A STRATEGY FOR MODELLING AND CONTROLLING A SINGLE PHASE MULTILEVEL INVERTER IN MEAN VALUES

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Abstract: - In this article, we present a control strategy for a single-phase multilevel d.c./a.c. converter. The main interest of this power inverter is the generation of a multilevel a.c. voltage which changes in five levels and, therefore, generates less high frequency harmonics as compared with classical inverters. We will bring to the fore the causal input output process which is a graphic description of the relations between implicated quantities. A method for controlling this matrix multilevel power inverter is presented by using an equivalent average value modelling. Therefore, the control system is deduced by inversion of this mathematical equation within taking into account dynamical performance criteria. The control of switching states will be detailed and the obtained mean value is used as a duty cycle in the generation of transistor gate signals.

Key-words: PWM, multilevel operating, direct converter, switch matrix, connection controller.
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1 Introduction

Fig.1 shows the structure of the studied single-phase voltage-source inverter.

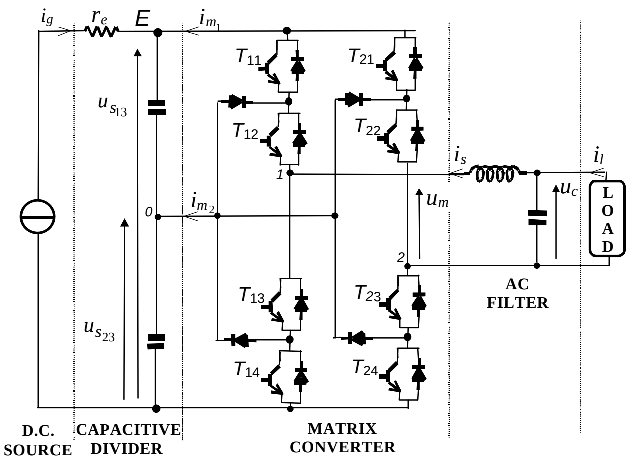

Fig.1: d.c. voltage-source multilevel inverter

The d.c. volatge-source comes from a rectifier which is placed in upstream. This voltage source is divided in two d.c. voltage sources by serial equal capacitors. The switching matrix chops these two d.c. voltage sources and produces a modulated multilevel voltage (u_m) which has five levels**.** An a.c. filter retains the fundamental component of this modulated voltage to fed the load.

The interest is double : the minimisation of the number of commutations which enables to reduce switches heating and the design of a simple control by taking all constraints into account. The control system feature is to force the filtered voltage (u_c) tracking an a.c. voltage reference.

This paper will be organised as follows. In section 2, the mathematical modelling of the inverter is presented. Section 3 is devoted to the modelling in mean values. The modelling of the a.c. filter and the capacitive divider are explained in section 4 and 5. In section 6, the control system is obtained by inversion of the mathematical modelling and is decomposed in a serial assembling of elementary control functions. Section 7 is devoted to the study of average connection generator and simulation results are presented in section 8.

2 Modelling of Switchings and conversions

The modulated a.c. voltage u_m is the potential difference between the point 1 and the point 2 (fig.1). u_m may be written as :

$u_m = u_{10} - u_{20}$

The output voltage u_{10} of the first clamped commutation cell may be switched to $E/2$ by switching the transistors T_{11} and T_{21}, $-E/2$ by switching the transistors T_{13} and T_{14}, *0* by switching the transistors T_{12} and T_{13} [2]. Therefore, this commutation cell is equivalent to a commutation cell which consists of three ideal switches (fig.2). This last may be associated to functions which are called switching functions : f_{cl} =1 if the corresponding switch is on, otherwise f_{cl} =0. The index *c* corresponds to a commutation cell and *l* to a row .

The corresponding values between the switching functions and the transistor gate signals are given in table 1.

T_{11}	T_{12}	T_{13}	T_{14}	u_{10}	f_{11}	f_{12}	f_{13}
1	1	0	0	$\frac{1}{2}.E$	1	0	0
0	1	1	0	0	0	1	0
0	0	1	1	$-\frac{1}{2}.E$	0	0	1

Table 1

By identification, we deduce the correspondence between the switching functions f_{2l} and the transistors T_{2l} for the second commutation cell.

