

Statistical Analysis Of Sewing Threads Breaking

JIŘÍ MILITKÝ, VLADIMÍR KOVAÈIÈ, PETR VOLF and ALEŠ LINKA
Department of Textile Materials and Applied Mathematics
Technical University of Liberec,
Liberec, Hálkova Street No. 6
CZECH REPPUBLIC

Abstract: Technology of clothing production is still based on quality of sewing process and sewing threads. The properties of sewing threads are important for realization of seams on sewing machines and for durability of this type of joints. For modern automatic sewing machines the threads breakage represents crucial problem. Quality of sewing threads for these machines is therefore dependent on their intensity of breakage at the specified sewing conditions (sewing ability). There are a lot of variables describing the failure of threads at sewing process e.g.:

- **number of thread breaks** per some length of thread N_T
- **number of thread breaks** per some time interval N_t
- **time between two successive thread breaks** T

In this contribution the basic methods of sewing ability evaluation are briefly described. The main part is devoted to modeling and analysis of experimentally determined values T_i , $i = 1, \dots, N$ for typical sewing threads. The system of exploratory data analysis based on the concept of quantile estimation is proposed. For selection of the suitable distribution of T the combination of nonparametric density and Quantile-Quantile graphs are used. For evaluated sewing yarns the Gumbell distribution is chosen as optimal. The parameter estimates of Gumbell distribution is computed from the quantile regression by using of least squares criterion and from selected quantiles. For characterization of sewing ability the median value computed from parameter estimates is proposed. The program package ADSTAT for realization of the above mentioned techniques is briefly described.

For modeling of survival of sewing threads the model of rope composed from m strands are adopted. It is assumed that at each moment the force applied to the rope is divided equally among the unbroken strand. For such a set of strands broken practically at the same moment we actually do not know the precise level of the strength causing the break of some of them, and we are not able to register the order in which they broke. Thus a part of data is interval-censored. Fortunately, if we observe a sufficient number of breaks, we register also a sufficiently large set of uncensored data. For estimation of cumulative hazard function the Nelson-Aalen estimator is useful. We were analyzed the asymptotic properties of this estimator. The main result is that the number of observed unbroken strand is of order n . The uniform consistency and asymptotic normality has been proved as well. This approach are used for Monte Carlo study simulating breaks of sewing thread composed from 10 filaments (strands) having survival distribution with constant hazard rates (exponential survival distribution).

Key-Words: Statistics of failure, Sewing ability estimation, Hazard rate simulation, Exploratory data analysis

CSCC'99 Proceedings: - Pages 2381-2388

1. Introduction

Technology of clothing production is still based on quality of process and sewing threads. The properties of sewing threads are important for realization of seams on sewing machines and for durability of this type of joints.

For modern automatic sewing machines the threads breakage represents crucial problem. Quality of sewing threads for these machines is therefore dependent on their intensity of breakage at the specified sewing conditions (sewing ability).

There are a lot of variables describing the failure processes at sewing process e.g.:

- **number of thread breaks** per some length of thread N_T
- **number of thread breaks** per some time interval N_t
- **time between two successive thread breaks** T

In this contribution the basic methods of sewing ability evaluation are briefly described.

The main part is devoted to modeling and analysis of experimentally determined values T_i , $i = 1, \dots, N$ for typical sewing threads. The system of exploratory data analysis based on the concept of quantile estimation is proposed. The program package ADSTAT for realization of the above mentioned techniques is briefly described.

2. Sewing Ability Evaluation

Methods for sewing ability evaluation differ in parameters describing this characteristics, conditions of measurements and techniques of data treatment.

Three typical representatives of sewing ability evaluation are summarized in this section.

A. Wieszlak [1] has defined sewing ability as a mean length of sewing (seam) without break. Measurements are realized on Textima 8332 sewing machine at rate 3960 ± 60 rpm. and defined brake. For sewing the two layers of cotton fabric are used. Minimum number of measurements is twenty. For treatment of results the Gumbell type distribution is assumed.

B. Nestler [2] has defined sewing ability as a number of thread breaks per 100 m of seam. Measurements are realized on Textima 8332 sewing machine at 5000 ± 100 rpm at defined brake. For sewing the three layers of cotton fabric ($260-300 \text{ gm}^{-2}$) are used. Sewing process has got defined pauses for cooling of sewing needle. For treatment of results the Poisson type distribution is assumed. The mean number of thread breaks per 100 m of seam is compared with standard one.

