Iterative Evaluation of Regularization Parameters in Regularized Image Restoration with Wavelet Filter Banks

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Abstract: - A novel approach of iterative regularized restoration of images based upon multirate representations is proposed in this paper. Regularized restoration is a method of solving the ill-posed inversion problem of image restoration. Inversion of usual degradations without regularization enhances the noise in the degraded image and may not be employed in practice. Wavelet filter-banks designed upon arbitrarily sampling lattices are proposed in order to replace the conventional regularization operator (usually a Laplacian filter) in regularized restoration of images. The proposed method employs a regularization parameter for each of the decomposition filters in the wavelet filter-bank. Thus it differs from standard regularized restoration methods which define just one regularization parameter corresponding to the smoothing filter. The regularization parameters should be estimated in advance or iteratively. A good estimate guaranties a good quality of the restored image. Statistical techniques like Generalized-Cross-Validation (GCV) may be used for estimating the regularization parameters in advance whereas the current estimate of the restored image may be used in estimating the regularization parameters in an iteratively solution of the regularization equation.

A perfect reconstruction filter-bank can be used to represent the degradation filter. Factorizations of unitary matrices using Givens rotations allow for efficient representations for a variety of degradations. Should both the degradation and the smoothing filter be replaced by multirate systems, the restoration problem may be split into independent restoration problems in each transformation channel. Regularization parameters are evaluated iteratively in each channel using image information from the corresponding subband and other channel dependent parameters. Numerical results indicate better ISNR (Improvement in Signal-to-Noise-Ratio) figures than conventional iterative regularization methods.

Key-Words: - Wavelets; Multiresolution; Filter banks; Regularized image restoration; Iterative methods.

1 Introduction

Wavelet functions exhibit very good localization properties in frequency and in spatial/time domain and provide an efficient way of representing 1-D and 2-D signals. Construction of wavelet and scale functions (mother wavelets) of finite support is possible by iterative filtering and subsampling [1]. Wavelet decomposition of signals allows for processing in separate subbands [2]. Several methods based on multiresolution analysis via wavelets and orthogonal wavelet filter-banks have appeared recently in the literature [3] for such applications as image/speech coding and compression [2], motion estimation in video sequences, neural wavelet networks [4], speech recognition and image restoration.

Digital images are degraded due to motion blur, defocusing, atmospheric turbulence and long exposure times, [5]. The linear degradation model assumes that the degraded image, denoted as $\mathbf{y}$ is related to the original image $\mathbf{s}$ through the degradation matrix $\mathbf{K}$ and additive white Gaussian noise $\mathbf{n}$

$$\mathbf{y} = \mathbf{K}\mathbf{s} + \mathbf{n} \tag{1}$$

where vectors $\mathbf{y}$, $\mathbf{s}$ and $\mathbf{n}$ are produced by raster scanning and stacking of the pixel values of the corresponding images. These vectors have N^2 elements for images having size (NxN) pixels. $\mathbf{K}$ is a block Toeplitz matrix which expresses the convolution of the original image with the degradation filter $K(m_1,m_2)$ and has dimensions N^2 by N^2.

Regularized restoration techniques [6] tackle the ill-posed problem of image restoration by assuming a smoothing filter as regularization operator and minimizing a Lagrangian functional which involves the error and the outcome of the filtering with the smoothing filter.

$$\min_{\hat{\mathbf{s}}} J_{\lambda}(\hat{\mathbf{s}}) = \min_{\hat{\mathbf{s}}} \left(\|\mathbf{y} - \mathbf{K}\hat{\mathbf{s}}\|^2 + \lambda \|\mathbf{C}\hat{\mathbf{s}}\|^2 \right) \qquad (2)$$

where $\mathbf{C}$ is the filtering matrix with the smoothing filter and λ is the so-called regularization parameter. There are several methods for estimating the regularization parameter λ, like iterative methods [7] and statistical methods [8],[9]. This paper proposes an iterative method of estimating regularization parameters in wavelet channels. The block diagram of the method is presented in Figure 1.

