Comparative analysis of the cost/performance ratio between conventional and Fuzzy controllers. A practical case

ANTONIO ROMEO TELLO _ JORGE SICILIA TIL
Escuela Universitaria de Ingeniería Técnica Industrial.
Universidad de Zaragoza
Corona de Aragón,42.50009 Zaragoza
SPAIN

Abstract: - The aim of this paper is to show a comparison between two control design strategies applied to particular case. The controller must allow to maintain the desired position of a ball on a bar whose inclination can settle down by a cc motor coupled in its center. The paper shows and evaluates the cost and the performance of two control strategies:

- State space based controller
- Fuzzy controller

And it compares and discuss the choose of both strategies applied to this particular case. In the University of Zaragoza a system's prototype has been developed. Over this prototype has been implemented the controllers designed according both strategies.

Key-Words: - State-space controller, Fuzzy controller, controller adjust, cost/performance ratio, rule-based model, formal model
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1 Introduction

The two more classic examples of unstable systems presented in the study of control theory are probably the ball on the bar and the inverted pendulum. their control is approached in diverse ways that allow to evaluate the different used strategies. In the University of Zaragoza a system's prototype has been developed. Over this prototype has been implemented the controllers designed according both strategies.

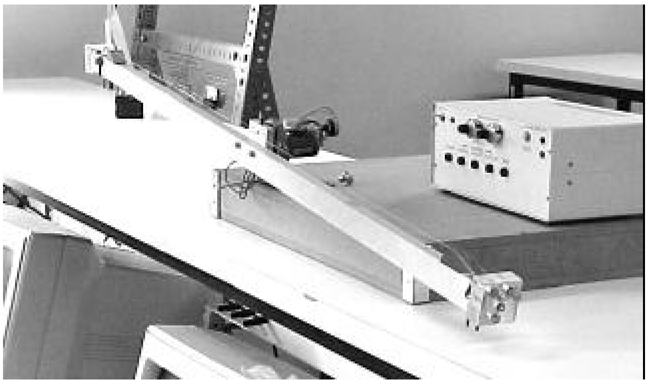

Fig. 1. *Prototype they are carried out the tests*

2 System identification. State space model

With purpose of simplifying the process of obtaining an acceptable mathematical model, certain simplifications have been assumed:

- The system has been linearized for small angles.
- Dynamics of the ball and the bar are considered non-coupled.
- Angular looseness of the reductor has been rejected ($\pm 2^{o}$ aprox.)

These simplifications allow to identify in a simple way (via non parametric methods) the two subsystems in those that the global system can be divided:

- The first subsystem is formed by the cc motor and the bar. Its entrance is the motor tension and its exit is the angular position of the bar. By means of a simple experiment the following transfer function has been determined:

$$G_1(s) = \frac{q(s)}{U(s)} = \frac{1.8447}{s \cdot (s + 7.5)}$$

- The second subsystem is formed by the bar and the ball. Their entrance is the bar's inclination and its exit is the ball's position. Experimentally the following transfer function has been determined:

$$G_2(s) = \frac{X(s)}{q(s)} = \frac{3.6383}{s^2}$$

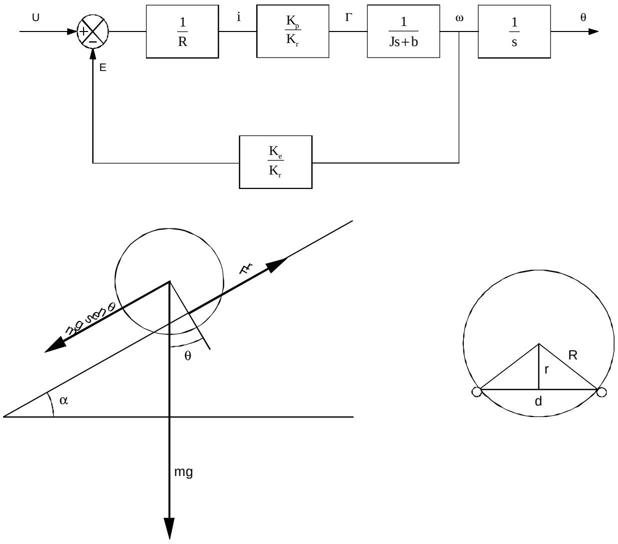

Fig. 2. *motor-bar and bar-ball subsystems*

Starting from the differential equations that define the system in the time domain, the following group of state variables is chosen:

$$x_1 = q \qquad x_2 = \frac{dq}{dt} = \dot{x}_1$$

$$x_3 = x \qquad x_4 = \frac{dx}{dt} = \dot{x}_3$$

Expressing these differential equations in matricial form, the state space model is obtained:

$$\dot{x}(t) = A \cdot x(t) + B \cdot u(t)$$
$$y(t) = C \cdot x(t) + D \cdot u(t)$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -7.5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 3.6383 & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1.8447 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \qquad D = [\,0\,]$$

3. State space based controller

Using the pole placement with integrator design technique, the matricial form of the system's model is the following one:

$$\begin{bmatrix} x_s(k+1) \\ x_i(k+1) \end{bmatrix} = \begin{bmatrix} Fs & 0 \\ -Gs_i \cdot C & Fs_i \end{bmatrix} \cdot \begin{bmatrix} x_s(k) \\ x_i(k) \end{bmatrix} + \begin{bmatrix} Gs \\ 0 \end{bmatrix} \cdot u(k) + \begin{bmatrix} 0 \\ Gs_r \end{bmatrix} \cdot r(k)$$

$$y(k) = \begin{bmatrix} C & 0 \end{bmatrix} \cdot \begin{bmatrix} x_s(k) \\ x_i(k) \end{bmatrix}$$

Where $x_s(k)$ represents the state of the system and $x_i(k)$ represents the state of the integrative one.

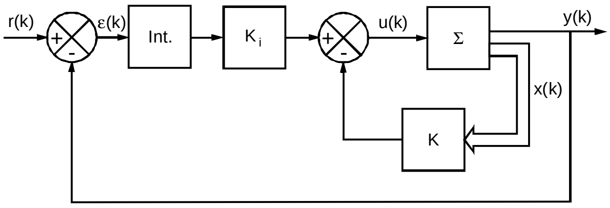

Fig. 3: *State space controller*

If closed-loop poles are placed so that system dynamics matches with a second order with a double real pole and three dominated poles, and fixing a settle time *tr*, desired closed-loop poles will be:

$$p = e^{\frac{-4.75}{tr} \cdot 0.1} \qquad pd = e^{\frac{-4.75}{tr}}$$

Applying the Ackermann' formula, we can obtain the feedback gains in function of the settle time, shown in the first table.

tr	3	4	5	6	7
K_1	361.6430	183.7239	101.8628	60.5961	38.1514
K_2	176.6126	116.0502	79.1575	55.9736	40.8615
K_3	124.6055	99.3881	80.2440	65.7531	54.6721
K_4	8.2114	7.26441	6.4033	5.6358	4.9562
K_i	-22.7864	-8.9115	-4.0106	-2.0065	-1.0897

Table 1. *Feedback gains in function of response time*

Since the ball's speed is not available for direct measurement, a minimum-order state observer has been implemented to estimate this speed. The pole of the observer is chosen so that its dynamics is four times quicker than the closed-loop controlled system. So, the vector gains of the observer will be:

$$K_e = \begin{bmatrix} 0 & 0 & \left(1 - e^{\frac{-4.75}{tr/4} \cdot 0.1}\right) \cdot 10 \end{bmatrix}$$

The different values of the observer gain in function of settle time are shown in the following table:

tr	2	3	4	5	6	7
K_e	6.1326	4.6918	3.7811	3.1614	2.7143	2.3771

Table 2. *Gain of velocity observer*

4. Fuzzy controller

Fuzzy logic control design methodologies are justified because imprecision of the mathematical model used previously. Rule-based controllers try account the human's knowledge about how to control a system without requiring a mathematical model. The model is given by a group of inference rules that operate on fuzzy sets.