C. Nemeth [3] has defined sewing ability as a mean length of sewing (seam) up to defined number of thread breaks. Measurements are realized on Textima 8332 sewing machine. The rate of sewing and sewing

material are not specified. Some sewing techniques are defined. At simple sewing the number of breaks per 100 m is evaluated. At sewing with defined pause after some seam length the number of breaks per 100 m is evaluated. At sewing of button holes the number of short seams without break is evaluated. For treatment of results the exponential distribution is assumed.

Our test of sewing ability evaluation is described in the experimental part.

3. Exploratory Data Analysis

From statistical point of view leads the analysis of sewing ability tests results to the identification of probability model and estimation of corresponding parameters. Due to well known fact that yarn breakage distribution is positively skewed the classical analysis based on the normality assumption cannot be used. Sewing threads are strongly non-homogeneous and sewing process is influenced by many random events. The results of sewing ability tests are therefore often corrupted by the outliers (dirty data). Techniques that allow to isolate certain basic statistical features and patterns of data are therefore necessary.

Special distribution free robust methods of this type are collected under the name exploratory data analysis (EDA) According to Tukey [8] the EDA is a "detective work". It uses as tools various descriptive and graphically oriented techniques which are free of strict statistical assumptions. These techniques are based on the assumptions of the continuity and differentiability of underlying density only.

In this contribution the set of selected computationally assisted EDA methods suitable for analysis of sewing ability test results are discussed. The computationally assisted exploratory data analysis system is described in the book [2].

The special variants of the *quantile plot* are proposed for graphical visualization of data and evaluation of dirty data. The construction of sample distribution i.e. the estimation of probability density function will be carried out by the *kernel estimation* of probability density function and by the *quantile-quantile* plot.

For practical realization of these techniques the ADSTAT package is useful.

3.1 Some Basic Concepts

The EDA techniques for small and moderate samples are based on so called order statistics

$$X_{(1)} < X_{(2)} < \dots < X_{(N)}$$

which are the sample values (assumed to be distinct) arranged the in increasing order.

Let $F_e(x)$ is the distribution function from which values x_i have been sampled. It is well known that the transformed random variable

$$z_{(i)} = F_e(x_{(i)}) \quad (1)$$

has independently on distribution function F_e the Beta distribution $Be [i, N-i+1]$. Its mean value is given by relation

$$E(z_{(i)}) = \frac{i}{N+1} \quad (2)$$

where $E(.)$ is operator of mathematical expectations. The elements V_{ij} of covariance matrix V for all pairs $z_{(i)}, z_{(j)}$, $i, j=1, \dots, N$ are simple functions of i, j and N only. Using back transformations of $E[z_{(i)}]$ the relation

$$E(x_{(i)}) = F_e^{-1}(z_{(i)}) = Q_e(P_i) \quad (3)$$

is obtained.

In (3) the $Q_e(P_i)$ denotes quantile function and

$$P_i = \frac{i}{N+1}$$

is cumulative probability.

Description of quantile function properties and its advantages for constructing of empirical sample distribution contains paper of Parzen [11].

From (3) is obvious that the order statistic $x_{(i)}$ is raw estimate of the quantile function $Q_e(P_i)$ in position of P_i . For estimation of quantile x_p $Q_e(P)$ at value $i/(n+1) < P < (i+1)/(n+1)$ the piecewise linear interpolation

$$x_{(p)} = (N+1) \left(\frac{PN + P - i}{N+1} \right) (x_{(i+1)} - x_{(i)}) + x_{(i)} \quad (4)$$

can be used. Variance $D(x_p)$ can be computed from equation

$$D(x_p) = \frac{P(1-P)}{N f^2(x_p)} \quad (5)$$

Symbol $f_e(x_p)$ means the probability density function corresponding to distribution function F_e .

The interpolation (4) can be used for estimation of sample quantiles x_{P_i} or x_{1-P_i} for $P_i=2^{-i}$ $i=1, \dots, n$. These quantiles are called letter values [23]. All letter values except for $i=1$ (median) are in pairs. For example we can estimate lower quartile $x_{0.25}$ ($P_i=0.25$) and upper quartile $x_{0.75}$ ($P_i=0.75$) etc.

