THE FAST ALGORITHMS FOR SOLVING OPTIMAL DISCRETE CONTROL PROBLEMS USING THE DIOPHANTINE POLYNOMIAL EQUATIONS

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Abstract .
A procedure for synthesizing fast algorithms with minimal estimates of multiplicative complexity is described. Their application to problems of optimal discrete control of dynamic objects is illustrated using the Diophantine polynomial equations.
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1. Introduction

The application of the method of polynomial equations is described in [1]-[10] for some problems of optimal discrete control. The theory of polynomial equations is a mathematical tool synthesizing control systems for processes that are tractable analytically by the differential calculus [2]. Stated differently, if controlled objects are describable by differential equations, then the controllers are describable by polynomial equations.

The present paper proposes a procedure for fast computational algorithms with minimal estimates of multiplicative complexity intended for solving polynomial equations in problems of optimal control.

2. Method of polynomial equations in problems of synthesizing discrete control systems

The transfer functions of linear dynamic systems (LDS) $H(z)=A(z)/B(z)$ are regular rational functions of z, that is, $deg H(z) < 0$ The correctness of $H(z)$ follows from the fact that the transitions of physical LDSs are described by ordinary nongeneralized time functions [2], [3] which leads to the condition $p > q$, where $p=deg\,B(z)$ and $q'= deg A(z)$ are the degrees of polynomials $B(z)$ and $A(z)$, respectively.

Computerized linear controllers also are described by discrete transfer functions (programs) $W(z)$ in the form of rational functions $W(z) = A'(z)/B'(z)$ Unlike the discrete transfer function of LDS, $W(z)$ can be irregular, that is, deg $W(z) > 0$ or < 0 [2]. The following constraint is imposed on $W(z)$ from the point of view of the physical realizability of a discrete controller:

$$def\, W(z) \geq 0, \tag{1}$$

where $def\, W(z)$ is the defect of polynomial $W(z)$ (the number of zeros at the point $z=0$).

The type of automatic control system (direct control, parallel action, feedback, combined, etc.) depends on the method of connecting the controlled object to the controller [1]-[10]. A certain type of dependence of the system transfer function $H(z)$ on the transfer functions of the controlled object $G(z)$ and controller $W(z)$ (denoted by $H(z) = H[G(z), W(z)]$) corresponds to each type of control systems.

The choice of controller programs $W(z)$ for synthesizing control systems for a given function $G(z)$ is limited by the following conditions [2]:

- the controller must be physically realizable, that is, its transfer, function $W(z)$ must satisfy relationship (1) and
- the synthesized system of equations must be operable, that is, its transfer function $H(z)$ must satisfy the analytical conditions of operability

$$(H(z))_{-}=0; \tag{2a}$$

$$(\delta H(z))_{-}=0; \tag{2b}$$

$$(\delta^2 H(z))_{-}=0,\ldots; \tag{2c}$$

where $(H(z))\_$ is the function obtained by separating the rational function $H(z)=(H(z))_-+(H(z))_+$ so that the poles of left-hand side $(H(z))\_$ are in the domain $C-=\{|z| < 1| z \in C\}$ and those of the right-hand side $(H(z))_+$ are in $C_+=\{|z| > 1| z \in C\}$ and $\delta H(z)=\sum_{n=-\infty}^{\infty}\delta h_n z^n$ is a variation of the rational function defined by the Laurent power series $H(z)=\sum_{n=-\infty}^{\infty}\delta h_n z^n$. Condition (2a) amounts to the condition of **stability** of the automatic control system, and conditions (2b) and (2c) amount to the conditions of **roughness** in the sense of A. A. Andronov [2]. For the synthesized system to be operable, both the stability and roughness conditions must be satisfied.

Using conditions (1) and (2a)-(2c), L. N. Volgin [2] defined the classes of admissible functions $H(z)$ for various control systems.

If, for a suitable choice of W(z), a system can be driven to the required state by finite control actions in a finite time [2], [11], then it is referred to as a practically controllable system in the sense of R. Kalman. Similarly, if a system can be driven to the zero state by finite control actions in a finite time [2], then it is referred to as a practically invariant system. If a system $Y(z)=H(z)X(z)$ *is* practically controllable, then its complementary system $E(z)=(1-H(z))X(z)=X(z)-Y(z)$ is practically invariant. The image $E(z)$ of the output of the complementary system is the discrepancy between the input and output of the original practically controllable system $X(z)-Y(z)$. Since processes of finite duration have images in form of polynomials in z, the system is practically controllable if the image of the output of its complementary system can be reduced to a polynomial in z.