T_{21}	T_{22}	T_{23}	T_{24}	u_{20}	f_{21}	f_{22}	f_{23}
1	1	0	0	$\frac{1}{2}.E$	1	0	0
0	1	1	0	0	0	1	0
0	0	1	1	$-\frac{1}{2}.E$	0	0	1

Table 2

Switching functions in the equivalent commutation cells depend on transistor gate signals as :

$$f_{c1} = T_{c1}.T_{c2}\,,\; f_{c2} = T_{c2}.T_{c3}\,,\; f_{c3} = T_{c3}.T_{c4}$$

By replacing reactive elements by their equivalent sources and transistors by ideal switches, we find an equivalent switch matrix representation of the inverter (fig.2).

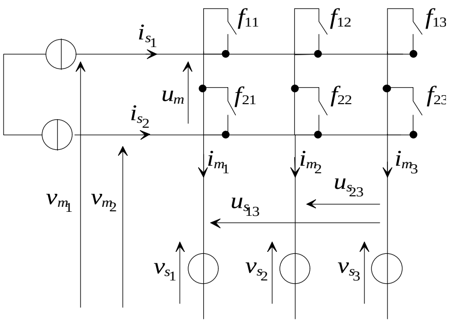

Fig.2: Matrix representation of the inverter

The current sources are connected to the voltage sources by a matrix of switches. Voltages are linked by the relation :

$$\begin{bmatrix} v_{m_1} \\ v_{m_2} \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \end{bmatrix} . \begin{bmatrix} v_{s_1} \\ v_{s_2} \\ v_{s_3} \end{bmatrix} \qquad (1)$$

$$v_{s_1} = \frac{E}{2}\,,\; v_{s_2} = 0 \text{ and } v_{s_3} = -\frac{E}{2}$$

As the sum of modulated voltages is null as well as the sum of voltage sources, we can write :

$$u_m = v_{m_1} - v_{m_2} =$$

$$[(f_{11} - f_{21}) \quad (f_{12} - f_{22})] . \begin{bmatrix} u_{s_{13}} \\ u_{s_{23}} \end{bmatrix} \qquad (2)$$

Currents are linked by the following relation :

$$\begin{bmatrix} i_{m_1} \\ i_{m_2} \\ i_{m_3} \end{bmatrix} = \begin{bmatrix} f_{11} & f_{21} \\ f_{12} & f_{22} \\ f_{13} & f_{23} \end{bmatrix} . \begin{bmatrix} i_{s_1} \\ i_{s_2} \end{bmatrix} \qquad (3)$$

As the sum of modulated currents is null, as well as the sum of source currents, one current may be mistaken and so we will consider :

$$\begin{bmatrix} i_{m_1} \\ i_{m_2} \end{bmatrix} = \begin{bmatrix} (f_{11} - f_{21}) \\ (f_{12} - f_{22}) \end{bmatrix} . [i_{s_1}] \qquad (4)$$

By using vectorial quantities, the relations (2) and (4) can be written as:

$$[I_m] = [M].i_s \text{ with } i_{s_1} = i_s \qquad (5)$$

$$u_m = [M]^{\mathrm{T}}.[U_s] \qquad (6)$$

$[M]=\begin{bmatrix} m_1 \\ m_2 \end{bmatrix}$ is the conversion vector and contains three level functions which are defined as:

m_1=f_{11}-f_{21} and m_2=f_{12}-f_{22}. (7)

In a scalar form, we get :

$$u_m = m_1.u_{s_{13}} + m_2.u_{s_{23}} \qquad (8)$$

$$i_{m_1} = m_1.i_s\,,\; i_{m_2} = m_2.i_s \qquad (9)$$

3 Modelling in mean values

The inverter consists of continuous quantities (i_s , u_c , $u_{s_{13}}$, $u_{s_{23}}$) and discrete quantities (m_l, f_{cl}, u_m, i_{m_l}), therefore its control system will consist of continuous regulators for continuous quantities and discrete regulators for discrete quantities. In order to design the continuous quantitie slaveries, a mathematical continuous model of the whole system has been established [3],[4].

