Low Complexity Methods for Blind Estimation of Up-link Multipath Channels in Long Code CDMA Communications [1]

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Abstract: - Signal processing techniques for CDMA systems employing long spreading codes have gained significant interest recently. Due to the time-varying nature of users' signatures, direct design of blind multiuser CDMA receivers is intractable. However, in the up-link channel, the multipath parameters for all users are time-invariant and are embedded in the correlations of the directly received signal as well as the outputs of matched filters for different users. It is shown in this paper that the channel parameters can be estimated from the sample averages of those correlations. When the second and fourth order moments of the random spreading codes for all users are available, closed form solutions with low computational complexity can be obtained. Our method is still applicable when the statistics of the spreading codes are unknown, but at the expense of more computations. It turns out that the identifiability of the channel parameters only depends on the non-singularity of a constant matrix determined by known system parameters.

Key-Words: - CDMA, Long codes, channel estimation, up-link channels

1 Introduction

Code-division multiple-access (CDMA) techniques have demonstrated improved capacity and have been proposed for future wireless digital networks. CDMA systems can employ either short spreading codes with period equal to the symbol duration, or long spreading codes with much longer period. The use of short codes implies a time-invariant structure for the interference and facilitates the design of multiuser receivers. Due to their analytical tractability, short code CDMA systems have been extensively studied. Various algorithms to detect a desired user have been developed and analyzed. Typically, two different approaches have been proposed: one is to build the receiver based on the estimated channel parameters from different methods [1],[6]; the other one is to directly design multiuser detectors [3],[8],[9], obviating the channel estimation step. However, the current IS-95 standard for direct sequence CDMA systems employs long spreading codes. The time-varying nature of signatures renders previous channel estimation and multiuser

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detection methods not applicable.

For these reasons, signal processing techniques for CDMA systems with long spreading codes have gained interest recently. A number of studies for such systems have appeared; [7] presents an iterative way to estimate the FIR channels based on the finite alphabet property of the input. In [5] and [10], subspace concepts are adopted to identify the multipath channel. A correlation matching approach to multiuser channel estimation has also been proposed in [11]. The design of blind receivers to suppress the interference from other cells is discussed in [2].

In this paper, we propose low complexity methods to estimate the multipath parameters for all users of up-link CDMA channels. Observe that the correlations of the received chip rate signal and the bit rate outputs of matched filters are explicitly parameterized by the channel parameters. By matching these correlations with their corresponding sample averages, the time invariant multipath parameters can be estimated within a scalar ambiguity. When the second and fourth order moments of the random spreading codes for all users are available, closed form solutions may be derived with low computational complexity. Even if those statistics are unknown, they can be estimated from the given spreading codes. Therefore our methods are still applicable but at the expense of more computations. The identifiability of all channels is shown to depend only on the non-singularity of a constant matrix. This closed form matrix is determined by known parameters such as the statistics of the long codes, the number of active users, the delay of arriving signals from different users and the assumed maximum channel order. Simulations are performed to verify the applicability of the proposed methods.

2 Problem Formulation

Consider an up-link CDMA system employing long spreading codes with J mobile stations (or J users) communicating with a base station. User j ($j = 1, \cdots, J$) transmits the information bit stream $w_j(n)$ through a multipath channel with chip rate coefficients $g_j(m)$. All channels are assumed to have maximum order q. During the nth bit period user j is assigned a random spreading