Since the control action $U(z)$ is related to the output of the controlled object $Y(z)=G(z)U(z)$, where $G(z)=P(z)/Q(z)$ is the transfer function of the object, the relationship between the discrepancy $E(z)$, control $U(z)$, and desired state of the object $X(z)$ is representable as follows:

$$P(z)U(z)+Q(z)E(z)=Q(z)X(z). \tag{3}$$

To satisfy the condition of practical controllability, $U(z)$ and $E(z)$ must be polynomials; if $U(z)$ and $E(z)$ are indefinite power series, then either infinitely large efforts or an infinite time are required to attain the desired state. Thus, since $P(z)$, $Q(z)$, $U(z)$, and $E(z)$ are polynomials in $\mathbf{z}$, the left-hand side and, consequently, the right-hand side of Eq. (3) are polynomials. Therefore, the object states that are admissible in terms of practical controllability have images $X(z)=A(z)/Q(z)$, where $A(z)$ is an arbitrary polynomial [2]. As follows from (3), for the admissible states of an object in a practically controllable system the polynomials $U(z)$ and $E(z)$ are related by the polynomial Diophantine equation

$$P(z)U(z)+Q(z)E(z)=A(z). \tag{4}$$

Equation (4) has a solution with respect to the polynomials $U(z)$ and $E(z)$ if the GCD of $(P(z), Q(z))=1$, that is, if the polynomials $P(z)$ and $Q(z)$ are relatively prime.

Since polynomials $P(z)$, $Q(z)$, and $A(z)$ are known, all solutions of the polynomial equation (4) are expressed in terms of its particular solution $\{U'(z), E'(z)\}$ as

$$U(z)=U'(z)+Q(z)S(z), \qquad E(z)=E'(z)-P(z)S(z), \tag{5}$$

where $S(z)$ is an arbitrary polynomial. A correct polynomial equation (4) for which $degA(z)<degP(z)+degQ(z)$ has among the solutions of (5) a unique minimal solution $\{U_{min}(z), E_{min}(z)\}$ such that $degU_{min}(z)=degQ(z)-1$, $degE_{min}(z)=degP(z)-1$. The minimal solution of (4) with respect to $E(z)$ provides the shortest transient with duration

$$t_{min} = (1+degE_{min}(z))\,\Delta t=degP(z)\Delta t,$$

where Δt is the clock cycle.

3. Relationship between the polynomial equations and generalized K_N-convolutions

The optimal equation (4) is equivalent to the congruence

$$P(z)U(z) = A(z) \text{ Mod } Q(z) \tag{6}$$

(with Mod for the operation of finding the polynomial residues), which is known [7], [12]-[16] to describe the generalized K_N-convolution of the sequences $\{u_n\}$ and $\{p_n\}$ if $degQ(z)=N$ and $degP(z)=N-1$. This fact is not in conflict with the correctness condition for the transfer function of the controlled object, according to which $degQ(z) > degP(z)$.

Definition 1. *By a generalized* K_N*-convolution of two N-point sequences* $\{x_n\}$ *and* $\{h_n\}$ *is meant as N-point sequence* $\{y_n\}$ *described in vector and matrix terms*

$$\boldsymbol{y}^T = \boldsymbol{x}^T \boldsymbol{Z}_N(\boldsymbol{h}), \tag{7}$$

where

$$\boldsymbol{Z}_N(\boldsymbol{h}) = \begin{bmatrix} \boldsymbol{h}^T \boldsymbol{K}_N^0 \\ \boldsymbol{h}^T \boldsymbol{K}_N^1 \\ \cdots \\ \boldsymbol{h}^T \boldsymbol{K}_N^{N-1} \end{bmatrix}, \qquad \boldsymbol{K}_N^m = \begin{bmatrix} 0 & 1 & & 0 \\ & & \ddots & \\ 0 & & & 1 \\ q_{N-1} & q_{N-2} & \cdots & q_1 \end{bmatrix}^m,$$