For obtaining an equivalent continuous function, we apply the following integrator operation :

$$x_g = \frac{1}{T_m}\left(\int_0^{T_m} x(t)\,dt\right)_{T_m \to 0}\,, \qquad (10)$$

We deduce the equivalent average quantities, by applying this operation onto all modulated variables and by considering that electrical sources remain constant during the period T_m:

$i_{m_{1g}} = m_{1g}.i_s$ (R_3), $i_{m_{2g}} = m_{2g}.i_s$ (R_4), and $i_{m_{3g}} = -i_{m_{1g}} - i_{m_{2g}}$ (R_5).

Concerning the voltages, we get :

$u_{m_{1g}} = m_{1g}.u_{s_{13}}$ (R_6), $u_{m_{2g}} = m_{2g}.u_{s_{23}}$ (R_7), and $u_{mg} = u_{m_{1g}} + u_{m_{2g}}$ (R_8).

Concerning the conversion functions, we obtain:

$$m_{1g} = f_{11g} - f_{21g} \qquad (R_1)$$

and
$$m_{2g} = f_{12g} - f_{22g} \qquad (R_2)$$

The real time behaviour of continuous electrical variables from each side of the converter is now analysed.

4 A.C. filter

The simplified representation of the a.c. filter is obtained by replacing the switch matrix by multilevel voltage generator (u_m on fig.3).

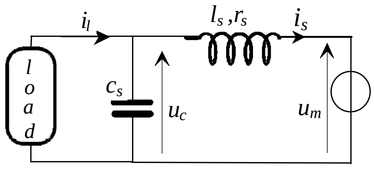

Fig.3 : a.c. filter

The differential equations of the a.c. filter are expressed by :

$$-\left(r_s.i_s + l_s.\frac{di_s}{dt}\right) = u_m - u_c \qquad (R_{11})$$

and
$$c_s.\frac{du_c}{dt} = i_l - i_s \qquad (R_{12})$$

The transfer function $G(s)$ of the multilevel voltage filter is found as follows :

$$\frac{u_c}{u_m} = \frac{\omega_n^2}{s^2 + 2.\zeta.\omega_n + \omega_n^2} \qquad (11)$$

With $\zeta = \frac{r_s}{2}.\sqrt{\frac{c_s}{l_s}}$ and $\omega_n = \sqrt{\frac{1}{l_s.c_s}}$.

The bandwidth is : $\omega_n.\left[1 - 2.\zeta^2 + \sqrt{2 - 4.\zeta^2 + 4.\zeta^4}\right]^{-1/2}$. Let us choose the bandwidth equal to the switching frequency (in order to attenuate the switching commutations) and considering available reactive elements, let us : $l_s = 50\text{mH}$, and $r_s = 1\Omega$, $c_s = 47\mu\text{F}$

5 Modelling of the capacitive divider

By erasing the switch matrix (in fig. 1) and drawing current generators for converted electrical variables, we can obtain the equivalent power circuit for the d.c. side of the converter (fig. 4).

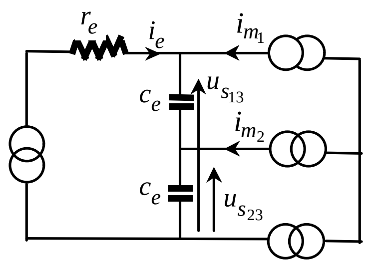

Fig. 4 : Modelling of the d.c. side

The d.c. voltages are described by equations :

$$c_e.\frac{du_{s_{12}}}{dt} = \frac{1}{r_e}E - \frac{1}{r_e}.u_{s_{12}} - \frac{1}{r_e}.u_{s_{23}} + i_{m_1} \qquad (R_9)$$

$$c_e.\frac{du_{s_{23}}}{dt} = \frac{1}{r_e}E - \frac{1}{r_e}.u_{s_{12}} - \frac{1}{r_e}.u_{s_{23}} + i_{m_1} + i_{m_2}$$

(R_{10}) and $u_{s_{13}} = u_{s_{12}} + u_{s_{23}}$ (R_{14}). The fig. 5 shows the average causal input output graph.