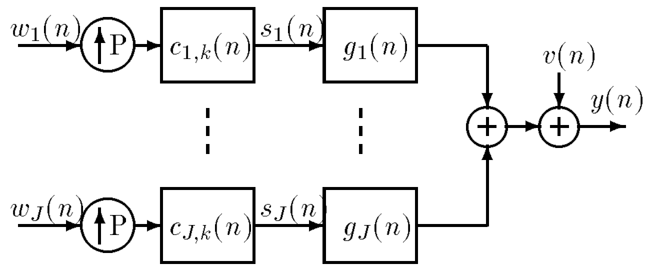

Figure 1: Long code CDMA system

code vector $\mathbf{c}_{j,n} = [c_{j,n}(0), \cdots, c_{j,n}(P-1)]^T$ with spreading factor P. The signal from user j arrives at the base station with delay δ_j (in chip periods). Then the received discrete-time signal can be written as (see Fig. 1 and [11])

$$y(n) = \sum_{j=1}^{J} \sum_{m=0}^{q} g_j(m) s_j(n - m - \delta_j) + v(n) \quad (1)$$

where

$$s_j(n) = \sum_{k=-\infty}^{\infty} w_j(k) c_{j,k}(n - kP) \quad (2)$$

where $v(n)$ is the zero-mean Gaussian noise with variance $\sigma_v^2 = E\{|v(n)|^2\}$ and $w_j(n)$ has power $\sigma_{w_j}^2 = E\{|w_j(n)|\}$. We will further assume the following:

(1) $w_j(n)$ is zero-mean i.i.d. random in j and n;
(2) $c_{j,k}(n)$ is zero-mean i.i.d. random in j, k and n, with variance $\sigma_c^2 = E\{|c_{j,k}(n)|^2\}$ and fourth order moment $m_{4c} = E\{|c_{j,k}(n)|^4\}$;
(3) $w_j(n)$, $c_{j,k}(n)$ and $v(n)$ are mutually independent;
(4) $J, P, q, \delta_j, c_{j,k}(n), y(n)$ are known.

Next we will estimate the channel parameters $g_j(m)$ and the noise power σ_v^2 based on $y(n)$ and the knowledge of all long spreading codes.

For clarity of presentation, we assume a quasi-synchronous system [4] where $\delta_j \ll P$. The extension to asynchronous interferers is straightforward. To eliminate the inter-symbol interference effect, we collect only $L = P - \mu$ samples around the nth bit interval and put them in a vector $\mathbf{y}(n) = [y(nP + \mu), \cdots, y(nP + P - 1)]^T$ with $\mu = \max(q + \delta_j)$ ($j = 1, \cdots, J$). Similarly, let the noise vector be $\mathbf{v}(n) = [v(nP + \mu), \cdots, v(nP + P - 1)]^T$ and the channel vector for user j be

$\mathbf{g}_j = [g_j(0), \cdots, g_j(q)]^T$. Then according to (1), a simple vector model follows (see also [11])

$$\mathbf{y}(n) = \sum_{j=1}^{J} \mathbf{C}_j(n)\mathbf{g}_j w_j(n) + \mathbf{v}(n) \tag{3}$$

where the code matrix $\mathbf{C}_j(n)$ is a truncated version from the $(\mu+1)$-st row to the P-th row of the filtering matrix

$$\tilde{\mathbf{C}}_j(n) = \begin{bmatrix} c_{j,n}(0) & & \mathbf{0} \\ \vdots & \ddots & c_{j,n}(0) \\ c_{j,n}(P-1) & & \vdots \\ \mathbf{0} & \ddots & c_{j,n}(P-1) \end{bmatrix} \tag{4}$$

i.e., $\mathbf{C}_j(n) = [\tilde{\mathbf{C}}_j(n)]_{\mu+1:P,1:q+1}$. Their relationship can thus be expressed by the transformation matrix $\mathbf{T}$

$$\mathbf{C}_j(n) = \mathbf{T}\tilde{\mathbf{C}}_j(n), \quad \mathbf{T} = [\mathbf{0}_{L\times\mu}\ \mathbf{I}_{P-\mu}\ \mathbf{0}_{L\times q}] \tag{5}$$

From (3), the output power can be easily obtained

$$\begin{aligned} E\{\mathbf{y}^H(n)\mathbf{y}(n)\} &= \sum_{j=1}^{J} E\{tr[\mathbf{C}_j(n)\mathbf{G}_j\mathbf{C}_j^H(n)]\} \\ &+ \sigma_v^2 L \end{aligned} \tag{6}$$

where $\mathbf{G}_j = \sigma_{w_j}^2 \mathbf{g}_j\mathbf{g}_j^H$, and "tr" represents the trace of a matrix.

