TUNING OF THE SELF-TUNING CONTROLLERS

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Abstract: The application of modern controllers depend on their ability to be tuned to fulfill specific practical requirements. LQ controllers offer optimum behaviour according to a chosen criterion. The problem is to find out such a criterion that leads to desired behaviour. The paper concentrates on the problem of tuning discrete LQ based adaptive controllers. These are based on identified input-output model of the plant. The optimization uses the state space approach with the (nonminimum) state containing delayed inputs and outputs. This form of state permits to interpret the state space terms directly in terms of filters or transfer functions. The technique was developed for dynamic input and output penalizations and considering additional dynamics in the loop. Special attention is paid to the possibility to guarantee a safe startup of an adaptive controller. Even if the results are primarily intended for a rather specific controller mentioned above, they can be directly applied in various forms of model based predictive controllers and even in classical state space LQ controlles.

Key-Words: LQ optimization, predictive controllers, adaptive control

1. Introduction

The title looks strange. Why to tune something that can tune itself. The explanation is very simple. The self-tuning controllers (adaptive) are able to tune its behaviour depending on the changing properties of the plant, the environment, generally on anything that could be measured and finally identified, it still remains to the user to tune (usually manually) the properties of a controller. The requirements are usually antagonistic and trade offs are necessary. This makes the tuning difficult and subjective.

To cope with all possible requirements it is advantageous to use many different tuning parameters. On the other hand practical applications of more complex controllers, e.g. LQ controllers shows that there is a lack of knowledge(experience) how to tune the controller to satisfy specific practical requirements. Only rarely the physical background permit to formulate the control requirements in terms of penalizations in the quadratic criterion. More frequently the criterion is used as a "tuning knob" and suitable values are searched for to obtain control process that satisfy user's requirements and possible constraints.

The adaptive (predictive) controllers are mainly based on input-output system models. The standard optimization based on these models defines only the input and output penalizations which are then the only natural tuning parameters (For SISO case only their relation). It will be shown later in the example, that this freedom is insufficient to reach reasonable responses. The situation improves when state space formulation with observers is used instead. However, the it is still difficult to define the rules how the choice of the state penalization matrix influences the behaviour of the controller as it depends on the physical meaning of the states.

The approach used in the paper is based on the state space optimization with the (nonminimum) state containing delayed inputs and outputs. Then it is possible
- to assume the state penalization matrix composed of a weighted sum of rank one matrices each having specific meaning.
- Simply augment the plant by a desired transfer function.
- Force the optimization process to generate optimal control law which is in some respect close to a predefined fixed control law.

By this technique it is possible.
- Influence the amplitude off the input variable.
- Influence the smoothness of the input and output.
- Minimize the steady state errors.
- Guarantee the bump-less startup of the adaptive controller.

First LQ optimization considered in this paper will be reminded and then each technique will be explained and demonstrated on simple examples.

2. Discrete time LQ optimization based on input-output models

The optimum controller is defined by the criterion, model of the system and by applied optimization process. The adopted technique is described in (1,).

2.1 Criterion

The paper considers a discrete time LQ adaptive controller based on on-line parameter estimation of a regression model of a specified order followed by a synthesis of a controller based on a minimization of the quadratic criterion

$$\Psi = \sum_{k=1}^{N}(y(k) - y0(k))'Q_y(y(k) - y0(k)) + \quad (1)$$

$$(u(k) - u0(k)'Q_u u(k) - u0(k)$$

Here the variable $y0(k)$ represents an output reference signal to be followed by the system output and $u0(k)$ possible input reference and in this paper will be assumed constant over the horizon of the minimization. $Q_y,\ Q_u$ are weighting matrices (scalars in considered SISO case)

2.2 Model

The assumed input-output models is a regression model of the form

$$y(k) = -\sum_{i=1}^{n} a_i y(k-i) + \sum_{i=0}^{n} b_i u(k-i) + e_k \quad (2)$$