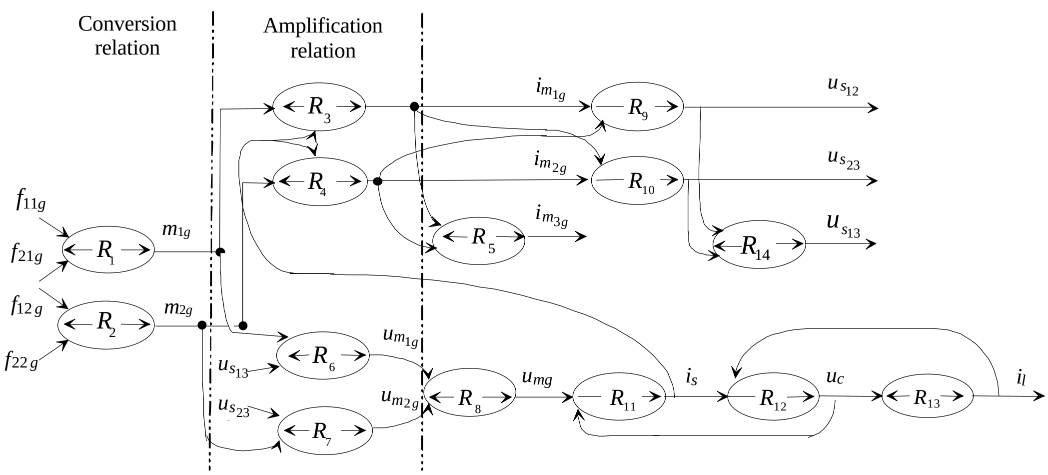

Fig. 5 : Average causal input-output graph

6 Control scheme

The control system architecture is obtained by inversion of the causal input output graph.

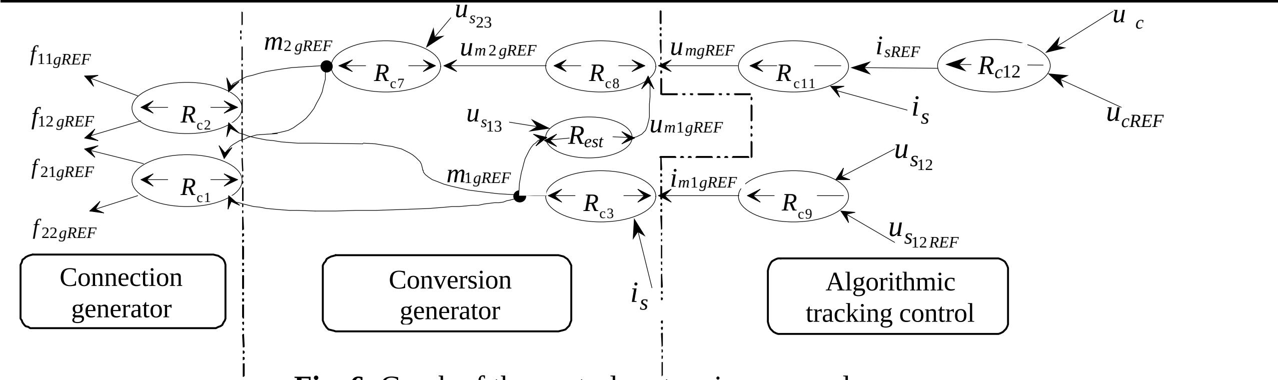

Fig. 6: Graph of the control system in mean value

A double regulation is implemented in order to make equal the load voltage u_c in magnitude and phase to a reference voltage u_{cREF} and in order to maintain the d.c. voltages to constant values, for example $u_{s_{13REF}} = E$ and $u_{s_{12REF}} = \frac{E}{2}$.

