Study of the DPCM Transmission System with a Periodic Input

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Abstract: - This paper deals with the study of bifurcation structure of a DPCM transmission system. Previous publications treated the case of a constant input. Here we tackle the case of sinusoidal input. This permits a better study of the DPCM behavior in real condition use. The DPCM with a constant input is described by a two-dimensional non invertible map. The use of a sinusoidal input gives a non autonomous map. The study of bifurcations by classical methods is not possible in this case. The Poincaré mapping method is used to obtain an autonomous map. For the obtained map, two aspects are studied: the parameter plane, which gives the bifurcation curves, and the state plane, which permits the study of basins of attraction. The critical curves tool is used. The obtained map is then studied for some values of the input frequencies.

Key-Words: - DPCM, bifurcation, signal transmission, non autonomous maps, Poincaré map

1 Introduction

The DPCM, Differential Pulse Code Modulation transmission system, is made up of two parts: a coder and a decoder as shown in fig. 1. Both the coder and the decoder contain a prediction filter, which is a linear filter. Here it is taken transverse of order 2, this means that B≡0 and A is a second order linear filter. The coder includes a quantizer. Its characteristic $Q(e)$ is non linear, bounded and often discontinuous.

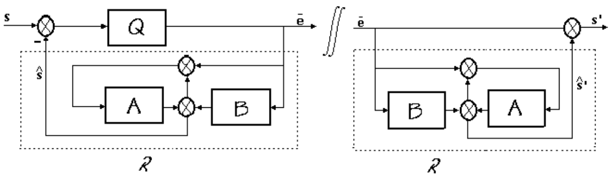

Fig. Error! Unknown switch argument. : The DPCM parts – (left) coder, (right) decoder. **R** is the prediction filter

$\hat{s}$ is the signal prediction. When the parameters of the prediction filter are well chosen, the difference between the signal prediction $\hat{s}_n$ and the input s_n, called prediction error, is tiny. The prediction error is quantized and transmitted by the coder.

The coder is described by a two dimensional non invertible map. Such map is of the following form:

$$(x_{n+1},y_{n+1})=f(x_n,y_n,n) \tag{1}$$

where x_n and y_n are the state variables.

(x_{n+1},y_{n+1}) is called the image of (x_n,y_n) by f and (x_n,y_n) is called the pre-image of (x_{n+1},y_{n+1}). The map f is said to be autonomous if n doesn't appear explicitly in f. If the map f is non autonomous, it is said to have an effective dimension equal to 3.

If each point (x_n, y_n) has one and one only pre-image, the map f is said invertible. In other case it is non invertible.

The state variables are chosen as follow:

$$x_n = \hat{s}_{n-1}\,,\; y_n = \hat{s}_n$$

The model of the DPCM coder is (2) [3]:

$$x_{n+1}=y_n$$
$$y_{n+1}=a_1.y_n+a_2.x_n+a_1.Q(s_n-y_n)+a_2.Q(s_{n-1}-x_n)$$

where: a_1 and a_2 are the parameters of the prediction filter, p is called the compression rate or signal-noise ratio, s_n is the input signal and Q the quantization function. This function has been chosen of the form:

$$Q(x)=Tanh(p.x) \tag{3}$$

In previous publications, authors studied the system behavior for an order 1 prediction filter [1] or with constant input $s_n=s$ [2,3]. In this case, the map obtained is autonomous.

The study of the behavior of this system with a constant input is not sufficient. For a non-constant input, the behavior may vary significantly. In this paper, the following input signal is taken:

$$s_n=A.\cos(2.\pi.f_e.n+\varphi) \tag{4}$$

where A is the input amplitude, f_e its frequency. In this case, the coder model is:

$$x_{n+1} = y_n$$
$$y_{n+1} = a_1.y_n + a_1.Q(A.\cos(2.\pi.f_e.n+\varphi)-y_n) + a_2.x_n + a_2.Q(A.\cos(2.\pi.f_e.(n-1)+\varphi)- x_n)$$

This is a non-autonomous map. It is not possible to study bifurcation phenomenon in this system with classical methods. To obtain an autonomous map, we apply the Poincaré mapping described in the next section. In sect. 3, the application of Poincaré mapping to DPCM is explained. In sect. 4, The parameter plane is studied and in sect. 5, the state plane is analyzed, using the critical lines tool.

