig.1. PV generator current-voltage characteristics

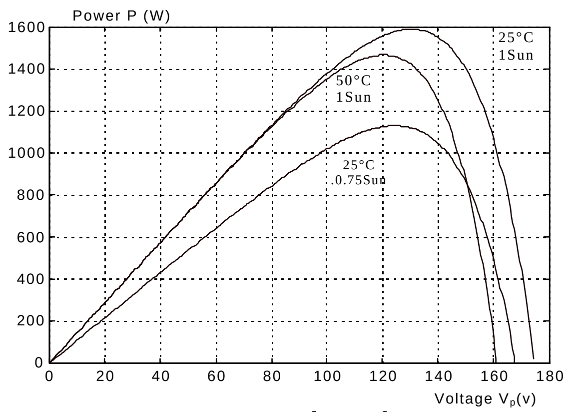

Fig.2. PV generator power-voltage characteristics

The condition of a maximum power point is given by:

$$\frac{d}{dV_p}[V_p \quad I_p] = 0 \qquad (2)$$

The corresponding voltage V_{po} and current I_{po} are determined by solving equation (2) with an iterative method.

Reference [1] show that for maximum power the current drawn from the PV generator, noted I_{mp}, must be directly proportional to the photo-current I_{ph} which is it self directly proportional to insolation level, and the linear relationship is approximated by

$$I_{mp} = 0.85 \ I_{ph} \qquad (3)$$

This remarkable property of the PV generator is useful in achieving an adaptive matching of the DC motor and its mechanical load to the PV generator so that matching is optimum at any insolation level.

3 Design Procedure of Variable Structure Controller

Consider the following linear time-invariant system:

$$\dot{x} = Ax + Bu \qquad (4)$$

where x(t) $\in \mathbb{R}^n$ is the state vector, u(t) $\in \mathbb{R}^m$ is the control input. The constants matrices A and B are of appropriate dimensions. We suppose that the pair (A,B) is controllable, B has full rank m, and n>m.

The two steps of the design approach, namely the existence step and the reaching step, will now be presented.

2.1 Design of the sliding surface

The switching function has the form $s = C\tilde{x}$, where C is an (mxm) matrix, and $\tilde{x} = x - x_R$ is the error between the system's state vector and the reference (constant) trajectory.

The equation $s = 0$ defines the m-dimensional sliding surface.

Differentiating with respect to time and inserting (4) gives $u_{eq} = -(CB)^{-1}CAx$, when $(CB)^{-1}$ exists. As a result, the dynamics $\dot{x} = (I - (CB)^{-1}C)Ax = A_{eq}x$ describes the motion on the sliding surface which is independent of the actual value of the control and depends only on the choice of the matrix C.

The hyperplane design methods [3,4,8] can be used to select the gain matrix C which gives a good and stable motion of the system during the sliding mode.

By assumption, the matrix B has full rank m, as a result, there exists an (nxn) transformation matrix T such that: $TB = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}$, where B_2 is (mxm) and non-singular.

Note that the choice of an orthogonal matrix T avoids inverting T when transforming back to the original system.

The transformed state variable vector is defined as $y = Tx$, in term of which the state equation becomes $\dot{y} = TAT^t y + TBu$ and the sliding condition $C\bar{x} = CT^t\bar{y} = CT^t(y - y_R) = 0$.

If the transformed state is partitioned as

$$y^t = \begin{bmatrix} y_1^t & y_2^t \end{bmatrix};\ y_1^t \in \Re^{n-m};\ y_2^t \in \Re^m$$

then

$$\begin{cases} \dot{y}_1 = A_{11}y_1 + A_{12}y_2 \\ \dot{y}_2 = A_{21}y_1 + A_{22}y_2 + B_2 u \end{cases} \quad (5)$$

and

$$C_1(y_1 - y_{1R}) + C_2(y_2 - y_{2R}) = 0 \quad (6)$$

with

$$TAT^t = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix};\ CT^t = \begin{bmatrix} C_1 & C_2 \end{bmatrix}$$

The assumption that the product matrix CB is non-singular implies that C_2 must also be non-singular and the condition defining the sliding mode becomes

$$y_2 = -C_2^{-1}C_1 y_1 + C_2^{-1}C_1 y_{1R} + y_{2R}$$
$$y_2 = -Fy_1 + Fy_{1R} + y_{2R}$$

with F being an mx(n-m) matrix.