For EDA purposes the modified definition of cumulative probability

$$P_i = \frac{i - 0.375}{N + 0.25} \quad (6)$$

proposed by Blom is often used. Some proposals for definitions of P_i are presented in paper [12].

3.2 Data Visualization

For graphical visualization of data many simple techniques as stem-leaf plot, box plot, dot plot [1] and dig-dot plot [13] have been proposed. Only the simple quantile plot (QP) and his variant is here described.

Symmetry and tail length can be characterized by using of $g - h$ distribution system (see [4]).

Empirical (sample) quantile plot $Q(P)$ is constructed as dependence of $x_{(i)}$ on P_i . From patterns of points some statistical features of data as a symmetry, local concentration and rough normality can be simply recovered. Detailed interpretation of QP is described in the book [8].

For better interpretation the quantile functions of normal distribution

$$Q_N(P) = \mu + \sigma u_p \quad (7)$$

are superimposed to QP. In (7) the u_p are quantiles of standard normal distribution $N(0, 1)$. Parameters μ and σ are estimators of location and scale.

Two various normal quantile functions are graphed. The first one is based on the moment estimators i.e. sample mean x_M and sample standard deviation s . The second one uses robust quantile estimators median $x_{0.5}$ and quartile based standard deviation

$$s_M = \frac{x_{0.75} - x_{0.25}}{1.349}$$

This variant of QP enables to compare deviation of sample values from assumed normal distribution. For data from sewing ability tests can be normal quantile function replaced by assumed distribution. For frequent exponential distribution is quantile function in the form

$$EX(P) = A + B \ln(1 - P) \quad (8)$$

The parameters of threshold A and scale B can be estimated as $A = x_{(1)}$ and $B = x_M - x_{(1)}$. Quantile functions for other types of positively skewed distributions are summarized in book [4]. QP graphs with superimposed theoretical quantile function enables to identify the outliers (dirty data) as well.

For complex visualization of data the quantile box plot (QBP) proposed by Parzen [5] is useful.

3.3 Building of Sample Distribution

As an estimator of the empirical probability density function histogram with variable bins is often constructed. Smooth kernel type density estimator is natural generalization of histogram.

Histogram is piecewise constant estimator of sample probability density. Histogram height in j -th class bounded by values (t_{j-1}, t_j) is calculated from the relationship

$$f_H(x) = \frac{C_N(t_{j-1}, t_j)}{N h_j} \quad (9)$$

where the function $C_N(a, b)$ denotes the number of sample elements within interval $\langle a, b \rangle$ and

$$h_j = t_j - t_{j-1}$$

is the length of the j -th interval. Now, the problem encountered is the choice of boundary values $\{t_j\}$ $j=1, \dots, M$, the number of class intervals M and their lengths h_j with respect to the histogram quality. In our

ADSTAT programs the simple data based two-stage technique is used.

In the first stage the number of class intervals

$$M = \text{int}[2.46(N - 1)^{0.4}] \quad (10)$$

is computed Here $\text{int}[x]$ is integer part of number x .

In the second stage the individual lengths h_j are determined. The estimation of h_j is based on the requirement of equal probability in all classes. For this purpose the empirical quantile function $Q(P)$ based on the order statistics $x_{(i)}$ is used.

In practice the P-axis is divided into identical intervals having the size of $1/M$. For these intervals the corresponding quantile estimates $t_j = x_{(j/M)}$ are constructed by using of (4) where $P = j/M$. Practical experiences have hitherto proven that this construction be suitable even for strongly skewed sample distributions.

The kernel type nonparametric estimator of sample probability density $f(x)$ can be constructed on the basis of Lejenne-Dodge-Kaelin procedure [11]. The final estimator has the form

$$\hat{f}(x) = \frac{1}{N} \sum_{i=1}^N K \left[\frac{x - x_i}{h_i} \right] \quad (11)$$

Selection of kernel function $K[x]$ and computation of bandwidths h_i is described in [4].

3.4 Selection of Sample Distribution

The main goal is to approximate the empirical sample distribution by suitable theoretical one. The comparison of these distributions the variants of Q - Q plot are suitable.