2 Regularized Image Restoration Using Filter Banks as the Smoothing Operator

Regularized image restoration using wavelet filter banks as the smoothing operator is formulated in the sequel as an optimization problem. Wavelet factorizations via unitary matrices are suggested to decompose the degradation matrix $\mathbf{K}$ into independent frequency channels. The equation of the regularized restoration method is solved iteratively in each of these channels.

2.1 Formulation of Regularized Restoration as an Optimization Problem

Generalization of regularized image restoration in multiresolution spaces has been presented in [10]. The restored image $\hat{\mathbf{s}}$ is found by minimizing the following Lagrangian cost,

$$\begin{aligned} &\min_{\hat{\mathbf{s}}} J_{(\lambda_0,\lambda_1\ldots\lambda_{D-1})}(\hat{\mathbf{s}}) = \\ &= \min_{\hat{\mathbf{s}}} \left(\|\mathbf{y} - \mathbf{K}\hat{\mathbf{s}}\|^2 + \lambda_0 \|\mathbf{W}_0^{\mathrm{T}}\hat{\mathbf{s}}\|^2 + \ldots + \lambda_{D-1} \|\mathbf{W}_{D-1}^{\mathrm{T}}\hat{\mathbf{s}}\|^2 \right) \end{aligned} \qquad (3)$$

where λ_0, λ_1, … λ_{D-1} are the regularized restoration parameters and $\mathbf{W}_0^{\mathrm{T}}$, $\mathbf{W}_1^{\mathrm{T}}$ … $\mathbf{W}_{D-1}^{\mathrm{T}}$ are the matrices of spatial filtering with the wavelets for the D distinct channels of image decomposition. Matrices $\mathbf{W}_0^{\mathrm{T}}$, $\mathbf{W}_1^{\mathrm{T}}$ … $\mathbf{W}_{D-1}^{\mathrm{T}}$ have dimensions $\frac{N^2}{D}$ rows by N^2 columns for N by N images. They obey the orthonormality conditions by construction, i.e.

$$\begin{aligned} &\mathbf{W}_0\mathbf{W}_0^{\mathrm{T}} + \mathbf{W}_1\mathbf{W}_1^{\mathrm{T}} + \ldots + \mathbf{W}_{D-1}\mathbf{W}_{D-1}^{\mathrm{T}} = \mathbf{I} \\ &\mathbf{W}_p^{\mathrm{T}}\mathbf{W}_q = \mathbf{0} \text{ if } p \neq q \text{ or } \mathbf{W}_p^{\mathrm{T}}\mathbf{W}_q = \mathbf{I} \text{ if } p = q . \end{aligned} \qquad (4)$$

One derives the regularized restoration equation by solving the equation, i.e. $\frac{\partial J_{(\lambda_0,\lambda_1\ldots\lambda_{D-1})}}{\partial \hat{\mathbf{s}}} = \mathbf{0}$, and taking into account the orthonormality conditions,

$$\left(\mathbf{K}^{\mathrm{T}}\mathbf{K} + \lambda_0\mathbf{W}_0\mathbf{W}_0^{\mathrm{T}} + \ldots + \lambda_{D-1}\mathbf{W}_{D-1}\mathbf{W}_{D-1}^{\mathrm{T}}\right)\hat{\mathbf{s}} = \mathbf{K}^{\mathrm{T}}\mathbf{y} . \qquad (5)$$

$\mathbf{K}^{\mathrm{T}}$ denotes the transpose degradation matrix. The solution of Eq. (5) yields the restored image $\hat{\mathbf{s}}$. Further simplification of this relationship is possible by making certain assumptions regarding the degradation matrix $\mathbf{K}$. The iterative solution of Eq. (5) independently in each decomposition channel is proposed in the sequel should the degradation matrix be decomposed by a perfect reconstruction wavelet filter bank.