For the minimization of 1 a state space form of this regression model is needed

$$x_t = P_x x_{t-1} + P_u u_t = P z_{t-1} \quad (3)$$

where $x_t' = [u_t, u_{t-1}, ..., u_{t-nb+1}, y_t, ...y_{t-na+1}, 1]$

$$z_t' = [u_t, x_{t-1}, \bar{v}_{t-1}]$$

Matrices will have the form:

$$P_x = \begin{bmatrix} 0 & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & ... & 0 & ... & 0 & 0 & 0 \\ b_1 & ... & b_{nb} & a_1 & ... & a_{na} & k \\ 0 & ... & 0 & 1 & 0 & ... & 0 \\ 0 & ... & 0 & ... & 0 & 0 & 0 \\ 0 & ... & 0 & ... & 0 & 0 & 0 \\ 0 & ... & 0 & ... & 0 & 0 & 1 \end{bmatrix}$$

$$P_u = \begin{bmatrix} 1 \\ 0 \\ b_0 \\ 0 \\ ... \\ 0 \end{bmatrix}$$

2.3 Optimization

The criterion (1) can be written in the form

$$J = \sum_{t=t_0+1}^{t_0+T} \tilde{z}_t' Q \tilde{z}_t \quad (4)$$

where $\tilde{z}_t' = [x_t, y0_t, u0_t]$

and for standard penalization (1) the penalization matrix has the form

$$Q = \begin{bmatrix} Qu & ... & 0 & ... & 0 & -Q_u \\ 0 & ... & 0 & ... & 0 & 0 \\ 0 & ... & Q_y & ... & -Q_y & 0 \\ 0 & ... & 0 & ... & 0 & 0 \\ 0 & ... & -Q_y & ... & Q_y & 0 \\ -Q_u & ... & 0 & ... & 0 & Q_u \end{bmatrix}$$

The optimization then proceeds considering only the last term in (4) as the other terms cannot be influenced by $u(t_0+T)$. The process of optimization now proceeds in such a way that variables which can be influenced by $u(t)$ (it is $x(t)$) are removed by substitution from the model (3) and then the form is minimized by $u(t_0+T)$. Optimum value of this partial minimization is

$$\tilde{z}(t_0+T-1)'S\tilde{z}(t_0+T-1)$$

This term is used in the next step of optimization. In the optimization process two operations alternate from the final term of the criterion to the beginning:

1.

$$\tilde{z}(t)'(S+Q)\tilde{z}(t) \rightarrow \quad (5)$$

$$z(t-1)'P'(S+Q)Pz(t-1)$$

2.

$$min_{u(t)}z(t-1)'P'(S+Q)Pz(t-1) \rightarrow \quad (6)$$

$$\tilde{z}(t-1)'Sz\tilde{-}1(t-1)$$

The evolution of a matrix S is equivalent with iterations of corresponding Riccati equation. The proposed algorithm is in reality realized in square root form (square root factors of matrices$S$, Q are used) but for convenience in the paper there is used a standard quadratic form.

3 Techniques of tuning

State space approach has enough freedom to reach various form of control performance. Some help in the choice of penalization can be obtained from the latest results in the relation between the closed loop roots allocation and corresponding LQ penalizations for state feedback discussed in (3,). Using these results it is possible to compute a penalization (state penalization matrix) that will cause the optimum control will lead to a prespecified closed loop pole position. Similar possibility exist for the output feedback with observer.

Another possibility is proposed in this paper. The approach is directly based on a specific form of pseudo-state vector composed of delayed inputs and outputs used in the optimization and the possibility to interpret state penalizations as a specific filter or transfer function. There are three modifications of these ideas.

3.1 Dynamic penalizations

The state space form of the criterion 4 suggests the possibility to use more general penalizations. To be able to construct such penalization, the following proposition is used **Proposition 1.** Any nonnegative definite matrix can be written as a weighted sum of rank one matrices.

$$\Psi = \sum_{t_0+1}^{t_0+T}(\tilde{z}_t'Q1\tilde{z}_t + \tilde{z}_t'Q2\tilde{z}_t + \tilde{z}_t'Q3\tilde{z}_t + \ldots) \quad (7)$$

where $Q_i = \alpha_i f_i' f_i$ and f_i is an arbitrary vector. As the elements of $\tilde{z}$ are delayed inputs and outputs vector f_i can be partitioned to a part corresponding to inputs f_{iu} and part f_{iy} corresponding to outputs. These parts represent FIR (finite impulse response) filters such that instead of penalizing $y(t)(u(t))$ $\bar{y(t)} = f_{iy}(z^{-1}y(t)$ (similarly $\bar{u(t)} = f_{iu}(z^{-1}u(t))$ is used in the criterion with a weight α_i which express relative importance of particular penalization.