Classical Q - Q plot is based on comparison of empirical quantile function $Q(P_i) = x_{(i)}$ with chosen theoretical quantile function $Q_T(P_i)$. For theoretical distribution functions of type $F_T((x-T)/S)$ is attractive to use standardized quantile function $Q_{TS}(P_i)$ (see[4]). When empirical and theoretical distributions are in coincidence, the relationship

$$x_{(i)} = T + S Q_{TS}(P_i) \quad (12)$$

is valid. Here usually T is the location parameter and S is the parameter of scale. For some three parameter distribution the shape factor is usually a parameter of the plot. Our programs (ADSTAT) select such shape factor value that straighten the individual points best.

Due to strong dependence among order statistics and their nonconstant variance the Q - Q plot gives a very patterned appearance and the degree of linearity is often hard to quantify.

In the work of Michael [18] the stabilized probability plot is introduced. Kafander and Spiegelman [13] propose the conditional Q - Q plot. For EDA purposes we use the empirical probability plot (EPP) (see also [9]).

3.5 Power Transformation of Data

Power transformation is in context of the EDA used as a tool for simplifying of data distribution. Suitable power-law transformations may result in a distribution that is nearly symmetrical and perhaps more nearly normal. This is obviously not useful for data from sewing ability tests but in some specific cases (e.g. distribution of multifilament sewing threads failure) the normality assumption can be adopted as well.

Power transformation enables to select of suitable location estimators for skewed distribution and construction of corresponding asymmetrical confidence bands. For these reasons is included to this section.

In many cases the using of simple power transformation

$$\begin{aligned} y &= g(x) = x^\lambda & \text{for } \lambda > 0 \\ y &= g(x) = \ln(x) & \text{for } \lambda = 0 \\ y &= g(x) = x^{-\lambda} & \text{for } \lambda < 0 \end{aligned} \quad (13)$$

leads to the improving of the data distribution.

This transformation is not scale invariant and is not continuous function of lambda. It requires the positive data only. Optimal transformation can be selected by minimizing of some robust measures of skewness

$$g_R = \frac{(y_{0.75} - y_{0.5}) - (y_{0.5} - y_{0.25})}{((y_{0.75} - y_{0.25}))^2}$$

As a diagnostic tool the selection graph can be simply constructed. This graph is based on the requirement of symmetry of quantiles about the median.

This requirement can be mathematically described by relationship [21]

$$\left(\frac{x_{P_i}}{x_{0.5}} \right)^\lambda + \left(\frac{x_{0.5}}{x_{1-P_i}} \right)^{-\lambda} = 2 \quad (14)$$

Letter values, where $P_i = 2^{-i}$ for $i = 2, 3, 4, \dots$ are usually chosen. Selection graph has on y-axis the quantities $x_{P_i}/x_{0.5}$ and on x-axis the quantities $x_{0.5}/x_{1-P_i}$. For comparison of computed points with ideal courses for constant lambda the solution of equation

$$y^\lambda + x^{-\lambda} = 2$$

is superimposed to graph.

Another exploratory technique for graphical estimation of optimal power is described by Emerson and Stoto [22]. After selection of optimal power the location parameter can be estimated from relation

$$x_{MR} = \left[\frac{\sum_{i=1}^N x_i^\lambda}{N} \right]^{1/\lambda} \quad \text{for } \lambda \neq 0$$

$$x_{MR} = \exp \left(\frac{\sum_{i=1}^N \ln(x_i)}{N} \right) \quad \text{for } \lambda = 0$$
(15)

Corresponding confidence interval is described in [4].

The Box-Cox power transformation family, which is continuous function of power lambda can be defined by equation

$$y = g(x) = \frac{x^\lambda - 1}{\lambda} \quad \text{for } \lambda \neq 0$$

$$y = g(x) = \ln(x) \quad \text{for } \lambda = 0$$
(16)

This transformation is limited for positive data only. After slight modification the range of applicability can be arbitrarily extended.

Properties of this transformation family are studied in the book [23]. Based on the assumption that for some power lambda the y variable is normally distributed $N(\mu_y, \sigma_y^2)$ the likelihood function can be constructed. Logarithm of likelihood function has the form

$$\ln L(\lambda) = (N/2) \ln(s_y^2) + (\lambda - 1) \sum_{i=1}^N \ln(x_i)$$

The s_y^2 is sample variance of transformed data.

The likelihood function can be plotted against lambda in suitable range (standard range is from -3 to 3). To this plot the 100(1- α)%th confidence interval of power

$$2[\ln(L(\lambda^*)) - \ln(L(\lambda))] < \chi^2(1) \quad (17)$$

The maximum likelihood estimator of power is here indicated by star.