2.2 Decomposition of Degradation Matrix into Wavelet Channels

Several wavelet factorizations like unitary matrices expressed by Givens rotations are possible for designing filter banks. This allows for variable degradations to be decomposed by a perfect reconstruction filter bank like the one shown in Figure 2. Matrix $\mathbf{K}$ is written in such a case as

$$\begin{aligned} \mathbf{K} = &\sum_{\mathbf{m}'\in\Lambda_{\mathbf{D}}} \mathbf{W}_0 d_0(\mathbf{m}')\mathbf{P}_{\mathbf{m}'}\mathbf{W}_0^{\mathrm{T}} + \ldots \\ &\ldots + \sum_{\mathbf{m}'\in\Lambda_{\mathbf{D}}} \mathbf{W}_{D-1} d_{D-1}(\mathbf{m}')\mathbf{P}_{\mathbf{m}'}\mathbf{W}_{D-1}^{\mathrm{T}} \end{aligned} \qquad (6)$$

where $\Lambda_{\mathbf{D}}$ is the sampling sublattice. $\mathbf{P}_{\Delta\mathbf{m}}$ is a permutation matrix whose elements take the values of zero and one and shifts the 2-D channel degradation filters $d_0(\mathbf{m}')$, $d_1(\mathbf{m}')$ … $d_{D-1}(\mathbf{m}')$ in Figure 2 by $\Delta\mathbf{m}$. Permutation matrices allow for expressing convolution as a sum of matrix products. The expression of degradation matrix $\mathbf{K}$ in Eq. (6) implies that the degradation filters in each channel of the decomposition are independent. Thus Eq. (5) may be split into D independent systems of equations as

$$\begin{aligned} \left(\mathbf{D}_{c,0} + \lambda_0\mathbf{I}\right)\mathbf{W}_0^{\mathrm{T}}\hat{\mathbf{s}} &= \mathbf{D}_0^{'}\mathbf{W}_0^{\mathrm{T}}\mathbf{y} \\ \left(\mathbf{D}_{c,1} + \lambda_1\mathbf{I}\right)\mathbf{W}_1^{\mathrm{T}}\hat{\mathbf{s}} &= \mathbf{D}_1^{'}\mathbf{W}_1^{\mathrm{T}}\mathbf{y} \\ &\vdots \\ \left(\mathbf{D}_{c,D-1} + \lambda_{D-1}\mathbf{I}\right)\mathbf{W}_{D-1}^{\mathrm{T}}\hat{\mathbf{s}} &= \mathbf{D}_{D-1}^{'}\mathbf{W}_{D-1}^{\mathrm{T}}\mathbf{y} \end{aligned} \qquad (7)$$

Regularized restoration parameters in each multiresolution channel are denoted as λ_0, λ_1, … λ_{D-1}. Matrices $\mathbf{D}_{c,0}$, $\mathbf{D}_{c,1}$…$\mathbf{D}_{c,D-1}$ are used to convolve, upon lattice $\Lambda_{\mathbf{D}}$, the 2-D series $d_{c,0}(\mathbf{m}')$, $d_{c,1}(\mathbf{m}')$, … $d_{c,D-1}(\mathbf{m}')$, (which are defined as $d_{c,0} = d_0 * \tilde{d}_0$, $d_{c,1} = d_1 * \tilde{d}_1$, … $d_{c,D-1} = d_{D-1} * \tilde{d}_{D-1}$) with the multiresolution coefficients of the restored image, $\mathbf{W}_q^{\mathrm{T}}\hat{\mathbf{s}}$. Channel filters $\tilde{d}_0$, $\tilde{d}_1$ … $\tilde{d}_{D-1}$ are the reconstruction filters defined as $\tilde{d}_0(\mathbf{m}') = d_0(-\mathbf{m}')$, $\tilde{d}_1(\mathbf{m}') = d_1(-\mathbf{m}')$, … $\tilde{d}_{D-1}(\mathbf{m}') = d_{D-1}(-\mathbf{m}')$, $\mathbf{m}'\in\Lambda_D$. Convolution matrices $\mathbf{D}_0^{'}$, $\mathbf{D}_1^{'}$ … $\mathbf{D}_{D-1}^{'}$ convolve the channel reconstruction filters with the multiresolution coefficients of the degraded image, $\mathbf{W}_q^{\mathrm{T}}\mathbf{y}$, upon $\Lambda_{\mathbf{D}}$.