2 The stability of the prediction filter

The stability of the prediction filter is a necessary condition to obtain a bounded output of the decoder. The stability of the prediction filter is given by the figure bellow. However, the optimal parameters, in term of static error, are such as [2][3]:

$$a_1+a_2=1 \qquad (5)$$

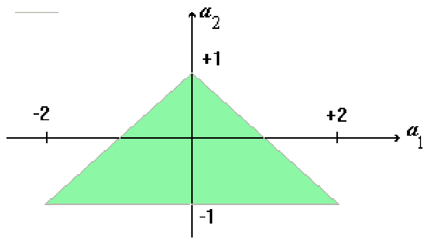

Fig. Error! Unknown switch argument.**:** Stability domain of the prediction filter

3 Poincaré Mapping

The Poincaré mapping is usually used for systems described by differential equations. It permits to decrease the system effective dimension: from a dimension N non autonomous differential equation, which has an effective dimension of N+1, it gives a dimension N autonomous map. Let be a system described by the differential equation (6). This system will be modelized by the map F:

$$X' = h(X,t) \qquad (6)$$

A surface of section is first defined in the state space. The image by F of a point X_n of this surface is the point X_{n+1}, the next intersection (with respect to time) between the state trajectory and the surface of section, as shown in fig. 3.

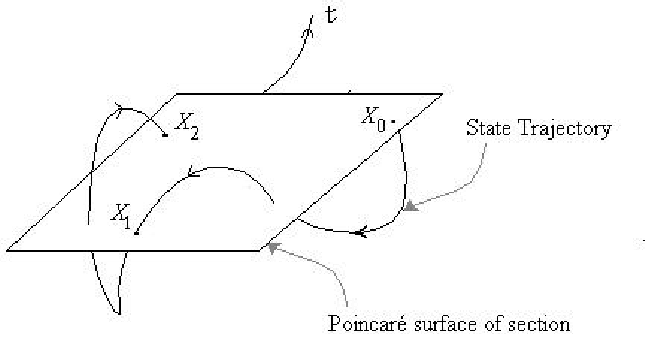

Fig. Error! Unknown switch argument. **:** Poincaré mapping

4 DPCM model

The above method is now adapted to study the DPCM with a periodic input.

In some cases, it is possible to apply the Poincaré mapping to obtain an autonomous map which describes the DPCM with periodic input.

The Poincaré surface of section is defined as follow:

$$\cos(2.\pi.f_e.n)=1 \qquad (7)$$

which is possible only if f_e is rational. This comes down to take the points $(x'_n,y'_n)=(x_{q.n},y_{q.n})$ such as $f_e=p/q$ where p and q are two integers.

Fig. 4 gives an example for f_e=1/50. In this case, a periodic solution of period 50 for (x_n,y_n) gives a fixed point for (x'_n,y'_n).

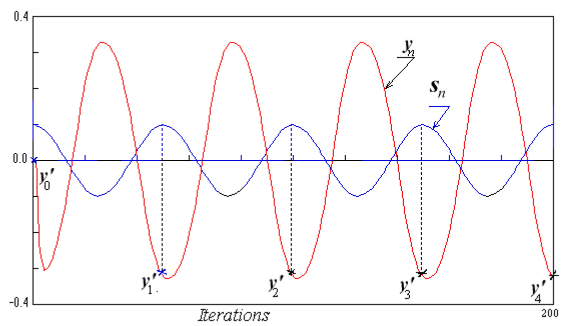

Fig. Error! Unknown switch argument. **:** Poincaré mapping applied to the DPCM with sinusoidal input

5 Study of the Parameter Plane

Different studies have been made concerning the map (2) with s_n=s=constant [2][3]. The study of the parameter plane permits to identify regions of the parameter plane for which the system has a certain qualitative behavior.

5.1 The DPCM with constant input

The parameter plane chosen is (a_1,a_2). The other parameters are fixed. Fig. 5 gives the structure of the structure of the (a_1,a_2) plane for a constant input s=0.01 and p=1.2

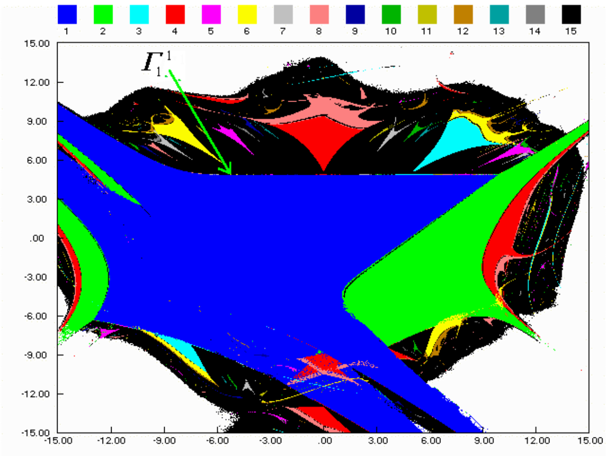

Fig. Error! Unknown switch argument. **:** Bifurcations in the DPCM with constant input s_n=0.01 and p=1.2

The region of different grey levels indicate the domain of existence of a particular behavior: a fixed point (color labeled 1), order 2 cycle (color labeled 2)… the color labeled 15 indicate a periodic behavior of period greater than 14 or a chaotic behavior. The border of each region represents a bifurcation curve. It is worth noting that some of these regions may overlap. In this case, the system has two or more possible behaviors depending on the initial condition. The bifurcation curve labeled Γ_1^1 corresponds to a Neïmark-Sacker bifurcation. It generates a structure called "Arnold Tongues". The bifurcation structure obtained is described in [4], and is also called "boxes in files".