The simplicity and power of this approach is demonstrated in the example

Example 1. One of the transfer functions describing the benchmark problem in (2,) has the form

$$G(z^{-1}) =$$

$$\frac{0.28z^{-3}+0.51z^{-4}}{1-2.0z^{-1}+2.20z^{-2}-1.84z^{-3}+0.89z^{-4}}$$

The root locus of closed loop poles for varying $0 < Q_u < \infty$, $Q_y = 1$ is shown in Fig.(2a). It is seen that for all penalizations the dominant close loop poles are oscillatory and the step response has overshoot.

The oscillatory character suggests that a penalization of output increments would improve the behaviour. If an additional term is added to the criterion penalizing the output increments

$$Q_2 = \alpha f_{2y}' f_{2y}$$

$$f_{2y} = [0., \ldots 0.1, -1, 0, \ldots, 0]$$

the position of closed loop pole will change as seen in Fig. 2b for the weights $\alpha = .1, 1, 10$. Now e.g $Q_u = .01$ and $\alpha = 1$ gives an aperiodic step response. See Fig.3. For larger penalization of inputs Q_u the offset in a step response will be obvious. It is an inherent property of the standard criterion where the difference between the output and setpoint one term and the input forms the second term. the penalization of input increments instead of input itself will remove the offset. See Fig.4 for $Q_u = .3$ and $\alpha = .3$. The dotted step response is a result of the optimization of the following criterion:

$$\Psi = \sum_{t_0+1}^{t_0+T} (\tilde{z}_t' f_{1y}' f_{1y} \tilde{z}_t + \tilde{z}_t' f_{2y}' f_{2y} \tilde{z}_t + \tilde{z}_t' f_u' f_u \tilde{z}_t)$$

where

$$f_{1y} = [0., \ldots 0.1, 0, 0, \ldots, 0]$$
$$f_{2y} = [0., \ldots 0.1, -1, 0, \ldots, 0]$$
$$f_u = [1, -1, 0 \ldots, 0, 0]$$

As it was seen from the example, the dynamic penalization is able to change the closed loop location but it is not able to add new open loop dynamics. For example using the penalization of input increments (u(t)- u(t-1)) instead of u(t) remove the offset, (due to correct feedforward) but no integrator was introduced in the open loop. If the system is affected by a step load disturbance, it will not be fully compensated.

Adding dynamics

Introduction of additional dynamics to the open loop is very important and frequently necessary. This task is very simple and consists of three steps.

1. The transfer function of the plant model is just multiplied by a transfer function of desired dynamics.

$$\frac{\bar{B}}{\bar{A}} = \frac{P}{Q}\frac{B}{A} = \frac{PB}{QA}$$

By this step the dynamics is introduced to the optimization 2. Optimization is done with a new model. 3. Resulting controller transfer function is augmented by this dynamics.

$$\frac{\bar{S}}{\bar{R}} = \frac{P}{Q}\frac{S}{R} = \frac{PS}{QR}$$

Here the dynamics is actually added to the loop

Typical example, is adding an integrator to the open loop to remove the the step steady-state error.

Combining control laws

When probing newly designed controller it can occur that it will not work properly. Usual reaction is to switch over to some standard controller, change new controller and try again. Similar problem is even more apparent when using an adaptive controller as the unsatisfactory behaviour can take place also due to identification.

All problems of this type can be easily removed if the following fact is used.

Proposition 2: Minimization of a quadratic criterion with a penalization matrix

$$Q_A = \alpha \begin{bmatrix} 1 & L_A \\ L_A' & L_A' L_A \end{bmatrix}$$

leads to the control law $u^*(t) = L_A x(t-1)$

If this term is added to the standard criterion so that

$$J = \sum_{t_0+1}^{t_0+T} \tilde{z}_t'(Q + \alpha Q_A \tilde{z}_t \qquad (8)$$

then, depending on the weight α the optimization will give a control law which is some mixture of the standard law resulting from the first part of the criterion and the control law L_A. For $\alpha \to 0$ this will be standard LQ control law, for $\alpha \to \infty$ it will be alternative control law L_A.