3 Iterative Solution of the Regularization Equation

The block diagram of the proposed iteration is shown in Figure 1. The iterative solution of Eqs (7) is proposed as

$$
\begin{aligned}
&\mathbf{W}_0^{\mathrm{T}}\hat{\mathbf{s}}(0) = \beta_0 \mathbf{D}_0^{'}\mathbf{W}_0^{\mathrm{T}}\mathbf{y}\\
&\mathbf{W}_0^{\mathrm{T}}\hat{\mathbf{s}}(t+1) = (\mathbf{I}-\beta_0\lambda_0)\mathbf{W}_0^{\mathrm{T}}\hat{\mathbf{s}}(t)+\beta_0\left(\mathbf{D}_0^{'}\mathbf{W}_0^{\mathrm{T}}\mathbf{y}-\mathbf{D}_{c,0}\mathbf{W}_0^{\mathrm{T}}\hat{\mathbf{s}}(t)\right)\\
&\vdots \\
&\mathbf{W}_{D-1}^{\mathrm{T}}\hat{\mathbf{s}}(0) = \beta_{D-1} \mathbf{D}_{D-1}^{'}\mathbf{W}_{D-1}^{\mathrm{T}}\mathbf{y}\\
&\mathbf{W}_{D-1}^{\mathrm{T}}\hat{\mathbf{s}}(t+1) = (\mathbf{I}-\beta_{D-1}\lambda_{D-1})\mathbf{W}_{D-1}^{\mathrm{T}}\hat{\mathbf{s}}(t)+\\
&\qquad +\beta_{D-1}\left(\mathbf{D}_{D-1}^{'}\mathbf{W}_{D-1}^{\mathrm{T}}\mathbf{y}-\mathbf{D}_{c,D-1}\mathbf{W}_{D-1}^{\mathrm{T}}\hat{\mathbf{s}}(t)\right)
\end{aligned}
\tag{8}
$$

This is an extension of the general iterative solution of the regularized restoration equation presented in [11],[12],[13]. Regularization parameters are evaluated iteratively according to the relationship,

$$
\lambda_q(t) = \frac{\varepsilon^2}{\left\|\mathbf{W}_q^{\mathrm{T}}\hat{\mathbf{s}}(t)\right\|^2+\delta_q} \tag{9}
$$

where ε^2 equals $\mathbf{n}^{\mathrm{T}}\mathbf{n}$ and δ_q is a small positive constant. It is necessary for convergence that the iteration parameter β_q, $q\in\{0,1\ldots D-1\}$, satisfy the following equation

$$
0<\beta_q<\frac{2}{\max_i\left(\lambda_q\mathbf{I}+\mathbf{D}_{c,q}\right)} \tag{10}
$$

where $\max_i\left(\lambda_q\mathbf{I}+\mathbf{D}_{c,q}\right)$ stands for the maximum eigenvalue of matrix $\left(\lambda_q\mathbf{I}+\mathbf{D}_{c,q}\right)$. Thus iterative parameters β_q are channel dependent.

4 Experimental Results

The degradation is assumed to be decomposed by a perfect reconstruction filter bank upon the quincunx sampling lattice with sampling matrix $\mathbf{D}=\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$. The filter bank has the following factorization,

$$
\mathbf{K}_p(\boldsymbol{\omega}) = \tilde{\mathbf{H}}_p(\boldsymbol{\omega})\,diag\{d_0,d_1\}\mathbf{H}_p(\boldsymbol{\omega}), \quad \text{where}
$$

$$
\tilde{\mathbf{H}}_p(\boldsymbol{\omega}) = \begin{pmatrix}\cos(\theta_0) & \sin(\theta_0)\\ -\sin(\theta_0) & \cos(\theta_0)\end{pmatrix}
\left(\prod_{n=1}^{N-2}\prod_{m=1}^{2}\begin{pmatrix}1 & 0\\ 0 & e^{j\omega_m}\end{pmatrix}\begin{pmatrix}\cos(\theta_{n,m}) & \sin(\theta_{n,m})\\ -\sin(\theta_{n,m}) & \cos(\theta_{n,m})\end{pmatrix}\right) \tag{11}
$$

where $\mathbf{H}_p(\boldsymbol{\omega})$ is the polyphase matrix of the filter bank. The factorization parameters are presented in Table 1. The original image of "Lena" along with its two polyphase components is presented in Figure 3.