5.2 The DPCM system with a periodic input

Now, a period 2 input is applied. The bifurcation diagram is given by fig. 6. We notice that bifurcation aspect don't change. For a value of the parameters a_1 and a_2, the cycle order is different from the case s_n=s=constant.

New bifurcations appeared on the system. Nevertheless, the domain of existence of a bounded solution, which corresponds to all the non-white area, is qualitatively the same than in the case of constant input, just smaller in fig. 7. The Arnold tongues structure for a periodic input is different form the standard one described in [4]. Some parts of it are repeated and some of them are inverted. This structure should be studied more precisely.

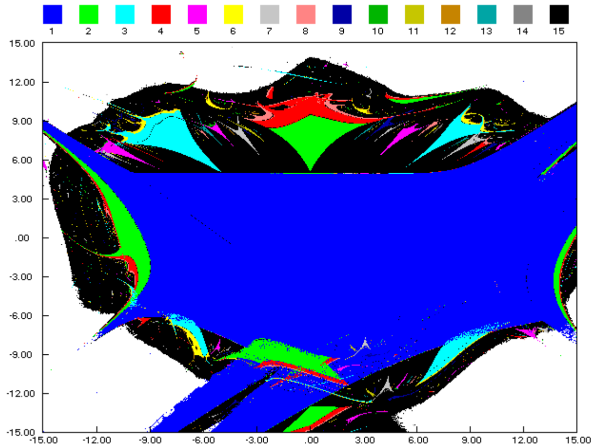

Fig. Error! Unknown switch argument. **:** Bifurcations in the DPCM system with periode 2 input s_n=0.01.cos(π.n) and p=1.2

The bifurcation diagram given by fig. 7 corresponds to an input of same amplitude, with frequency f_e=1/5,

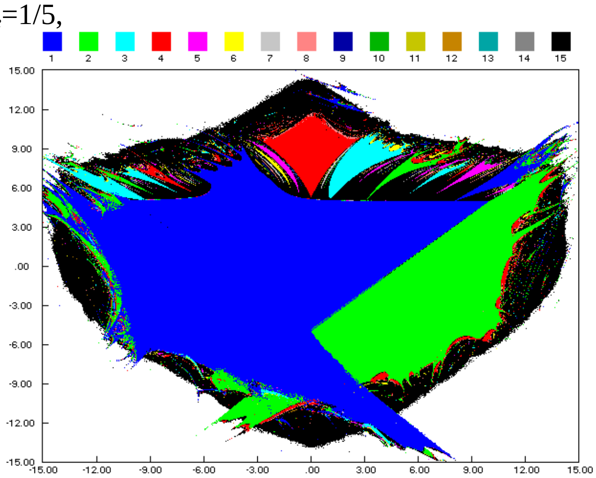

Fig. Error! Unknown switch argument. **:** Bifurcations in the DPCM system with periode 5 input: s_n=0.01.cos(2/5.π.n) and p=1.2

6 Study of the state space

The state space is the space of the state variables (x'_n,y'_n). Here the critical curves are determined and the basins of attraction studied.

6.1 Recalls on the critical curve tool

The basin of attraction of a cycle in the state space is the set of initial conditions that leads to this cycle. To study the transformation of basins of attraction, the tool of critical curves is used [4][5]. This tool is used in the case of non invertible maps, which is the case of the DPCM model.

Let LC^{-1} be a curve described by the following equation:

$$J=0$$

J is the Jacobian matrix of a map F, matrix of partial derivatives. The critical curve is defined by:

$$LC=F(LC^{-1})$$

LC divides the state plane into regions noted Z_i. Each one is characterized by the number i of pre-images of points in it.

One possible transformation is the appearance of holes in the basin of attraction. This happens when the basin boundary crosses through a critical curve.

6.2 Application to the DPCM

In the case of the DPCM with a constant input s:

$$LC^{-1}: \quad \frac{p}{Ch^2\left(p.(s-x_n)\right)}-1=0$$

In the case of a period 2 input (s_1,s_2), LC^{-1} is defined by:

$$\left(\frac{p}{Ch^2\left(p.(s_1-x_n)\right)}-1\right)\cdot\left(\frac{p}{Ch^2\left(p.(s_2-y_n)\right)}-1\right)=0$$

so:

$$x_n=s_1\pm\frac{Ch^2\left(\sqrt{p}\right)}{p},\ y_n=s_2\pm\frac{Ch^2\left(\sqrt{p}\right)}{p}$$

for a signal period greater than two, only implicit equation can be obtained.