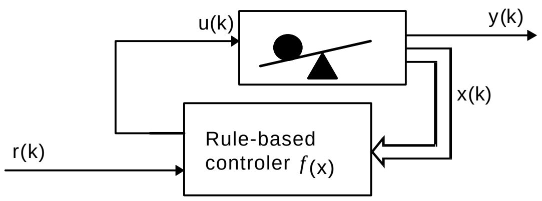

Fig. 4. *Control outline*

A base-rule that contains expert's linguistic description of how to achieve good control has been implemented. This base-rule operates on three linguistic variables that results from fuzzyfication of the ball's relative position, of its lineal speed, and of the bar's angular position.

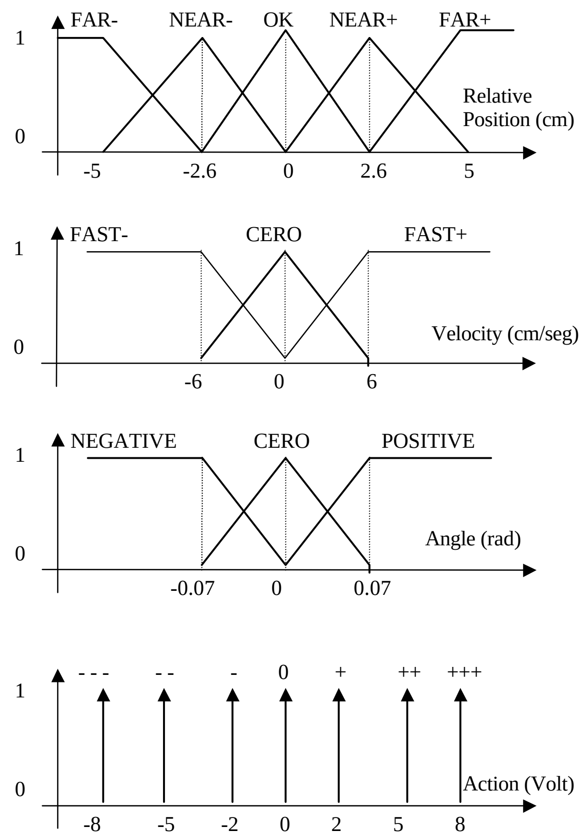

Fig. 5. *Fuzzy sets and its memberships functions*

In the table 3 the cubic base-rule is shown prepared in layers corresponding to the three possible angular position linguistic values.

Applying this base-rule previously to the defined fuzzy sets, and by means of a simple process of defuzzyfication (Center Average of Maximum [COX 92], [PASSINO 98]). the control surfaces particularized for the different angular positions of the bar are shown in the figure 6.

NEGATIVE	FAST-	CERO	FAST+
FAR-	+++	+++	++
NEAR-	+++	++	++
OK	++	++	++
NEAR+	++	++	+
FAR+	+	+	0

CERO	FAST-	CERO	FAST+
FAR-	+++	++	0
NEAR-	+++	+	--
OK	++	0	--
NEAR+	++	-	---
FAR+	0	--	---

POSITIVE	FAST-	CERO	FAST+
FAR-	0	-	-
NEAR-	-	--	--
OK	--	--	--
NEAR+	--	--	---
FAR+	--	---	---

Table 3. *Inference base-rule disposed in angle layers.*

Stability and robustness

Stability and robustness analysis of systems controlled by a non-linear rule-based controller can be carried out in an approximated way using the system's state space model ([ARACIL 89], [HOLGADO 95]):

Suppose that the system's model is the following state equation:

$$\dot{x} = f(x) + bu \qquad x \in R^n, u \in R^m$$

and that the rule-based controller is described by the following state function:

$$u = f(x) \qquad f(0) = 0$$

It is desired that the origin is a equilibrium point. The stability in this point will be insured if this point is an attraction point and it is the only equilibrium point. Hence:

$$|f'(0)| > |f'(0)|$$
$$|f(x)| > |f(x)|$$

The first condition is related with the loss of stability when real or complex eigenvalues cross the imaginary axis toward positive values of its real component, while the second is related with the uniqueness of the attraction point. In this case these conditions implies the non-negativity of three indexes I_1,I_2 and I_3. Both first they are associated to the first condition, and they will be obtained from the jacobian matrix of the controlled system:

$$J=\begin{bmatrix} \partial\dot{q}/\partial q & \partial\dot{q}/\partial\dot{q} & \partial\dot{q}/\partial x & \partial\dot{q}/\partial\dot{x} \\ \partial\ddot{q}/\partial q & \partial\ddot{q}/\partial\dot{q} & \partial\ddot{q}/\partial x & \partial\ddot{q}/\partial\dot{x} \\ \partial\dot{x}/\partial q & \partial\dot{x}/\partial\dot{q} & \partial\dot{x}/\partial x & \partial\dot{x}/\partial\dot{x} \\ \partial\ddot{x}/\partial q & \partial\ddot{x}/\partial\dot{q} & \partial\ddot{x}/\partial x & \partial\ddot{x}/\partial\dot{x} \end{bmatrix}=$$

$$=A+B\begin{bmatrix} \dfrac{\partial f}{\partial q} & \dfrac{\partial f}{\partial\dot{q}} & \dfrac{\partial f}{\partial x} & \dfrac{\partial f}{\partial\dot{x}} \end{bmatrix}=A+B\begin{bmatrix} -71.4 & 0 & -76.9 & -83.3 \end{bmatrix}$$

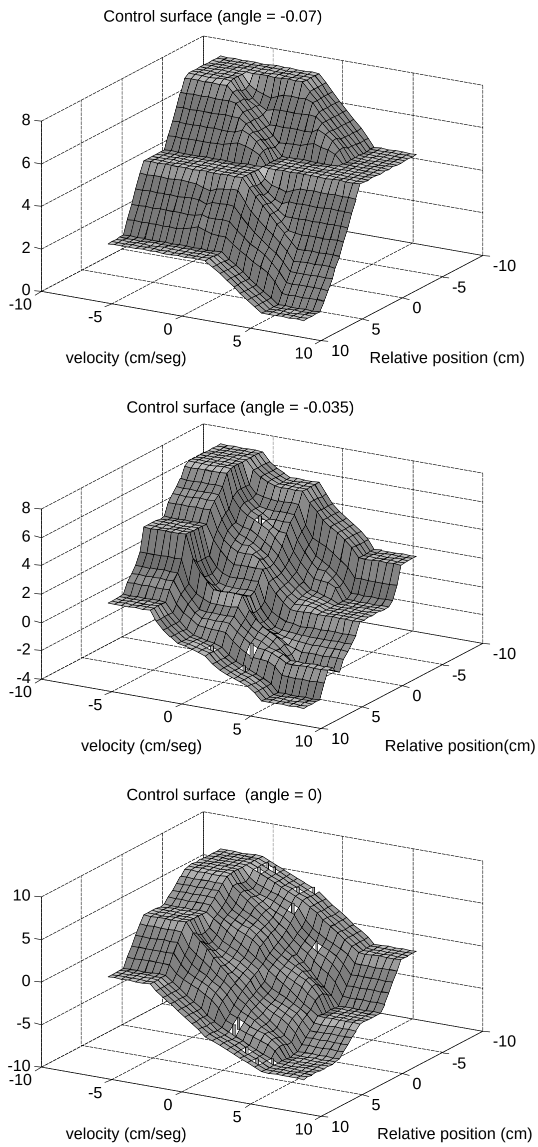

Fig. 6. *Control surfaces for some negative angle positions of the bar. The case of positive angles keeps symmetry.*

The evaluation of those partial derivatives in the origin can be carried out from the cut of the control surface with the vertical planes of null position and null speed, as well as of the limits of the fuzzy sets for the angular position and the rule-base. This process allows to find the characteristic equation, and the indices I_1 and I_2:

$$s^4+7.5s^3+131.7646s^2+559.2977s+516.2748=0$$
$$I_1=\det(J)=516.2748>0$$
$$I_2=-a_0+\frac{a_1a_2}{a_3}-\left(\frac{a_1}{a_3}\right)^2=3748.7>0$$