Degradation parameters

d_0	θ_0	$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	$\theta_{4,1}$	$\theta_{5,1}$	$\theta_{6,1}$
0.65	-60°	55.5°	42.5°	-20.5°	4.5°	4°	1°
d_1		$\theta_{1,2}$	$\theta_{2,2}$	$\theta_{3,2}$	$\theta_{4,2}$	$\theta_{5,2}$	$\theta_{6,2}$
0.30		-26.5°	-71°	162.5°	6°	1°	4°

Table 1: Filter bank factorizations according to Eq. (11) for the degradation

The linear degradation model of Eq. (1) is assumed. The standard deviation of additive noise is σ=0.01. The degraded image $\mathbf{y}$ is shown in Figure 4a. An estimate of the s.t.d. of the noise is obtained from the flat regions of the corrupted image. This estimate is used in evaluating ε^2 in Eq. (9) which yields λ(t) vs iteration. Iterative regularized restoration is carried out as described by Eq. (8) (see Figure 1) for various iterative parameters β_0 and β_1. The algorith is terminated by the if $\frac{\left\|\hat{\mathbf{s}}(t+1)-\hat{\mathbf{s}}(t)\right\|^2}{\left\|\mathbf{s}(t)\right\|^2}\leq 10^{-6}$. The results are compared against standard iterative solutions of the regularized restoration equation that appear in the literature. The Laplacian is used as the smoothing filter in conventional methods. The plot of the Mean-Square-Error (MSE) for all cases is presented in Figure 5 versus iteration. Table 2 gives the values of the parameters of the iteration as well as the final improvement in Signal-to-Noise-Ratio (ISNR), which is defined as $10\log(\frac{\left\|\mathbf{y}-\mathbf{s}\right\|^2}{\left\|\mathbf{y}-\hat{\mathbf{s}}\right\|^2})$, for all cases. Iterative regularized image restoration in multiresolution channels yields the same final ISNR for two different pairs of iterative parameters β_0 and β_1. This value is better than the ISNR value obtained for conventional iterative regularized restoration. The restored image of "Lena" with β_0=3 and β_1=2 is shown in Figure 4b. Convergence rates differ in each case. The evolution of MSE and the values of the regularization parameters obtained from Eq. (9) are presented in Figure 7 and Figure 8 for both multiresolution channels.

5 Conclusion

A novel approach of iterative regularized image restoration in multiresolution channels is proposed. A perfect reconstruction filter bank which is defined upon arbitrarily sampling lattices decomposes the degradation filter as well as the smoothing filter into independent wavelet channels. The corresponding systems of regularization equations are solved iteratively. The regularization parameters are evaluated at each iteration step. Restoration results obtained with the proposed method are better than results obtained with conventional regularization methods.

Case	Laplacian	Filter bank	
Iteration parameters	$\beta=0.5$ $\delta=0.01$	$\beta_0=0.5$ $\beta_1=1.0$ $\delta_0=0.1$ $\delta_1=0.1$	$\beta_0=3.0$ $\beta_1=2.0$ $\delta_0=0.1$ $\delta_1=0.1$
Number of iterations	33	45	25
ISNR [dB]	9.3	12.4	12.4

Table 2: Iteration parameters and final improvements in Signal-to-Noise-Ratio (ISNR) after convergence