The sliding mode is then governed by the equations

$$\begin{cases} \dot{y}_1 = A_{11}y_1 + A_{12}y_2 \\ y_2 = -Fy_1 + Fy_{1R} + y_{2R} \end{cases} \quad (7)$$

representing an (n-m)th order system with y_2 playing the role of a state feedback control. The closed loop system will then have the following dynamics: $\dot{y}_1 = (A_{11} - A_{12}F)y_1 + A_{12}Fy_{1R} + A_{12}y_{2R}$. This indicates that the design of a stable sliding mode ($y \to 0$ as $t \to \infty$) requires the selection of the matrix F such that $(A_{11} - A_{12}F)$ has (n-m) left-half-plane eigenvalues. This may be achieved by using any standard design method giving a linear feedback controller for a linear dynamical system, including the quadratic performance approach, eigenstructure assignment and assignment of eigenvalues in certain regions [5,8].

Upon determining F, the matrix C can be calculated using the fact that $F = C_2^{-1}C_1$. This leads to $C = K_R B_2^{-1}[F \quad I_m]T$ where K_R is an (mxm) design matrix, and I_m is the (mxm) identity matrix [3].

The simplest approach to avoid the amount of calculation in the selection of the matrix K_R is to let $K_R = B_2$, which is equivalent to specify F, I_m, and gives

$$C = \begin{bmatrix} F & I_m \end{bmatrix} T \quad (8)$$

2.2 Reaching mode design and control law

Once the existence problem has been solved that is the matrix C has been determined, attention must be turned to solving the reaching problem. This involves the selection of a non-linear feedback control function *u(x)* which ensures that trajectories are directed towards the switching surface from any point in the state space. Therefore, if $V(x,t) = s^t s$ is a Lyapunov function for system (1), a suitable control u(x) must be chosen to guarantee that the reaching condition, namely, $s^t\dot{s} < 0$, is satisfied.

Several types of control structures insuring the reaching condition are given in the literature [3,4,5,8]. The classical and the simplest one is the relay control law, defined by the following function:

$$u(x) = \begin{cases} k_{max} & \text{when } s(x) > 0 \\ k_{min} & \text{when } s(x) < 0 \end{cases} \quad (9)$$

The relay gain may be either constant or state dependent.

4 Application to MPPT of a PV DC-Water Pump System

4.1 System and control scheme description

The motor and pump used in this study have the following parameters:

- Motor type: Permanent-magnet DC motor
- Pump type: Centrifugal
- Armature circuit resistance: $R_m = 1\Omega$
- Armature circuit inductance: $L_m = 0.04H$
- Inertia: $J = 0.001$ Kg - m^2
- Friction coefficient: $Fv = 0.001$ N - m/rd/s

The applied shaft torque Γ is assumed [5,10] to be proportional to angular velocity Ω

$$\Gamma = K_T \Omega \text{ and } E_m = K_b \Omega \quad (10)$$

where $K_T = 0.1$ and $K_b = 0.5$ are the proportionality factor between shaft torque, back emf (E_m) and angular velocity respectively.

To attempt the optimal point, the DC motor is matched to the solar array by means of maximum power point tracker (MPPT), which consists of a power processing circuit. The block diagram of the proposed MPPT variable structure control scheme is given in the following figure:

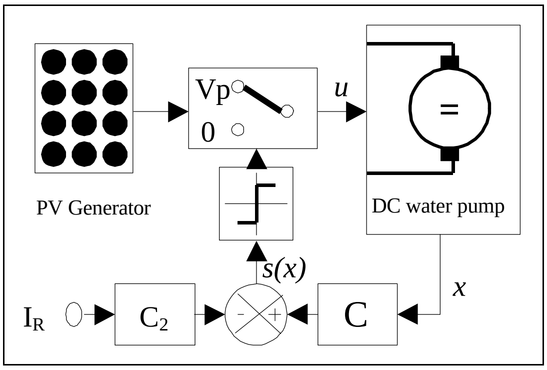

Fig.3. Block diagram of the control scheme

The DC water pump system can be described by the following linear state equation model:

$$\dot{x} = Ax + Bu$$

where

$$A = \begin{bmatrix} -1 & 500 \\ -12.5 & -25 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 25 \end{bmatrix} \quad x = \begin{bmatrix} \Omega \\ i_m \end{bmatrix}$$

The input of this system is the motor control signal u. The states x_1 and x_2 being the angular speed of the motor shaft Ω and the current i_m, respectively.