Besides the directly received chip rate vector $\mathbf{y}(n)$, we will further employ the bite rate outputs from matched filters $\mathbf{y}_k(n) = \mathbf{C}_k^H(n)\mathbf{y}(n)$ for $k = 1, \cdots, J$, by correlating $\mathbf{y}(n)$ with code matrices $\mathbf{C}_k^H(n)$. For the new output $\mathbf{y}_k(n)$, their correlations $\mathbf{R}_k = E\{\mathbf{y}_k(n)\mathbf{y}_k^H(n)\}$ can also be computed

$$\begin{aligned} \mathbf{R}_k &= \sum_{j=1}^{J} E\{\mathbf{C}_k^H(n)\mathbf{C}_j(n)\mathbf{G}_j\mathbf{C}_j^H(n)\mathbf{C}_k(n)\} \\ &+ \sigma_v^2 E\{\mathbf{C}_k^H(n)\mathbf{C}_k(n)\} \end{aligned} \tag{7}$$

Noticing that (6) and (7) are linearly parameterized by $\mathbf{G}_j$ and σ_v^2, we will focus on estimating $\mathbf{G}_j$ and σ_v^2 based on these second order statistics of the output. Then SVD can be performed on the rank one matrix $\mathbf{G}_j$ to obtain the estimate for $\mathbf{g}_j$ within a scalar ambiguity.

3 Channel Estimation with Low Complexity

Our unknowns are embedded in the correlation $\mathbf{R}_k$ and output power $E\{\mathbf{y}^H(n)\mathbf{y}(n)\}$. In order to obtain a closed form solution, we introduce the *vec* operation which stacks all columns of a matrix into a vector. To incorporate this operation on both sides of (7), let's first define $\mathbf{d}_j = vec(\mathbf{G}_j)$ and express $\mathbf{R}_k$ as

$$\mathbf{R}_k = E\{\mathbf{C}_k^H(n)\mathbf{y}(n)\mathbf{y}(n)\mathbf{C}_k(n)\}$$

according to its definition. Therefore based on properties of *vec*, the operation on $\mathbf{R}_k$ yields

$$\mathbf{r}_k = vec(\mathbf{R}_k) = E\{\mathbf{Q}_k^H(n)vec[\mathbf{y}(n)\mathbf{y}^H(n)]\} \tag{8}$$

where

$$\mathbf{Q}_j(n) = \mathbf{C}_j^*(n) \otimes \mathbf{C}_j(n) \tag{9}$$

and "$\otimes$" denotes the Kronecker product. Performing the same operation on the right hand side of (7) and considering (8), we can obtain

$$\begin{aligned} &E\{\mathbf{Q}_k^H(n)vec[\mathbf{y}(n)\mathbf{y}^H(n)]\} = \\ &E\{\mathbf{Q}_k^H(n)\sum_{j=1}^{J}\mathbf{Q}_j(n)\mathbf{d}_j\} + \sigma_v^2 E\{vec[\mathbf{H}_k(n)]\} \end{aligned} \tag{10}$$

where

$$\mathbf{H}_k(n) = \mathbf{C}_k^H(n)\mathbf{C}_k(n) \tag{11}$$

Collecting (10) for $k = 1, \cdots, J$ together and exchanging two sides of those equations, we have

$$\begin{aligned} E\{\mathbf{Q}^H(n)\mathbf{Q}(n)\}\mathbf{d} &+ \sigma_v^2 E\{vec[\mathbf{H}(n)]\} \\ &= E\{\mathbf{Q}^H(n)vec[\mathbf{y}(n)\mathbf{y}^H(n)]\} \end{aligned} \tag{12}$$

where

$$\begin{aligned} \mathbf{d} &= [\mathbf{d}_1^T, \cdots, \mathbf{d}_J^T]^T \\ \mathbf{Q} = [\mathbf{Q}_1, \cdots, \mathbf{Q}_J], &\quad \mathbf{H} = [\mathbf{H}_1, \cdots, \mathbf{H}_J] \end{aligned} \tag{13}$$