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Figures

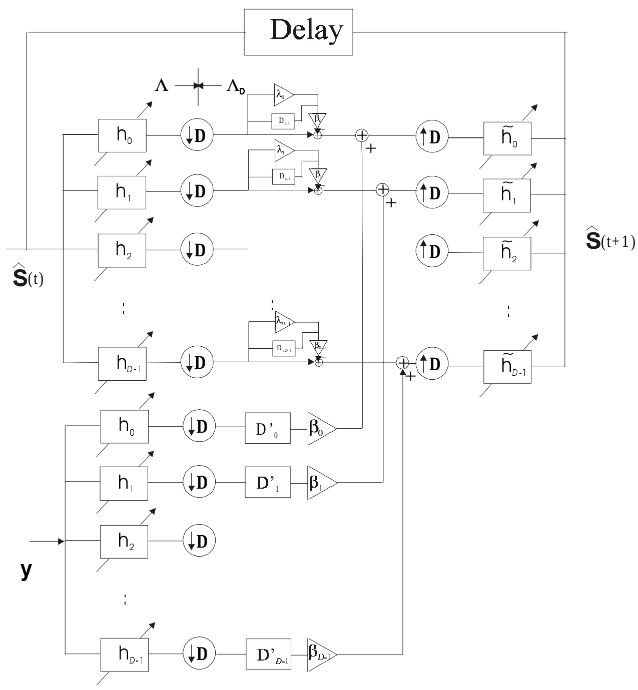


Figure 1: Block diagram of the proposed iterative method for solving the equation of regularized restoration

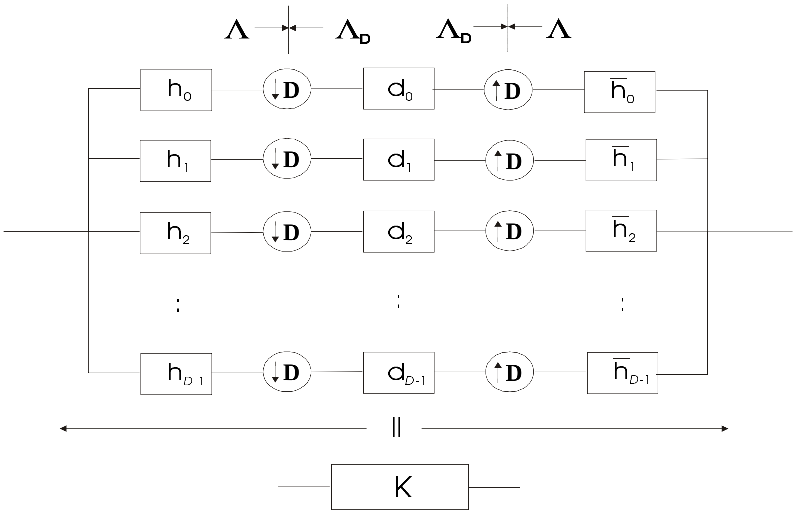


Figure 2: Decomposition of the degradation matrix by a perfect reconstruction wavelet filter bank

a: Original image

b: First polyphase component corresponding to coset vector $(0,0)^T$

c: Second polyphase component corresponding to coset vector $(1,0)^T$

Figure 3: Original and polyphase components of "Lena" for sampling upon the quincunx sublattice

a: Degradated image by filter bank degradation and additive noise

b: Iterative restored image with two regularization parameters (β_0=3 and β_1=2)

Figure 4: Degraded and restored image of "Lena"

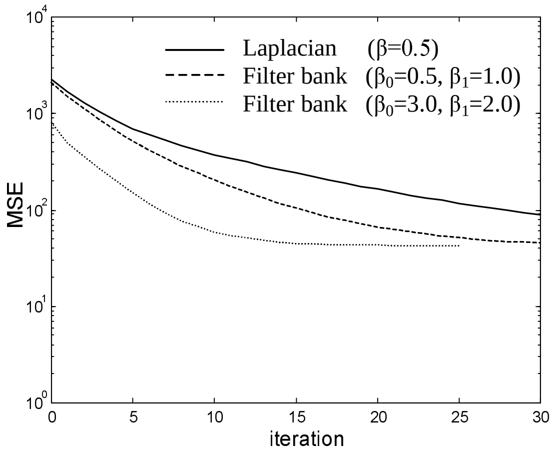


Figure 5: MSE for iterative restoration using the Laplacian and filter bank regularization operator