$m=0, 1,...,N-1$, $\boldsymbol{y}^T=[y_0 y_1 \dots y_{N-1}]$, $\boldsymbol{x}^T=[x_0\, x_1 \dots x_{N-1}]$, $\boldsymbol{h}^T=[h_0\; h_1 \dots h_{N-1}]$,

In polynomial terms, the generalized K_N-convolution (7) has the following form [13], [14]:

$$Y(z) \equiv H(z)X(z) \bmod q(z), \tag{8}$$

where $Y(z)=\sum_{n=0}^{N-1} y_n z^n$, $H(z)=\sum_{n=0}^{N-1} h_n z^n$, $X(z)=\sum_{n=0}^{N-1} x_n z^n$, $q(z) = det(K_N - zE_N) = (-1)^N (z^N - q_1 z^{N-1} - ... - q_{N-1}z - q_N)$.

An insight into the values involved in (8) can be gained by comparing (6) and (8):

- the image $U(z)$ of control action $\{u_n\}$ of finite duration $degU(z)+1=N$ applied to the object corresponds to the polynomial $X(z)$,
- the numerator $P(z)$ of the transfer function $G(z)$ of the controlled object corresponds to the polynomial $H(z)$,
- the denominator $Q(z)$ of the transfer function $G(z)$ of the controlled object corresponds to the polynomial modulo $q(z)$ (the characteristic polynomial of the K_N-shift operator), and
- the numerator $A(z)$ of the image of the desired state of the object corresponds to the polynomial $Y(z)$.

Congruence (8) describes a model of a K_N-stationary (K_N-invariant) LDS [7], [13]. If a finite-duration control action $\{u_n\}$ is fed into its input (subject to the condition that the impulse response $\{h_n\}$ is a sequence of coefficients of the numerator $P(z)$ of the transfer function of a physical object), then the given K_N-stationary LDS outputs the result of generalized K_N-convolution, that is, a finite-duration process characterizing the stale of this object. The polynomial equation corresponding to (8) is written as

$$H(z)X(z)+q(z)p(z)=Y(z), \tag{9}$$

where $q(z)$ is the characteristic polynomial of the K_N-shift operator and $H(z)$, $X(z)$, and $Y(z)$ are polynomial representations, respectively, of the impulse $\{h_n\}$, input sequence $\{x_n\}$, and output sequence $\{y_n\}$. Since the polynomials $H(z)$ and $X(z)$ and $q(z)$ and $p(z)$ in (9) are interchangeable, the polynomial $p(z)$ has the meaning of a polynomial modulo [10].

To solve the polynomial equation (9) with respect to $H(z)$ and $q(z)$, $H(z)$ and $p(z)$, and $X(z)$ and $p(z)$, it is necessary that the corresponding conditions be satisfied: GCD of $(X(z),p(z))=1$, GCD of $(X(z), q(z))=1$, GCD of $(H(z), p(z))=1$ or GCD of $((H(z), q(z))=1$. We note that the condition of relative primality of the polynomials amounts to the Kalman controllability conditions.

Proposition 1. The condition of relative primality of polynomials $H(z)$ and $q(z)$ for solving polynomial equation (9) with respect to the polynomials $X(z)$ and $p(z)$ amounts to the condition of linear independence of the row vectors of the operator of K_N-stationary LDS: $\boldsymbol{h}^T$, $\boldsymbol{h}^T\boldsymbol{K}_N^1$, $\boldsymbol{h}^T\boldsymbol{K}_N^2, \dots, \boldsymbol{h}^T\boldsymbol{K}_N^{N-1}$, where $\boldsymbol{h}^T$ consists of the coefficients of polynomial $H(z)$ and K_N is the companion matrix of polynomial $q(z)$.

The proof was given in [10]. According to Proposition I, the condition of relative primality of polynomials $X(z)$ and $p(z)$ for solving (9) with respect to $H(z)$ and $q(z)$ implies that the row vectors $\boldsymbol{x}^T$, $\boldsymbol{x}^T\boldsymbol{K}_{N'}^1, \dots, \boldsymbol{x}^T\boldsymbol{K}_{N'}^{N'-1}$ are linearly independent, where $N'=degp(z)$, and so on.