A tracking control of the a.c. voltage (u_c) to an a.c. reference sets the necessary average value of the modulated voltage u_{mgREF} (relations R_{c11} and R_{c12}) :

$$u_{mgREF} = k_{c11}.(i_{sREF} - i_s) \qquad (R_{c11})$$

$$i_{sREF} = k_{c12}.(u_{cREF} - u_c) \qquad (R_{c12})$$

In a similar way, a regulation of the d.c. voltage source ($u_{s_{12}}$) gives the necessary average value for the modulated current $i_{m_{1gREF}}$ (relation R_{c9}).

$$i_{m_{1gREF}} = k_{c9}.(u_{s_{12REF}} - u_{s_{12}}) \qquad (R_{c9})$$

After the tracking algorithmic control, the generation of average conversions is tackled. The conversion function reference m_{1gREF} is obtained by relation R_{c3}.

$$m_{1gREF} = \frac{i_{m_{1gREF}}}{i_s} \qquad (R_{c3})$$

The average modulated voltage reference can be synthesised by adding up two average modulated reference voltages $u_{m_{1gREF}}$ and $u_{m_{2gREF}}$:

$$u_{mgREF} = u_{m_{1gREF}} + u_{m_{2gREF}} \qquad (12)$$

The average modulated voltage reference $u_{m_{1gREF}}$ may be estimated through relation R_{est} :

$$u_{m_{1gREF}} = m_{1gREF}.u_{s_{13}}$$

The second modulated voltage reference $u_{m_{2gREF}}$ is therefore deduced :

$$u_{m_{2gREF}} = u_{mgREF} - u_{m_{1gREF}} \qquad (R_{c8})$$

By using the knowledge of the d.c. source voltage, we find the second conversion function reference :

$$m_{2gREF} = \frac{u_{m_{2gREF}}}{u_{s_{23}}} \qquad (R_{c7})$$

7 Average connection generator

The aim of the connection generator is to define the switching function references from the conversion function references (R_{c1}, R_{c2}). The relation (7) can be written in matrix form, as :

$$\begin{bmatrix} m_1 & m_2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \end{bmatrix}.\begin{bmatrix} f_{11} & f_{21} \\ f_{12} & f_{22} \end{bmatrix} \qquad (13)$$

and $f_{13} - f_{23} = -(m_1 + m_2)$ (14)

From the relations (13) and (14), we can bring to the fore the switching matrix :

$$\begin{bmatrix} m_1 & m_2 & -(m_1 + m_2) \end{bmatrix} = \begin{bmatrix} 1 & -1 \end{bmatrix}.\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \end{bmatrix}$$

Knowing the conversion function references in mean values, we can deduce the relation between these last and the switching function references, by the following expression :

$$\begin{bmatrix} m_{1gREF} & m_{2REF} & -(m_{1gREF} + m_{2gREF}) \end{bmatrix} = \begin{bmatrix} 1 & -1 \end{bmatrix}.\begin{bmatrix} f_{11gREF} & f_{12gREF} & f_{13gREF} \\ f_{21gREF} & f_{22gREF} & f_{23gREF} \end{bmatrix}$$

This relation may be written in other form, as :

$$\begin{bmatrix} m_{1gREF} & m_{2gREF} \end{bmatrix} . \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & -1 \end{bmatrix} . \begin{bmatrix} f_{11gREF} & f_{12gREF} & f_{13gREF} \\ f_{21gREF} & f_{22gREF} & f_{23gREF} \end{bmatrix}$$

In vectorial form, we get :

$$\left[M_{gREF}\right]^T . [N] = [E] . \left[F_{gREF}\right]^T \tag{15}$$

where $[N] = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$ and $[E] = \begin{bmatrix} 1 & -1 \end{bmatrix}$

$\left[M_{gREF}\right]^T$ is the conversion matrix reference which is defined as :

$$\left[M_{gREF}\right]^T = \begin{bmatrix} m_{1gREF} & m_{2gREF} \end{bmatrix}$$

$\left[F_{gREF}\right]^T$ is the switching matrix reference which contains the switching function references :

$$\left[F_{gREF}\right]^T = \begin{bmatrix} f_{11gREF} & f_{12gREF} & f_{13gREF} \\ f_{21gREF} & f_{22gREF} & f_{23gREF} \end{bmatrix}$$

The matrix E is not a squared matrix, therefore, it has no inverse. The set of closest solutions in linear algebra is :

$$\left[F_{gREF}\right]^T = [E]^+ . \left[M_{gREF}\right]^T . [N] + \left([I] - [E] . [E]^+\right) . \left[H_g\right]^T \tag{16}$$

where $[E]^+$ is the pseudo-inverse of the matrix $[E]$.