From the width of confidence interval the quality of power transformation can be indicated.

3.6 Program Systém ADSTAT

System ADSTAT contains 8 independent modules of statistical methods for univariate and multivariate data [4]. The manipulation with ADSTAT is very simple by using of pull down menu and panes. Individual program modules are built windows like environment. This environment includes the powerful block oriented data editor, context sensitive help and unified graphical presentation. Exploratory methods included in module "Basic Statistics" can be divided to three main parts.

A. Techniques for presentation of data.

B. Construction of empirical sample distribution and comparison with 12 theoretical ones.

C. Power transformation of data

The above mentioned and more complex EDA techniques described in [4] are used. By this program the computations in this contribution were realized.

4. Experimental Part On the base of above mentioned methods the modified sewing ability test has been proposed. Sewing ability is characterized by time between two successive thread breaks T during sewing at defined conditions. Measurements were realized on Minerva 72112-105Q sewing machine with Quick-stop digital motor. The rate of sewing was 4500 rpm. Braking was adjusted to good appearance of seam. For sewing the one layer of cotton fabric (160 gm²) was used.

For evaluation of sewing ability the following sewing threads were selected:

- core yarn of PES/cotton 70/30 sample **No 44**
- PES staple yarn sample **No 31**
- PES staple yarn sample **No 35**

Properties of sewing threads are given in table 1.

Table 1 Properties of sewing threads

Yarn	P	T _t	CV	E _L
No	N/tex	tex	%	GPa
31	0.319	29.84	8.80	3.95
35	0.287	32.00	15.00	5.44
44	0.423	38.20	6.25	5.81

Here P is strength, T_t is fineness, CV is unevenness and E_L is loss modulus.

The measurements of P and F were realized by standard methods.

Unevenness was characterized by the so-called quadratic unevenness CV measured on the Uster device.

Acoustical loss modulus E_L was measured on Rheovibron device at 110 Hz and temperature 20 °C.

For each thread the 30 values of times between two successive breaks T_i [s] were evaluated.

The statistical analysis of experimentally determined T_i, i = 1,...N was used for creation of nonparametric probability density function and selection of the suitable parametric one (see. chap.3). The sewing ability is characterized by the median value Me of time between successive threads breaks during sewing experiment.

5. Results and discussion

Statistical analysis of sample values T_i, i = 1,...N) consists from three parts:

- nonparametric density creation
- selection of suitable theoretical one
- parameter estimation for selected distribution

The main aim is creation of suitable probability model, evaluation of its parameters and selection of characteristics for expressing the sewing ability.

5.1 Nonparametric density estimation

It is well known, that for full description of random variables the corresponding probability density function is required.

From values $T_i, i = 1, \dots, N$ of times between two successive breaks the sample density estimator was constructed. For creation of density trace the kernel type nonparametric estimator has been used. Typical kernel type estimator (dotted line) is compared with density of normal distribution (solid line) on the fig 1.

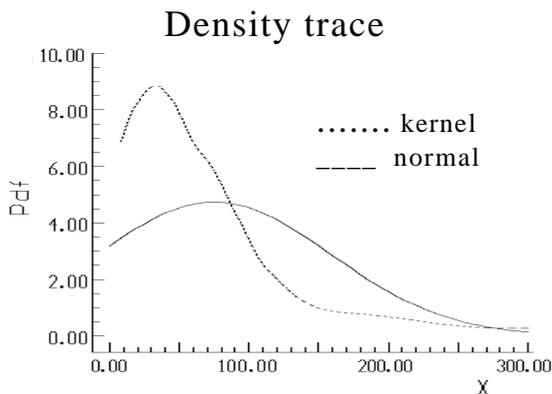


Fig. 1 Density traces for sample No 31

From these kernel type nonparametric estimator is evident, that the distribution of times between two successive breaks is skewed to the right and the form of density under modal value is not too sharp. This shape of sample density trace has been obtained for other sewing threads as well.

5.2 Selection of theoretical distribution

Time to failure or times between two successive breaks T are often modeled by exponential distribution. Nonparametric densities created from experimental data indicated that the exponential one is not acceptable for description of this random variable. Due to lack of theoretical explanation of behavior of T variable the data based approach has been used.