The previous results show the stability in the origin with the used controller. The global stability will be insured with the non negativity of the third index. This index is calculated as the minimum distance between the state function and the controller's action for a unidimensional space denominated auxiliary space that excludes an area around the equilibrium point. In this case, the auxiliary space corresponds to a horizontal axis in function of the position when the angular position of the bar and the speed of the ball are null. The index I_3 belongs together with the distance from the horizontal axis to the plane area of the resulting graph of intersecting the control surface for horizontal bar with the plane of null speed:

$$I_3=5$$

The robustness of fuzzy controller is increased with the value of these three indexes.

5. Results and comparative analysis

Figures 7 and 8 shown the behaviour of the ball when this part of a position far from the center of the bar (-45 cm) for the two developed controllers.

a) Transient and steady response: In both cases a oscilant behaviour is observed in steady state due to the existence of a serious angular looseness ($\pm 2^{\circ}$). The width of this oscillations will be related with the dynamics demanding. For the fuzzy controller the width of this oscillations is smaller than state space controller, but they are associated to greater oscillations in the angular position of the bar. This result is explained by the dynamics that the fuzzy controller demands in the area of lineal work around the equilibrium point ($-2.6<x<2.6$, $-6<dx/dt<6$, $-0.07<q<0.07$). This dynamics is comparable with the one imposed by a state space based controller designed by pole-placement method to provide a response time to the step of 2 seconds.

The transient response in the case of fuzzy controller is complex to adjust without affecting the steady response, and in this case it is worse than the transient response for state space controller.

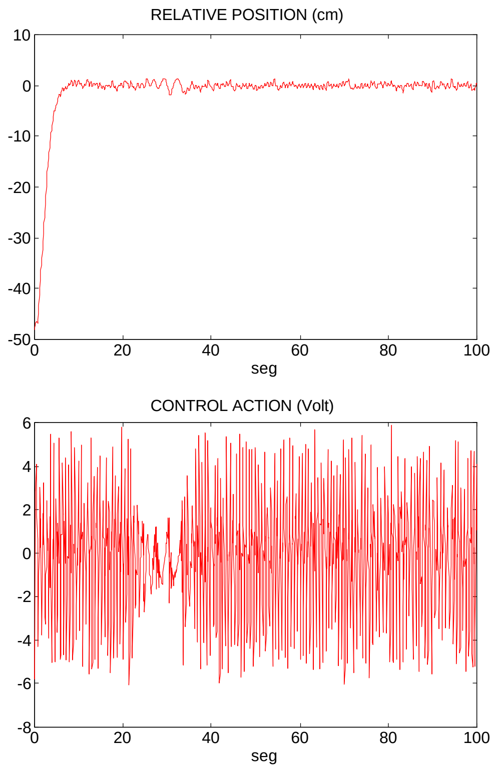

Fig. 6. *Relative position and control action for a reference steep of 0.45 m. State space based controller*

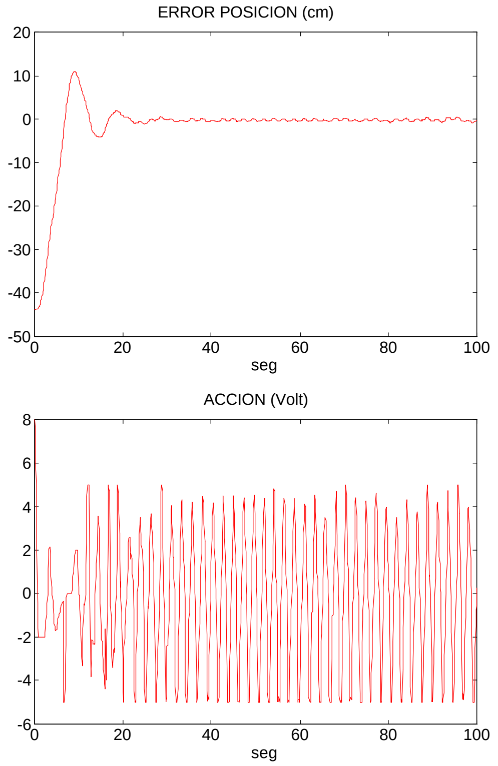

Fig. 7. *Relative position and control action for a reference steep of 0.45 meter. Fuzzy controller*