The matrix $[I]$ is an identity matrix and $\left[H_g\right]^T$ is any matrix of dimension (2,3). $\left[F_{gREF}\right]$ may be view as sum of two matrix :

- $\left[F_{MgREF}\right]^T = [E]^+ . \left[M_{gREF}\right]^T . [N]$ which depends of the conversion matrix reference :

$$\left[F_{MgREF}\right]^T = \frac{1}{2} . \begin{bmatrix} m_{1gREF} & m_{2gREF} & -m_{1gREF} - m_{2gREF} \\ -m_{1gREF} & -m_{2gREF} & m_{1gREF} + m_{2gREF} \end{bmatrix}$$

- $\left[F_{HgREF}\right]^T = \left(I - [E] . [E]^+\right) . \left[H_g\right]^T$ which maintains the switching function references in their variation domain. This matrix contains the homopolar components :

$$\left[F_{HgREF}\right]^T = \begin{bmatrix} h_{1g} & h_{2g} & h_{3g} \\ h_{1g} & h_{2g} & h_{3g} \end{bmatrix}$$

All constraints are now mathematically expressed :

- the first one is that we must never short-circuit the voltage sources and never let current sources opened. Then, for a commutation cell, only one switch should be closed. This can be translated by relations :

$$\begin{cases} f_{11} + f_{12} + f_{13} = 1 \\ f_{21} + f_{22} + f_{23} = 1 \end{cases} \tag{17}$$

In average values, this relation can be written as :

$$\begin{cases} f_{11g} + f_{12g} + f_{13g} = 1 \\ f_{21g} + f_{22g} + f_{23g} = 1 \end{cases} \tag{18}$$

- The second one consists in forcing the mean switching functions into their variation domain :

$$f_{clg} \in [0,1]$$

For the mean switching function references, we apply the same conditions :

$$\begin{cases} f_{11gREF} + f_{12gREF} + f_{13gREF} = 1 \\ f_{21gREF} + f_{22gREF} + f_{23gREF} = 1 \end{cases}$$

and $f_{clg\,REF} \in [0,1]$

These two constraints must be satisfied by a right setting of the homopolar component $[H_g]$. To obtain the switching functions in binary values, a modulator is placed just after the connection generator. The switching functions f_{clgREF} are compared with a triangular signal which varies in the domain [0, 1]. The figure 7 shows the transit from mean switching functions to discrete switching functions .

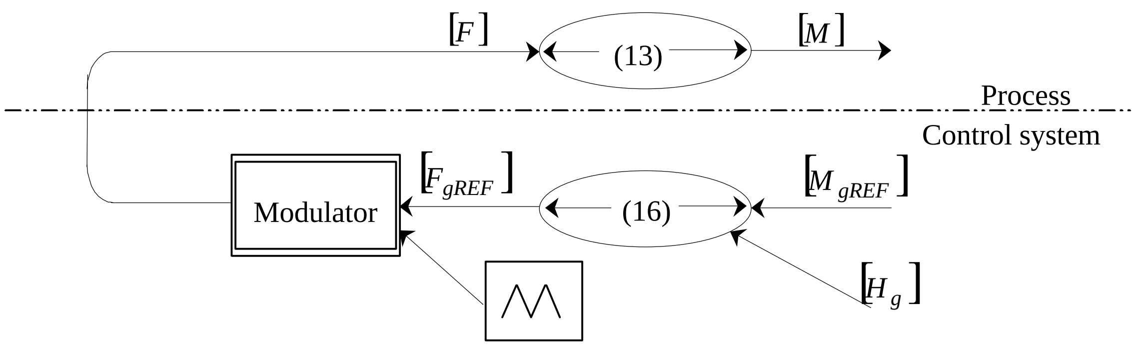

Fig.7 : Transit from mean switching functions to discrete switching functions .