4.2 Sliding surface design

In this current control system, $x_R = [x_{1R} \quad x_{2R}]^t = [0 \quad I_R]^t$ defines the reference input. The design procedure of the sliding surface $s = C\tilde{x} = 0$ is as follows. As B has the right form, there is no need for the transformation, *i.e.*, take T to be the identity matrix. Here, the closed loop systems dynamics are then described by the equation

$$\dot{y}_1 = (-1 - 500F)y_1 + 500I_R \qquad (11)$$

The eigenvalue of this first-order system can be conveniently selected. To this end, we choose to set $F = 0.02$, yielding a stable closed loop pole at -2. Now, from Equation (8), it follows that

$$C = [C_1 \quad C_2] = [0.02 \quad 1] \qquad (12)$$

4.3 Control law selection:

The relay control law approach is considered here, and leads a control function of the following general form:

$$u(x) = \begin{cases} V_p & \text{when} \quad s(x) > 0 \\ 0 & \text{when} \quad s(x) < 0 \end{cases} \qquad (13)$$

where the relay gain V_p is the PV generator voltage witch is not constant and depends on the operating point, but its value is not critical, as the reaching condition is guaranteed if $V_p \neq 0$.

The choice of this control structure turns out to be very interesting as it avoids the introduction of the non linear I-V characteristic of the PV generator in the system model, yielding linear state equation system and a very simple control scheme in comparison to the classical step-up converter driven by duty-ratio pulse. In fact, the proposed strategy is based on the sliding mode control theory and allows a natural non linear control of the power converter. Fig.3 shows that the power switch is controlled directly by the sign of the switching function *s(x)*.

Noting also that, in the sliding mode, the dynamics behaviour of the system is totally invariant with respect to a subset of uncertainties called matched uncertainties, and the dynamics are completely defined by the choice of the sliding surface. The propriety of invariance during the sliding mode is more strong that robustness.

5 Simulation Results

Since its hard to adjust the operating condition of the PV array, such as the insolating level and temperature in the real field test, the following simulations are carried out instead to confirm the performance of the MPPT-VSC technique by using Matlab with Simulink package.

The open circuit voltage and the short-circuit current of the PV generator used in this study are respectively $V_{PO} = 52$ V and $I_{CC} = 15$ A at 100% insolation (E=1000w/m^2). The numerical simulations were done for different level of insolation (small, medium and high).