However, (12) has $J(q+1)^2+1$ unknowns but only $J(q+1)^2$ equations. To solve $\mathbf{d}$ and σ_v^2, we apply the property of "tr" to (6) and obtain

$$E\{vec^H[\mathbf{H}(n)]\}\mathbf{d} + L\sigma_v^2 = E\{\mathbf{y}^H(n)\mathbf{y}(n)\} \tag{14}$$

If we define $\mathbf{x} = [\mathbf{d}^T, \sigma_v^2]^T$, then (12) and (14) can be combined in a compact form

$$E\{\mathbf{S}(n)\}\mathbf{x} = E\{\mathbf{z}(n)\} \tag{15}$$

where

$$\mathbf{S}(n) = \begin{bmatrix} \mathbf{Q}^H(n)\mathbf{Q}(n) & vec[\mathbf{H}(n)] \\ vec^H[\mathbf{H}(n)] & L \end{bmatrix} \tag{16}$$

$$\mathbf{z}(n) = \begin{bmatrix} \mathbf{Q}^H(n)vec[\mathbf{y}(n)\mathbf{y}^H(n)] \\ \mathbf{y}^H(n)\mathbf{y}(n) \end{bmatrix} \tag{17}$$

Therefore $\mathbf{x}$ can be written as

$$\mathbf{x} = [E\{\mathbf{S}(n)\}]^{-1}E\{\mathbf{z}(n)\} \tag{18}$$

In (18), $\mathbf{z}(n)$ is a data-related vector, and $E\{\mathbf{z}(n)\}$ can be estimated by its sample average $\hat{\mathbf{z}}_N = \frac{1}{N}\sum_{n=1}^{N}\mathbf{z}(n)$ for N symbol periods. Then the estimate for $\mathbf{x}$ can be obtained

$$\hat{\mathbf{x}}_N = \frac{1}{N}[E\{\mathbf{S}(n)\}]^{-1}\sum_{n=1}^{N}\mathbf{z}(n) \tag{19}$$

Noticing that $\mathbf{S}(n)$ only depends on code matrices, $E\{\mathbf{S}(n)\}$ can be theoretically pre-computed if the statistics of the long codes $c_{j,k}(n)$ are given. As will be shown next, $E\{\mathbf{S}(n)\}$ is determined by the second as well as fourth order moments of $c_{j,k}(n)$. Therefore the complexity to perform (18) is significantly reduced. However, for the case of unknown code statistics, the inversion of a sample average matrix $\hat{\mathbf{S}}_N = \frac{1}{N}\sum_{n=1}^{N}\mathbf{S}(n)$ has to be performed to estimate $[E\{\mathbf{S}(n)\}]^{-1}$.

4 Identifiability & Consistency

Our estimator in (19) exists only if $E\{\mathbf{S}(n)\}$ is nonsingular. Due to limited space, we only present the following result for the deterministic matrix $E\{\mathbf{S}(n)\}$ without proof. Under our assumptions in Section 2 we can obtain

$$E\{\mathbf{S}(n)\} = \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 & \cdots & \mathbf{B}_2 & \\ \mathbf{B}_2 & \ddots & \ddots & \vdots & \\ \vdots & \ddots & \ddots & \mathbf{B}_2 & \mathbf{b} \\ \mathbf{B}_2 & \cdots & \mathbf{B}_2 & \mathbf{B}_1 & \\ & & \mathbf{b}^T & & L \end{bmatrix} \tag{20}$$