8 Experimentation

The system is designed for a 5.5kVA drive operating. The rectifier is designed to yield a constant d.c. voltage $u_{s_{13}} = E = 220.\sqrt{2}$. This test is executed with a resistive load $r = 20\Omega$, and the voltage reference is set to $u_{cREF} = 220.\sqrt{2}.sin(\omega t)$, 50Hz. The d.c. voltage reference is : $u_{s_{12REF}} = 110.\sqrt{2}$, and the modulation frequency is equal to 5kHz.

We have effectively a modulated voltage u_m which changes in five levels (fig.8). The source current i_s and the output voltage u_c have sinusoidal waveforms. The d.c. voltages undulate around their mean values.

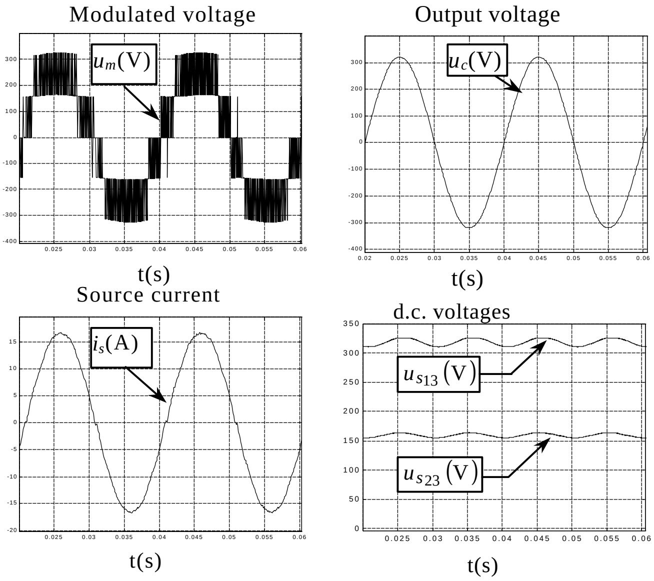

Fig. 8 : Steady-state test

Fig.9 shows a result for an a.c. voltage step from $u_{cREF} = 220.\sqrt{2}.sin(\omega t)$ to $u_{cREF} = 110.\sqrt{2}.sin(\omega t)$

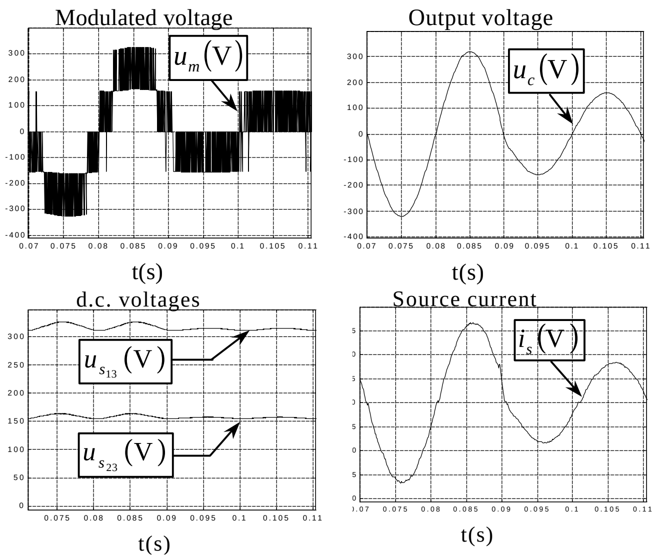

Fig.9 : a.c. voltage step

The results show the robustness of the regulator because the d.c. voltages are maintained constants and the output voltages u_c has a quick response dynamic due to our chosen dynamic regulator.The source current i_s is also affected. The undulation magnitude of d.c. voltages diminishes when we apply an a.c. reduced reference voltage. The switches are switched so that the modulated voltage changes in three levels $\left(-\frac{E}{2},0,\frac{E}{2}\right)$ when the u_{cREF} magnitude is less than $110.\sqrt{2}$. This principle enables to generate less high frequency harmonics.

9 Conclusion

An average process system modelling has been developed to establish an control system in average values. This enables to have all references in continuous. At the conclusion of the average control bloc, we have a modulation of the switching functions, contrary to the control method by modulation of the conversion functions [1]. We verified that this method gives satisfying results.

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