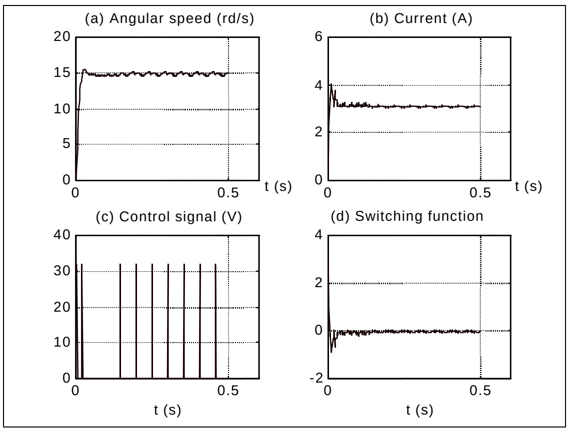

Fig.4. System response at E=250w/m^2 and I_R=3.2A

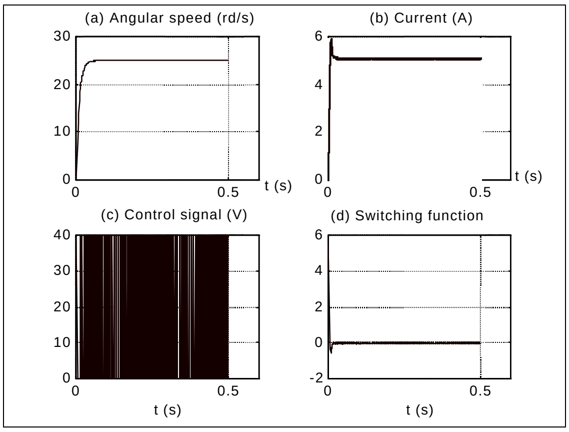

Fig.5. System response at E=400w/m^2 and I_R=5.2A

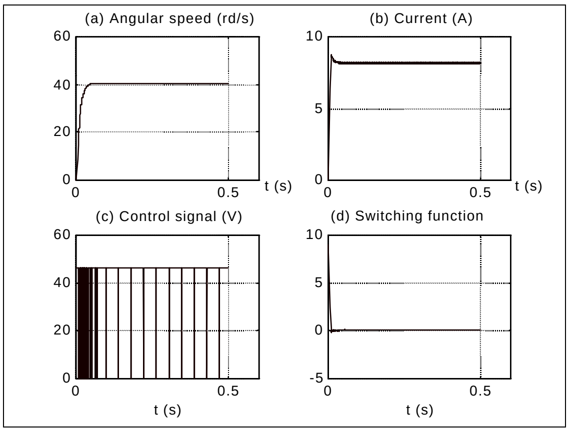

Fig.6. System response at E=650w/m^2 and I_R=8.4A

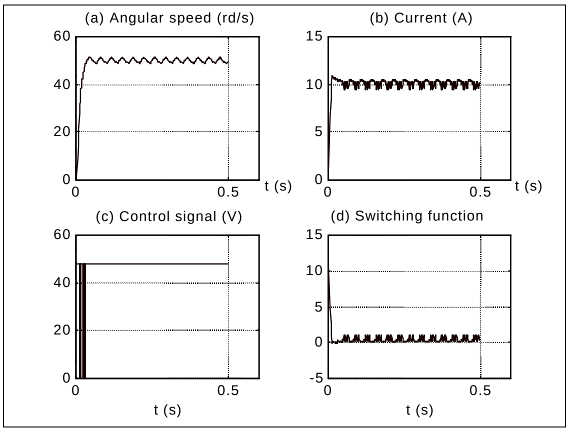

Fig.7. System response at E=800w/m^2 and I_R=10.1A

From Fig.4 to Fig.7 it is evident to see that an optimum matching of the DC water pump to the PV generator can be achieved at any insolation level. Furthermore, the control signal frequency is adapted to the operating conditions.

6 Conclusion

A novel MPPT control strategy based on the sliding mode theory has been shown. to significantly solve the problem of poor utilisation efficiency of PV generator coupled to DC water pump system. Its shown that maximum power output from a PV source is achieved at all insolation levels. The resulting control scheme is easily designed using only the linear system model and allowing a direct non linear control of the power converter.

The two steps of the design approach, namely the existence step in which we choose the sliding surface that gives good behaviour during the sliding mode, and the reaching step in which we choose the control to ensure that the reaching condition is met, are investigated. Stability and robustness issues are addressed. Numerical simulations have been presented showing the applicability and the efficiency of the proposed method.

References

[1] S.M. Alghuwainem,″Matching of a DC motor to a photovoltaic generator using a step converter with a current looked loop″. *IEEE transactions on Energy Conversion*, Vol 9 , N°1, 1994, pp. 192-198.

[2] R. Andoulsi, A. Mami, G. Dauphin-Tanguy, and M. Annabi, ″Bond graph modelling of a photovoltaic generator coupled to a DC motor″. *WMC'99, ICBGM'99* San Francisco, January 1999.

[3] V.I. Utkin, "*Sliding modes in control optimisation*". Springer-Verlag, 1992.

[4] A. Sellami, R. Mhiri, O. Boubaker, M. Ksouri, and J. Zrida, "Variable structure control and performance comparison". *IFAC-IFIB-IMACS Conference on CIS*, Vol. 2, Belfort-France,20-22 Mai 1997, pp.279-287.

[5] A. Sellami, D. Arzelier, R. M'hiri, J. Bernussou, M. Ksouri, and J. Zrida, "A discontinuous sliding mode control approach for systems in the presence of a norm bounded uncertainty". *IMACS-IEEE Multiconference on CES*A, Vol.1, Tunisia, 1-4 April 1998, pp.788-793.

[6] A. Sellami, F. Ghodbane, R. Andoulsi, and H. Ezzaouia, "An Electrical Performance Tester for PV Modules". *Renewable Energy*, Vol.15-16/4003, September 1998, pp.1717-1719.

[7] S. Singer, J. Appelbaum, ″Starting characteristcs of current motors powred by solar cells″, *IEEE Transactions on Energy conversion*, Vol.8, N°1, 1993, pp.47-53.

[8] A.S.I. Zinober, "*An introduction to sliding mode variable structure control*". Lecture Notes in Control and Information sciences 193, Springer-Verlag, 1994.