b) Development time.: The time spent in adjusting the base-rule, the limits of fuzzy sets and the magnitude of the control actions is much higher than the spent in the identification of the mathematical model and the state space controller design. It is complex to define rules that operate on three variables at the same time. The adjustment process would be simplified notably using state space controllers as a reference, because they can give us an idea of the weight of the different feedback, indicating us the guidelines to continue. This strategy can be very useful from the practical point of view, although it con be discussed from the theoretical point of view, because it uses the mathematical model of the system that the fuzzy logic seeks to ignore.

6. Conclusions

Fuzzy controllers offer in certain circumstances, clear advantages over conventional controllers:

- They materialise in form of inference rules the human reasoning, conjugating the inherent imprecision of natural language with the expert's knowledge about how to control a system.
- They allow to control non lineal systems in an appropriate form.
- They allow to obviate the systems' mathematical model. This is very useful in those cases that don't have these models, or the available models are very imprecise.

The comparison of the two control strategies makes sense because the environment of the fuzzy controllers' application is not well defined. The system purposed in this paper had some lacks in their mathematical model that made attractive the implementation of a fuzzy controller. However, some difficulties in its design and implementation can dissuade its use:

- It is complex to enunciate intuitively rules that relate three variables.
- Controller adjust is very time-consuming. If the system's mathematical model is ignored, it is possible to design incorrect controllers. A clear example of this is shown in the figure 8. An apparently acceptable behaviour is detected with a fuzzy controller in comparison with the corresponding state space controller designed to obtain a 6 seconds response time. but fuzzy controller is unstable in the equilibrium point because there is a repulsion point flanked by two attraction points in its proximity. This circumstance isn't accomplished if it isn't carried out a formal analysis using the mathematical model
- Serious difficulties in the adjustment of the transient response.

The key is to choose the appropriate application environment of both controller types. In this case, the cost of extracting a sufficiently precise mathematical model is notably inferior to the cost associated to design the inference base-rule and the adjustment of fuzzy controller.

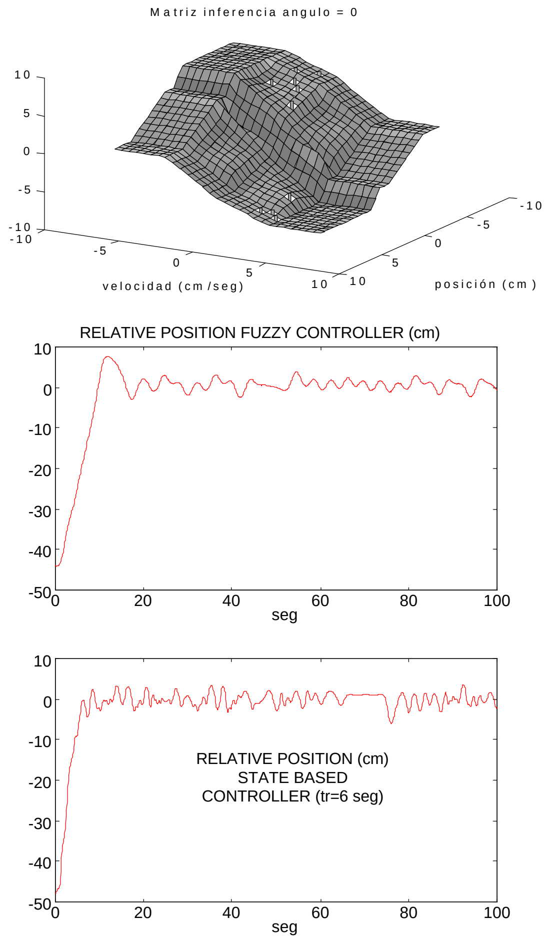

Fig. 8: *Control surface, (horizontal bar). seemingly acceptable fuzzy controller. Relative position for both control strategies.*

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