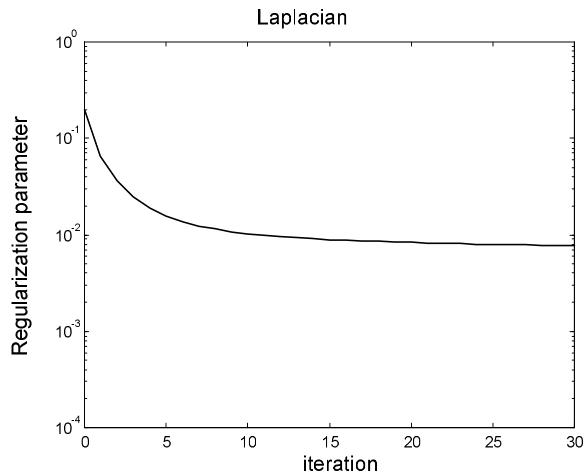
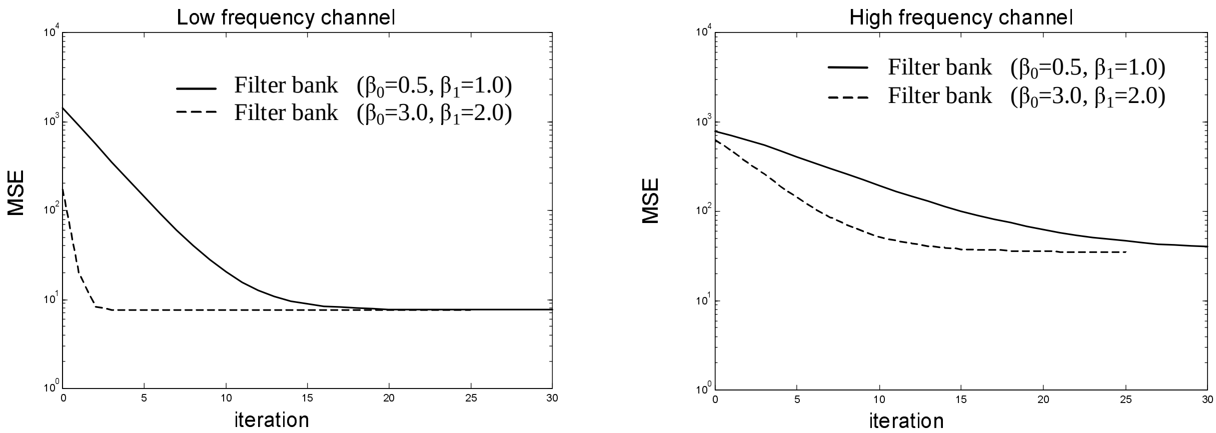


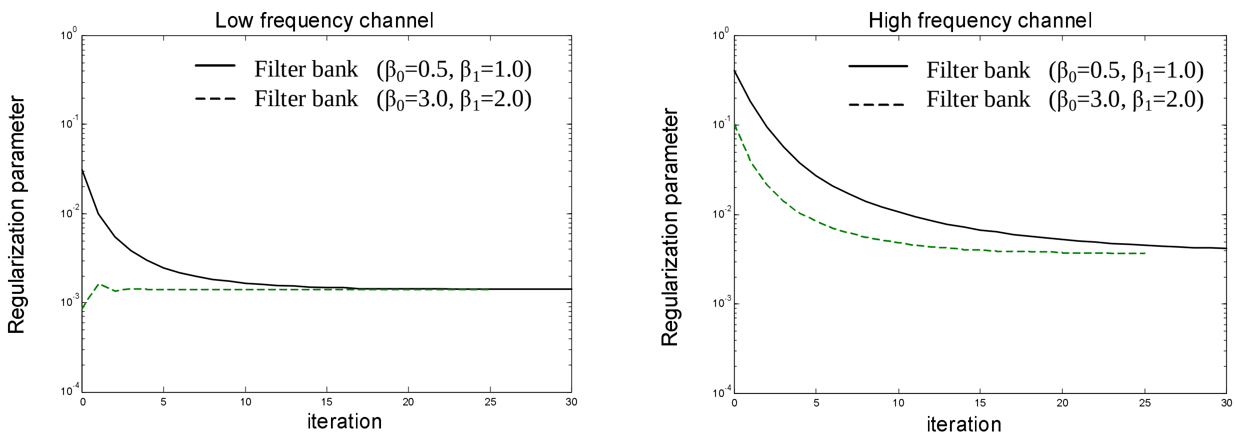
Figure 6: Regularization parameter for iterative restoration using Laplacian



a: Low frequency channel

b: High frequency channel

Figure 7: Mean Square Error for iterative restoration using wavelet filter bank



a: Low frequency channel

b: High frequency channel

Figure 8: Regularization parameters for iterative restoration using wavelet filter bank