Typical applications are

1. A standardly used fixed controller is used as an alternative control law. The control process is started with α high. The other penalization according to previous measures are designed. Value α is decreasing until zero if the controller works well or is increased if something goes wrong. Then the LQ controller can be redesigned and again verified its functionality.
2. The weight α can be controlled by the identification process such that at the beginning α is high and with the leaning of the system it goes down.
3. Even more alternative controllers can be used in the criterion with their specific weights. Each control law can represent e.g. suitable controller for specific product or working condition. Changing the weights the desired control law becomes dominant.

The advantage of such process is:
- The stability of any such combination of control laws is guaranteed by the same conditions as a standard LQ problem.
- The controller tuning using weights in penalization is smooth enough -Its implementation is very simple.

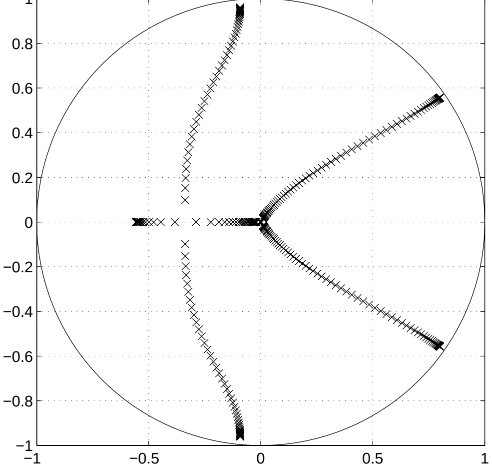

Fig. 1. Root locus for LQ design with $0 < Qu/Qy < \infty$

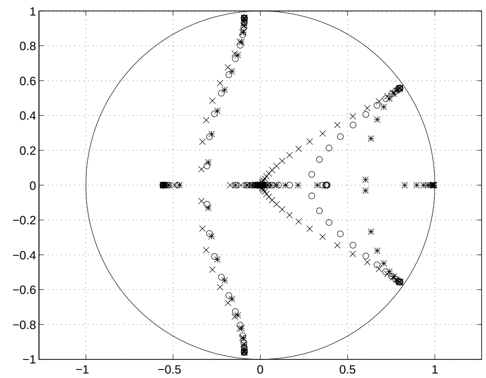

Fig. 2. Root locus for LQ design with $0 < Qu/Qy < \infty$and Q_{dy}=0.1;1;10

Conclusion

Described possibilities are mainly oriented to adaptive versions of controllers where the optimization repeats in each(generally) sampling period, can be, however, useful even in an off line iteration process of tuning.

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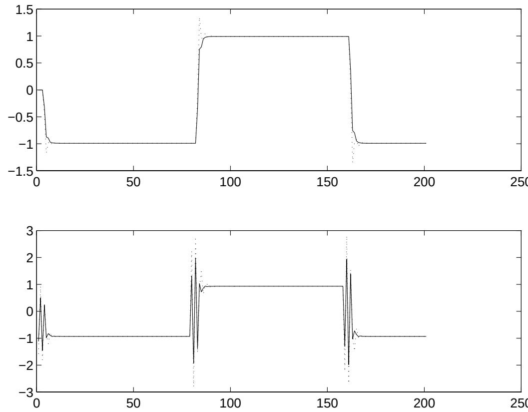

Fig. 3. Step responce and inputs with standard LQ (Qu=.01) dotted and with output increment penalization - full

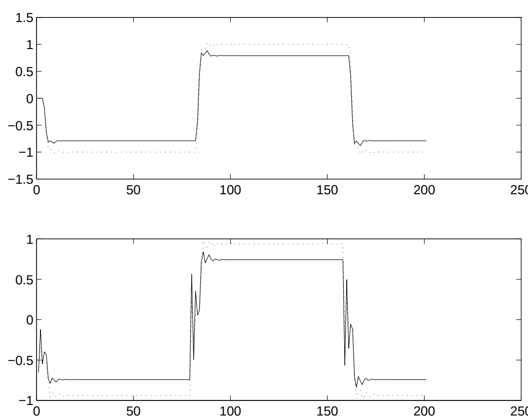

Fig. 4. Step responce and inputs with Qu=.3 full and with penalization of input increments -dotted