The graphical tools for selection of theoretical distribution well approximating the sample one are the Q-Q graphs (see chap. 3)). Empirical quantiles $Q_e(P_i)$ are approximated by the sample order statistics $T_{(i)}$. For known theoretical distribution the quantile function Q_T is simply the inverse function to cumulative density function.

Let the theoretical distribution is e.g. Gumbell one

$$F_T = \exp(-\exp(-z)) \quad (18)$$

where $z = (T-A)/B$, A is the modal value and B is the scale parameter. The corresponding quantile function has the form

$$Q_T(P_i) = A + B \ln(-\ln(P_i)) \quad (19)$$

If the sample distribution can be approximated by the Gumbell one the dependence of the $T_{(i)}$ on $-\ln[-\ln(P_i)]$ called Q-Q plot has to be linear. By the same way the Q-Q plots for other types of theoretical distributions can be created. For all tested threads the Q-Q graphs for normal, log-normal, rectangular exponential, Weibull, gamma, Pareto and Gumbell distribution were compared. The Gumbell Q-Q graph were in most cases the best ones. Typical Q-Q graph for Gumbell distribution is shown on the Fig 2.

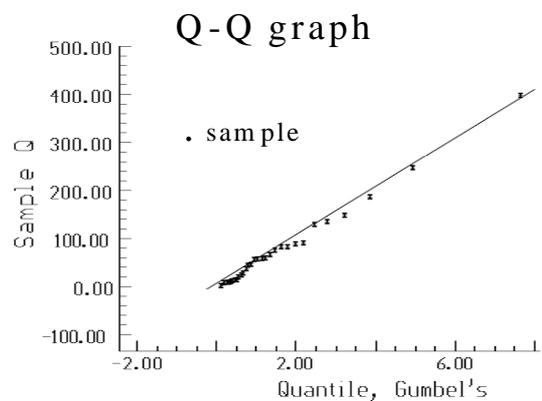


Fig. 2 Q-Q graph for sample No 31

5.3 Parameter estimation

Above described techniques lead to important conclusion that the Gumbell distribution can be used for modeling of times between two successive breaks T distribution. Probability density function of this distribution has the form

$$f(T) = \exp(-z) \exp\{-\exp(-z)\}/B$$

where $z = (x-A)/B$.

The location characteristic are defined by relations:

- **mean value $E(T)$**
 $E(T) = A + B * 0.57722 \quad (20)$

- **modal value $Mo(T)$**
 $Mo(T) = A \quad (21)$

- **median value Me**
 $Me = A + B * 0.36651 \quad (22)$

The **variance $D(T)$** is equal to
 $D(T) = 1.64493 * B \quad (23)$

and **variation coefficient** is defined by equation
 $v(T) = D(T)/E(T) = 1.28255/(0.57722 + A/B) \quad (24)$

These characteristics can be computed for known values of parameters A and B .

For estimation of parameters A, B the minimization of the least squares criterion

$$S = \sum_{i=1}^N (T_{(i)} - Q_T(P_i))^2 \quad (25)$$

can be used. For the Gumbell distribution the minimization leads to the two nonlinear equations containing unknown parameter estimates A^* , B^* . The parameter estimates A^* , B^* and values $E(T)$, Me and $v(T)$ are for investigated threads summarized in the table 2.

Table 2. Parameters of Gumbell distribution

No	A^*	B^*	$E(T)$	Me	$v(T)$
31	76.6	144	169	129	1.16
35	174	227	306	258	0.95
44	44	66	82	68	0.95

For small samples the estimates A^* , B^* are very rough. These estimates are sensitive to presence of outlying T_i values as well. The quick and robust estimates A^+ and B^+ can be obtained from selected quantiles $T_{(s)}$ and $T_{(r)}$

$$A^+ = [a T_{(s)} - b T_{(r)}] / (a - b) \quad (26)$$

$$B^+ = [T_{(s)} - T_{(r)}] / (a - b) \quad (27)$$

where $a = \ln(\ln(1/q_1))$ and $b = \ln(\ln(1/q_2))$. Parameters s/N and r/N are equal or just greater than q_1 and q_2 respectively and $r < s$. For estimation of A^+ the optimal are:

$$q_1 = 0.1789$$

$$q_2 = 0.6022.$$

For estimation of B^+ the optimal are:

$$q_1 = 0.0263$$

$$q_2 = 0.8327.$$

Details about this procedure are given in [11]

From above presented results follows that for specification of sewing ability the characteristics computed from parameters of the Gumbell distribution can be used. In according to clear interpretation and other properties we recommend the median $Me(T)$. Higher $Me(T)$ represents better sewing ability.