In view of the fact the Kalman conditions of practical controllability require that the column vectors $\boldsymbol{b}$, $(\boldsymbol{K}_N^1)^T\boldsymbol{b}, ..., (\boldsymbol{K}_N^{N-1})^T\boldsymbol{b}$ (where $\boldsymbol{b}$ is the state vector of the system under study [11]) be linearly independent, they are, by Proposition 1, equivalent to the condition of relative primality of polynomial $B(z)=\sum_{n=0}^{N-1} b_n z^n$ and the characteristic polynomial $q(z)$ of K_N. On the other hand, relative primality of the polynomials in (3) - the numerator and denominator in the transfer function of the object - is a necessary condition of practical controllability of the system; it results in the shortest transient time.

It follows from the foregoing that in problems of synthesizing discrete control systems by polynomial equations one has to compute repeatedly the products of polynomials with respect to an arbitrary polynomial modulo, that is, generalized K_N-convolutions. Therefore, it is of interest to establish the minimal estimate of multiplicative complexity of the generalized K_N-convolution [14]-[16].

4. Estimates of minimal multiplicative complexity of reduction modulo an arbitrary polynomial and of generalized K_N-convolutions

We assume that the elements $q_1, ..., q_N$ of matrix K_N do not belong to $\boldsymbol{\Omega} \subset \boldsymbol{V}$, where $\boldsymbol{\Omega}$ is a subfield of the constants of field $\boldsymbol{V}$ in the sense of Winograd [17], which means that the multiplications $q_n x_n$ $(q_n, x_n \in V)$ cannot be regarded as trivial. We consider the field in which the characteristic polynomial $q(z)$ is decomposable into linear factors as $\boldsymbol{V}$, for example $\boldsymbol{V}=\mathbf{C}$ is a field of complex numbers.

In view of the polynomial treatment of generalized K_N-convolution (8), we must assess the computations for reduction with respect to an arbitrary polynomial modulo $q(z)$ whose coefficients do not belong to the subfield of constants $\boldsymbol{\Omega} \subset \boldsymbol{V}$. We present here only the main results, because this estimate was obtained in [14]-[16].

Lemma 1 (on estimation of the number of multiplications required for computing the reduction modulo an arbitrary polynomial). A polynomial $A(z)$ over a field $\boldsymbol{V}$ of degree $\deg A(z)=L$ can be reduced modulo a fixed polynomial $q(z)$ over $\boldsymbol{V}$ of degree $\deg q(z)=N$, where N is $ent\,[L/2]$, that is, $N=L/2$ for even L and $N=(L\pm 1)/2$ for odd L, in $2L$ multiplications.

Lemma 2. The reduction of a polynomial $A(z)$ over a field $\boldsymbol{V}$ of degree deg $A(z)=L$ modulo a fixed polynomial $q(z)$ over V of degree deg $q(z)=N$ where $N<L$ and $2N \geq L$ can be executed in $3L—2N$ multiplications.

Theorem 1. The number of multiplications for computing a generalized K_N-convolution is $6N$-7, that is, $0(6N)$ if all coefficients of the characteristic polynomial $q(z)$ of the K_N-shift matrix operator have nontrivial coefficients over the field V.

5. Fast algorithms of generalized K_N-convolution for dynamic problems of optimal control

We show, using by way of example minimization of the total quadratic error in discrete control systems, how the synthesized algorithms of generalized K_N-convolutions can be applied to some problems of optimal control [10].

Let us consider a discrete direct-control system with an infinite transient $\{e_n\}$. To optimize it, we minimize the transient energy described by the functional $J=\sum_{n=0}^{\infty} e_n^2$ with $e_n \to 0$ for $n \to \infty$. The value J is called the total quadratic error [2], [3]. By the Parseval theorem, the equality

$$\sum_{n=0}^{\infty} e_n^2 = \frac{1}{2\pi j} \oint_{|z|=1} E(z)\hat{E}(z)dz/z, \quad j=\sqrt{-1}, \tag{10}$$

where $E(z)=\sum_{n=0}^{\infty} e_n z^n$ and $\hat{E}(z)=E(z^{-1})$ is an inverted function, is valid for the z-transformation. Alternatively, in the space of Laurent power series representing the rational functions $H(z)=\sum_{n=-\infty}^{\infty} h_n z^n$ in a ring containing a loop $|z|=1$, one can assign to each power series a constant term $h_0 = ct\,H(z)$:

$$h_0 = \oint_{|z|=1} H(z)dz/z \quad , \tag{11}$$

where *ct* stands for a constant term.