The critical curves tool permits to explain the bifurcations of the basin of attraction of singularities. For example, for A=0.05 and f_e=0.5, the figures 8-a and 8-b illustrate this process. For the first image, a_1=4.5 and a_2=4. The figure represents the basin of attraction, LC^{-1} and LC.

In the constant input case, there are only Z_1 and Z_3 regions. In our case, there are Z_1,Z_3,Z_5,Z_7,Z_9.

The parameter a_1 is changed to a_1=4.7 In this case, a small region H_0 bounded by LC and the boundary of the basin appear. The pre-image of H_0, noted H_0^{-1}, is a hole in the basin of attraction. H_0^{-1} possesses three pre-images H_0^{-2} ,which are also holes. These holes seem to have no pre-image in the basin of attraction.

If the parameter a_1 changes again, other similar bifurcations can appear. This often leads to basin with fractal boundary. Fig. 9 gives an example as in the constant input case.

For the case of the DPCM with a periodic input, critical curves have more branches than in the case of constant input. So there are more possible bifurcations in this case.

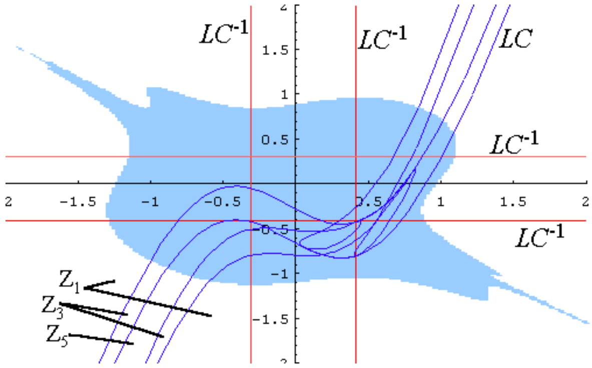

Fig. Error! Unknown switch argument.**-a : ** Basin of attraction for a_1=4.5 and a_2=4

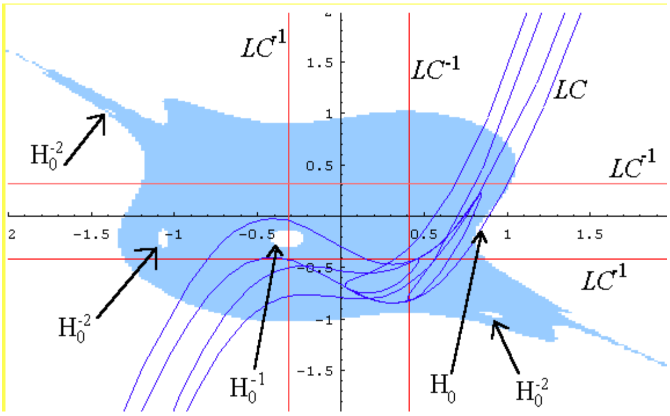

Fig. 8-b : Appearance of holes in the basin of attraction for a_1=4.7 and a_2=4. H_0 has one pre-image H_0^{-1}. H_0^{-1} has three pre-images H_0^{-2}

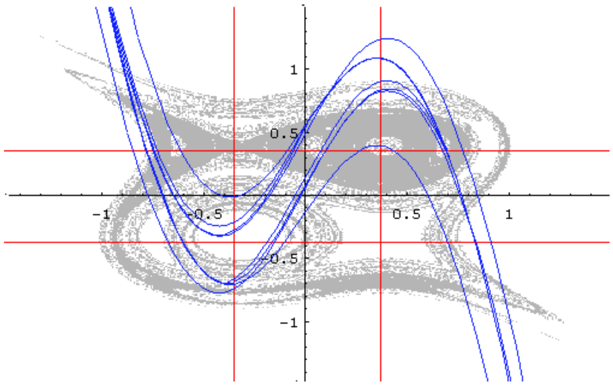

Fig. Error! Unknown switch argument. **:** Basin with fractal boundary for a_1=-11.73, a_2=-7.37, A=0.01, f_e=0.5, p=1.2

7 Conclusion

In this paper, we presented a tool to study the DPCM with a periodic input. The domain of existence of periodic solutions is reduced, with respect to the case of constant input, and new bifurcations appeared in the parameter plane (a_1,a_2). These bifurcations should be studied more precisely. In the state space, the critical curves have been determined. There are more different Z_i regions than in the constant input case. Basin fractalization has been illustrated.

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