For estimation of parameters A , B the quantile based technique is suitable.

Comparison of results from the table 1 and table 2 leads to the conclusion that sewing ability is not directly connected with loss modulus of sewing threads.

6. Survival of Sewing Threads

In this section is proposed method for estimation of reliability (survival) of sewing threads. The sewing threads is considered as rope composed from m strands (filaments). These strands forms system composed from parallel organized units. The reliability is understand as a resistance of the system against a load applied to it. It is assumed that reliability is tested in such a way that the load increases from 0 to the level causing the failure of all units or up to maximal load. Further it is assumed

that the experiment is relatively fast, so that the time of duration of the load does not influence the survival. These conditions are common for testing of the multifilament sewing threads.

The standard survival analysis approach and counting processes models are used, however, instead of time-to-failure, the breaking load of strands is variable of interest.

The concept and relevant theory of counting processes is described in the book[6]. Let the survival of strands is described by i.i.d. random variables U_j , $j=1..m$ with distribution given by $f(u)$, $F(u)$, $h(u)$, $H(u)$ denoting the density, distribution function, hazard function and cumulative hazard function, respectively. It is assumed that at each moment the force applied to the thread is divided equally among the (unbroken) strands (filaments). The global force stretching the thread is observed. However, as the break of strand leads to an immediate re-distribution of the force to the other strands (so that to the abrupt increase of the force affecting each individual strand), the consequence can be the break of several of remaining strands. For such a set of strands broken practically at the same moment the precise level of the strength causing the break of some of them is actually not know. Thus, a part of data is interval-censored. If the sufficient number of threads is observed the sufficiently large set of uncensored data are registered.

Let the n identical and independent threads are tested. Denote by U_{ij} random variables - survivals, by $N_{ij}(u)$, $I_{ij}(u)$ related individual counting and indicator processes for the j -th strand of the i -th thread ($j=1..m$, $i=1..n$). Further denote

$$N_i(u) = \sum_{j=1}^m N_{ij}(u), \quad I_i(u) = \sum_{j=1}^m I_{ij}(u)$$

$$N(u) = \sum_{i=1}^n N_i(u), \quad I(u) = \sum_{i=1}^n I_i(u)$$

The common estimator of the cumulative hazard function is the Nelson-Aalen one

$$\hat{H}_N(u) = \int_0^u \frac{dN(v)}{I(v)} \quad (28)$$

where is set $0/0=0$. The ability of the estimator to approximate well the true $H(u)$ depends on the indicator processes for all values of strength u in the interval of interest. Proof of asymptotic uniform consistency and asymptotic normality of estimator defined by eqn. (28) is derived in [7].

In the simple case the strands are considered to be only two types. Standard one has hazard function $h_0(s)$ and weaker one has hazard function $h_1(s)=c h_0(s)$ for $c>1$. (Koziol-Green scheme).

The proposed model can be used for prediction of the survival of threads when the survival distribution of strands is known. Though the overall survival can be derived from the order statistics distribution, its computation is generally complicated. Therefore, the Monte Carlo simulation is useful.

This simulation has been used for description of breaks of threads composed from $m=10$ (filaments) strands. The survival distribution of strands types were chosen to be $\exp(1)$ and $\exp(2)$ i.e. constant hazard rates $h_0=1$ and $h_1=2h_0$. Fig. 3 shows comparison of estimated hazard functions of survival of two sets of threads.

Threads in the first set are composed only from stronger strands(10+0), while the threads in the second set are composed form five strands of each type (5+5). For each set the 100 threads are simulated.

Based on the Kolmogorov-Smirnov test the hypothesis about equality of distribution of breaking strength for both sets of threads is rejected (on the 5% level of significance).