Denoting $H(z)=E(z)E(z^{-1})$ and taking into consideration (10) and (11), we write the criterion for the total error as [2]

$$J = ctE(z)\hat{E}(z). \tag{12}$$

Let us consider a direct-control system assuming that we are given the process of its motion $X(z)=A(z)/B(z)$, $degA(z)<degB(z)$ and the transfer function of the controlled object $G(z)=P(z)/Q^+(z)$ with $B(z)=B_0(z)V(z)$ and $Q^+(z)=Q_0^+(z)V(z)$, where $V(z)=GCD(B(z),Q^+(z))$. To minimize the total quadratic error (12), we seek a control of the object in form [2]

$$U(z) = Q_0^+(z)\,\theta(z)/(B_0(z)P^+(z)\,\bar{P}^-(z)), \tag{13}$$

where $\bar{P}^-(z)$ is a polynomial with an inverse order of coefficients as compared with the original polynomial $\bar{P}(z)$ and $\theta(z)$ is a polynomial to be defined below. In view of (13), the error image is

$$E(z)=X(z)-G(z)U(z)=(A(z)\bar{P}^-(z)-P^-(z)\,\theta(z))/(B(z)\bar{P}^-(z)).$$

If we form the polynomial equation

$$P^-(z)\theta(z)+B(z)\ddot{I}(z)=A(z)\bar{P}^-(z), \tag{14}$$

then the error image is described by

$$E(z)=\ddot{I}(z)/\bar{P}^-(z). \tag{15}$$

The minimal solution of polynomial equation (14) with respect to $\ddot{I}(z)$ amounts to computing the generalized K_N-convolution in polynomial form [10]

$$\ddot{I}_{min}(z)=B^{-1}(z)[A(z)\,\bar{P}^-(z)\,]Mod\,P^-(z) \tag{16}$$

if $deg\,\bar{P}^-(z)=N$. The product of polynomials $A(z)\bar{P}^-(z)$ is computed using the schemes for computation of the linear convolutions (LCs) (for example, by means of algorithms for real-valued fast Fourier transforms [18]-[20]). The inverse polynomial $B^{-1}(z)$ in the ring $\boldsymbol{R}[z]/(P^-(z))$ is determined by solving congruence $B^{-1}(z)B(z)\equiv 1ModP^-(z)$ by the algorithms for computing LCs [20]. According to (16), $\ddot{I}_{m\,in}(z)$ is computed after determining $A(z)\bar{P}^-(z)$ and $B^{-1}(z)$ by the algorithm of generalized K_N-convolution, and the degree of the polynomial is deg $\ddot{I}_{min}(z)=degP\text{-}(z)\text{-}1=N\text{-}1$.

To obtain an optimal control $U(z)$, one must find a polynomial $\theta(z)$ in conformity with (13) for which polynomial equation (14) is reduced to a polynomial variant of the generalized K_N-convolution [10]:

$$\theta(z)\equiv(P^-(z))^{-1}A(z)\bar{P}^-(z)\,ModB(z)$$

where $N'=degB(z)$. (17)

6. Conclusion

The present paper demonstrates that since the minimal solution of polynomial equations is equivalent to the solution of generalized K_N-convolutions, discrete control systems can be synthesized from an input-output representation by means of the theory of K_N-stationary LDSs [7]. As a result, the developed procedures for computing the polynomial residues and generalized K_N-convolutions enable one to synthesize fast control algorithms for discrete dynamic objects.

Furthermore, it follows from the method of state space [11] that the controllable (identifiable) states of a stationary LDS make up a K_N-invariant subspace (cyclic vector subspace). In view of this fact and the results of Sec. 3 (Proposition 1, in particular), it is likely that the model of K_N-stationary systems "bridges" the methods of input-output representations and of state space. A substantiation of this conclusion is the subject matter of further studies.

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