where $\mathbf{B}_1$, $\mathbf{B}_2$ and $\mathbf{b}$ are constants

$$\begin{aligned} \mathbf{B}_1 &= E\{\mathbf{Q}_j^H(n)\mathbf{Q}_j(n)\} \\ &= \sigma_c^4\sum_{l_1,l_2=0}^{P-1}(\mathbf{U}_{l_1,l_1,l_2,l_2}+\mathbf{U}_{l_1,l_2,l_1,l_2}) \\ &+ (m_{4c}-2\sigma_c^4)\sum_{l=0}^{P-1}\mathbf{U}_{l,l,l,l} \end{aligned} \tag{21}$$

$$\begin{aligned} \mathbf{B}_2 &= E\{\mathbf{Q}_k^H(n)\mathbf{Q}_j(n)\} \quad (k \neq j) \\ &= \sigma_c^4\sum_{l_1,l_2=0}^{P-1}\mathbf{U}_{l_1,l_2,l_1,l_2} \end{aligned} \tag{22}$$

$$\begin{aligned} \mathbf{b} &= vec[E\{\mathbf{H}(n)\}] \\ &= \sigma_c^2 L vec([\mathbf{I}_{q+1},\cdots,\mathbf{I}_{q+1}]) \end{aligned} \tag{23}$$

$$\mathbf{U}_{l_1,l_2,l_3,l_4} = (\mathbf{M}^T\tilde{\mathbf{X}}^{l_1}\mathbf{F}\mathbf{X}^{l_2}\mathbf{M})\otimes(\mathbf{M}^T\tilde{\mathbf{X}}^{l_3}\mathbf{F}\mathbf{X}^{l_4}\mathbf{M}) \tag{24}$$

$\mathbf{X}$ is a $(P+q)\times(P+q)$ Jordan matrix whose first sub-diagonal entries below the main diagonal are unity while all remaining entries are zeros, and

$$\mathbf{X}^0 = \tilde{\mathbf{X}}^0 = \mathbf{I}, \quad \tilde{\mathbf{X}} = \mathbf{X}^T \tag{25}$$

$$\mathbf{M} = [\mathbf{I}_{q+1}\ \mathbf{0}]^T, \quad \mathbf{F} = \mathbf{T}^T\mathbf{T} \tag{26}$$

Therefore, the non-singularity of $E\{\mathbf{S}(n)\}$ depends on the system parameters such as σ_c^2, m_{4c}, P, q. Notice that it does not depend on the channel parameters. For a set of given parameters, $E\{\mathbf{S}(n)\}$ is expected to be full rank for typical applications and the solution is expected to be unique. The rank of $E\{\mathbf{S}(n)\}$ can always be a-priori checked.

To establish the identifiability of channel parameters, $\frac{1}{N}\sum_{n=1}^{N}\mathbf{z}(n)$ in (19) has to be further examined. Since the spreading codes are random, we only investigate the asymptotic behavior of our estimator as $N\to\infty$. Under our assumptions in Section 2, it can be shown that as $N\to\infty$,

$$\lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N}\mathbf{z}(n) = E\{\mathbf{z}(n)\} \tag{27}$$

in the m.s.s. Based on (27) and the expression for $\mathbf{x}$ and $\hat{\mathbf{x}}_N$ in (18), (19), it is not hard to show that

$$\lim_{N\to\infty}\hat{\mathbf{x}}_N = \mathbf{x} \tag{28}$$

provided that $E\{\mathbf{S}(n)\}$ is full rank.

5 Discussion

Our estimator is derived from both (7) and (6). In fact the current solution can be interpreted as a correlation matching procedure. To further explore this point, let us arrange $\mathbf{r}_k$ (c.f. (8)) and $E\{\mathbf{y}^H(n)\mathbf{y}(n)\}$ in a vector

$$\mathbf{r} = [\mathbf{r}_1^T,\cdots,\mathbf{r}_J^T, E\{\mathbf{y}^H(n)\mathbf{y}(n)\}]^T$$