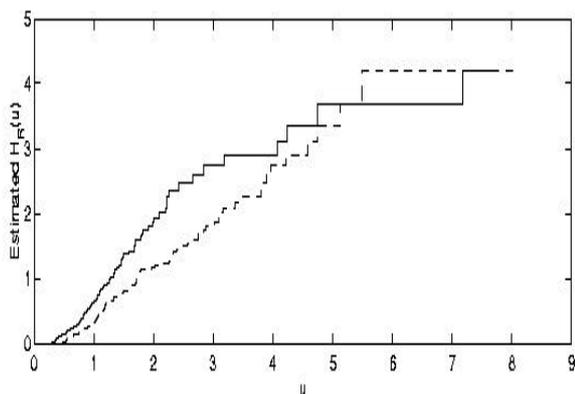


Fig.3 Estimated $H(u)$ for two sets of threads, 5+5 (full line) and 10+0 (dashed line)

This approach enables to simulate the survival of sewing threads based on the assumptions about filament types, number of filaments and corresponding distribution of filaments breaks.

7. Conclusion

For the evaluation of sewing ability the median value of time between successive break of sewing threads during sewing under standard conditions are proposed.

The methodology for statistical analysis of these data based on the exploratory data analysis principles are shown. By using of this methodology the statistical model of time between successive break is identified as Gumbell distribution. Two techniques for estimation of parameters of this distribution are presented. Program system ADSTAT is well suited for EDA of one sample problems on personal computers. Extensive description

of algorithms used in ADSTAT and examples of its utilization for analysis of chemical data is given in the book [4]. The Monte Carlo Study can be used for prediction of multifilament sewing threads breaks distribution. Corresponding statistical model is based on the theory of counting processes.

Acknowledgment: This work was supported by Czech Ministry of Education Grant VS 97084.

References

- [1] Wiezlak W.: *Textiltechnik* **30**, 388 (1980)
- [2] Nestler R., Kolbe W.: *Textiltechnik* **32**, 388 (1982)
- [3] Nemeth E.: *Textiltechnik* **30**, 446 (1980)
- [4] Meloun M., Militký J., Forina M.: *Chemometrics for Analytical Chemistry, voll PC Aided Statistical Data Analysis*, Ellis Horwood, Chichester 1992
- [5] Burry K.V.: *Statistical Models in Applied Science*, J. Wiley, New York 1975
- [6] Anderson P.K., Borgan O., Gill R.D. and Keiding N.: *Statistical models Based on Counting Processes*, Springer New York 1993
- [7] Volf P., Linka A.: *Two applications of Counting Processes*, Rept. UTIA Praha No,1935, 1998
- [8] Tukey J.W.: *Exploratory Data Analysis*, Addison Wesley, Reading, Mass. 1977
- [9] Militký J.: *System EXDAB*, Proc. Int. Conf. COMPSTAT'84, Prague, 1984, Poster section
- [10] Hinkley D.V.: *Appl. Statist.* **26**, 67 (1977)
- [11] Parzen E.: *J. Amer. Statist. Assoc.* **74**, 105 (1985)
- [12] Looney S. W., Gullledge T.R.: *The Statistician*, **34**, 297, (1985)
- [13] Hunter S.: *Amer. Statist.*, **42**, 54 (1988)
- [14] Chambers J. et al.: *Graphical Methods for Data Analysis*, Duxbury Press, Boston 1983
- [15] Hoaglin D. C., Mosteler F., Tukey J.W., Eds.: *Exploring Data, Tables, Trends and Shapes*, J. Wiley, New York 1985
- [16] Militký J.: *Package for Fitting Theoretical Distributions to Data*, Proc. Conf. COMPSTAT'88, Copenhagen 1988, Poster section
- [17] Lejenne M., Dodge Y., Kaelin E.: *Proc. Conf. COMPSTAT'82*, Toulouse, 1982
- [18] Michael J.R.: *Biometrika*, **70**, 11 (1983)
- [19] Kafander K., Spiegelman C.H.: *Comput. Statist. and Data Anal.*, **4**, 167 (1986)
- [20] Landwehr J.M., Pregibon D., Shoemaker A.C.: *J. Amer. Statist. Assoc.*, **79**, 61 (1984)
- [21] Hines W.G.S., Hines R.J.H.: *Amer. Statist.*, **41**, 21 (1987)
- [22] Emerson J. D., Stoto M. A.: *J. Amer. Statist. Assoc.*, **77**, 103 (1982)
- [23] Hoaglin D. C., Mosteler F. Tukey J.W., Eds. *Understanding Robust and Exploratory Data Analysis*, J. Wiley, New York, 1983