By introducing this new vector, eq. (15) can be written as

$$\mathbf{r} = E\{\mathbf{S}(n)\}\mathbf{x} = E\{\mathbf{z}(n)\}$$

Then the problem can be cast into an equivalent one, which minimizes the error between $\mathbf{r}$ and its sample average $\hat{\mathbf{r}}_N = \frac{1}{N}\sum_{n=1}^{N}\mathbf{z}(n)$. Therefore we can build our cost function as the norm of the matching error $\|E\{\mathbf{S}(n)\}\mathbf{x} - \hat{\mathbf{r}}_N\|^2$ and minimize it to obtain our estimates for all channel parameters. Since $E\{\mathbf{S}(n)\}$ is a square matrix, the least square solution to this problem is thus the same as (19). This embedded correlation matching idea is hardly new and has been employed in [11]. From this point of view, it is not surprising that similar result is obtained here (compared with eq. (14) in [11]) although the current method exploits the bit rate outputs of matched filters $\mathbf{y}_k(n)$ while [11] only manipulates the chip rate output $\mathbf{y}(n)$.

6 Generalization

The proposed method can be used to estimate the multipath parameters simultaneously for all users in the system. However, when the number of active users grows, the involved computation is still significant in the sense that all unknowns have to be solved. One may wonder if it can be simplified to be suitable for a single user receiver. We will extend the method in this direction next.

Consider a general communications scenario with M dominant users in the system[1]. Our chip rate output $\mathbf{y}(n)$ can also be written as a superposition of signals from these M users and a colored noise $\mathbf{u}(n)$ (interference plus thermal noise)

$$\mathbf{y}(n) = \sum_{j=1}^{M}\mathbf{C}_j(n)\mathbf{g}_j w_j(n) + \mathbf{u}(n) \tag{29}$$

Assume $\mathbf{u}(n)$ has autocorrelation $\mathbf{R}_{int}$. Taking similar steps as in (7) and (6), we obtain

$$\begin{aligned}\mathbf{R}_k &= \sum_{j=1}^{M} E\{\mathbf{C}_k^H(n)\mathbf{C}_j(n)\mathbf{G}_j\mathbf{C}_j^H(n)\mathbf{C}_k(n)\} \\ &+ E\{\mathbf{C}_k^H(n)\mathbf{R}_{int}\mathbf{C}_k(n)\}\end{aligned} \tag{30}$$

[1] $M \leq J$ and $M = 1$ corresponds to a single user case.

for $k = 1, \cdots, M$ and

$$\begin{aligned}E\{\mathbf{y}(n)\mathbf{y}^H(n)\} &= \sum_{j=1}^{M} E\{\mathbf{C}_j(n)\mathbf{G}_j\mathbf{C}_j^H(n)\} \\ &+ \mathbf{R}_{int}\end{aligned} \tag{31}$$

If we define $\mathbf{r}_{int} = vec(\mathbf{R}_{int})$ and

$$\bar{\mathbf{d}} = [\mathbf{d}_1^T, \cdots, \mathbf{d}_M^T]^T, \;\; \bar{\mathbf{Q}} = [\mathbf{Q}_1, \cdots, \mathbf{Q}_M],$$

then after vec operation on (30) and (31), we can obtain

$$E\{\mathbf{Q}_k^H(n)\bar{\mathbf{Q}}(n)\}\bar{\mathbf{d}} + E\{\mathbf{Q}_k^H(n)\}\mathbf{r}_{int} = \mathbf{r}_k \tag{32}$$

$$E\{\bar{\mathbf{Q}}(n)\}\bar{\mathbf{d}} + \mathbf{r}_{int} = E\{vec[\mathbf{y}(n)\mathbf{y}^H(n)]\} \tag{33}$$

From (33), $\mathbf{r}_{int}$ can be first expressed by $\bar{\mathbf{d}}$ and then substituted in (32), which yields

$$\begin{aligned}&[E\{\mathbf{Q}_k^H(n)\bar{\mathbf{Q}}(n)\} - E\{\mathbf{Q}_k^H(n)\}E\{\bar{\mathbf{Q}}(n)\}]\bar{\mathbf{d}} \\ &\quad = \mathbf{r}_k - E\{\mathbf{Q}_k^H(n)\}E\{vec[\mathbf{y}(n)\mathbf{y}^H(n)]\}\end{aligned} \tag{34}$$

Stacking (34) for $k = 1, \cdots, M$ together, $\bar{\mathbf{S}}\bar{\mathbf{x}} = \bar{\mathbf{z}}$ holds where

$$\bar{\mathbf{S}} = E\{\bar{\mathbf{Q}}^H(n)\bar{\mathbf{Q}}(n)\} - E\{\bar{\mathbf{Q}}^H(n)\}E\{\bar{\mathbf{Q}}(n)\} \tag{35}$$

$$\begin{aligned}\bar{\mathbf{z}} &= E\{\bar{\mathbf{Q}}^H(n)vec[\mathbf{y}(n)\mathbf{y}^H(n)]\} \\ &- E\{\bar{\mathbf{Q}}^H(n)\}E\{vec[\mathbf{y}(n)\mathbf{y}^H(n)]\}\end{aligned} \tag{36}$$

Therefore $\bar{\mathbf{x}}$ can be estimated from the sample $\mathbf{y}(n)$

$$\begin{aligned}\bar{\mathbf{x}}_N &= \frac{1}{N}\bar{\mathbf{S}}^{-1}\sum_{n=1}^{N}\bar{\mathbf{Q}}^H(n)vec[\mathbf{y}(n)\mathbf{y}^H(n)] \\ &- \frac{1}{N^2}\bar{\mathbf{S}}^{-1}\sum_{n_1,n_2=1}^{N}\bar{\mathbf{Q}}^H(n_1)vec[\mathbf{y}(n_2)\mathbf{y}^H(n_2)]\end{aligned} \tag{37}$$

Here only $M(q+1)^2$ equations instead of $L(q+1)^2$ need to be solved. Thus much lower computational complexity can be achieved especially when $M \ll L$, e.g. $M = 1$, at the expense of lower SNR.

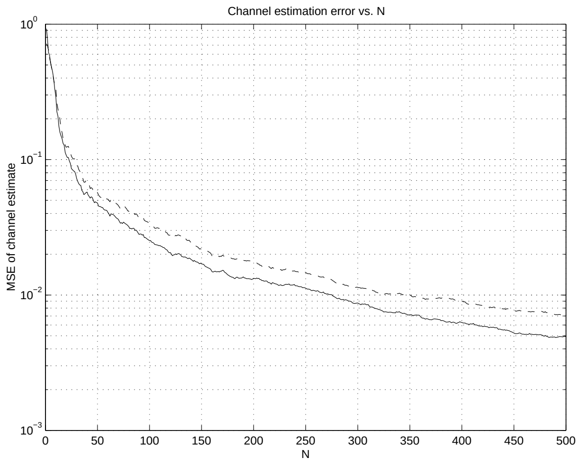

Figure 2: The channel estimation error for user 1 under different computational loads

7 Simulations

We test our method by simulating a CDMA system with $J = 8$ users in the computer. Each user has totally $N = 500$ i.i.d. complex input symbols to transmit. All users are equally powered. Different multipath channels are simulated with maximum order $q = 3$ chips. Each user is assigned a complex i.i.d. random spreading sequence, with both real part and imaginary part taking values from a set $\{\pm 1\}$. The spreading factor is assumed $P = 16$. Signals from different users arrive at the receiver simultaneously ($\delta_j = 0$). A $15dB$ white Gaussian noise is added to the input.

We adopt the mean square error (MSE) of the channel estimates as our performance measure. Since similar performance is observed for all users, the average results from 50 Monte Carlo runs are shown in Fig. 2 only for one user (user 1) versus the number of received bits: the dashed line is based on the pre-computed value of $E\{\mathbf{S}(n)\}$ according to (20), while the solid line is obtained in terms of $\hat{\mathbf{S}}_N$ estimated from given long codes. Close performance for these different approaches can be observed from this figure. However, the latter method achieves a slightly better result.

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