State Estimation By IMM Filter in the Presence of Structural Uncertainty [1]

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Abstract - A solution to the state estimation problem under structural uncertainty (unknown or changeable dimension of the system state space) is given by the Interacting Multiple Model (IMM) filter. The requirements for its applicability under structural uncertainty are formulated. The highest IMM model probability is an indicator for the true model order and it can be used for structural identification. Results from test examples with stationary systems and systems with structural nonstationarity (changeable structure in the course of the time) demonstrate the filter efficiency. The scalar and multivariable cases are investigated.

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1 Introduction

The present paper considers the state estimation problem subject to structural uncertainty - unknown or changing dimension of the system state space. The system state is estimated when *the structure and the true parameters of the system model* are unknown but they belong to an uncertainty domain. A solution to this problem by another MM algorithm is given in [7]. The requirements for its applicability under structural uncertainty are formulated in [7]. These requirements are here extended for the IMM filter - a powerful scheme [2,5] for estimation of hybrid (continuous-discrete) systems. The IMM filter belongs to the group of the multiple-model algorithms that recently are very popular [1, 2, 8, 10, 12]. In most cases the IMM estimator is applied under parametric model/noise uncertainty [1-5, 9, 11]. In contrast to this, here the problem with structural uncertainty is studied. The overall state estimate is a weighted sum of q partial estimates, generated by a bank of Kalman filters for q models with different structure from the uncertainty domain. At the same time the IMM model probabilities can be used for model order determination.

With the standard methods for structural identification [6] the structure (model order) selection is an iterative process. Usually, after the initial model order computation on the basis of the input-output data, the next obligatory step is the model adequacy verification with the help of different tests - by the zeros, poles and their standard deviations, or based on the comparison between the simulated/ predicted output with the measured output, the residual errors, etc. The presence of close poles and zeros is an indicator that the model order is artificially increased and the process of the structure selection is repeated until receiving of "enough good" results according to the verification criteria. In contrast to this standard approach for structural identification, the IMM estimator directly provides the model order - it corresponds to the model with the greatest probability, recursively computed by the estimator. The current system structure is detected based on the measurements of the global system, and not by the outputs of the separate subsystems.

2 IMM State Estimation under Structural Uncertainty

The state $x_k \in R^n$ of the system

$$x_{k+1} = F_k\left(k, M_{k+1}\right)x_k + G_k\left(M_{k+1}\right)v_k\left(M_{k+1}\right), \quad (1)$$

$$y_k = Cx_k + w_k \quad (2)$$

is estimated where $y_k \in R^r$ is the measurement vector, $v_k \in \Re^m$ and $w_k \in \Re^r$ are respectively the system and measurement noises, assumed to be white and mutually uncorrelated, with zero means and variances, respectively, Q_k and R_k. The system model (1) at time k is among q possible models (modes) that are depending on the parameter $M_k \in \{1,2,\ldots,q\}$. $M_k = i$ denotes that the *i-th* submodel is in effect during the sampling period k of length T. The model switching is described by a Markov chain with known initial model probabilities $m_i = P\{M_0 = i\}$ and transitional probabilities $p_{ij} = P\{M_k = j / M_{k-1} = i\}$, for $i, j = 1,2,\cdots,q$. The IMM state estimation algorithm is a

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Bayesian suboptimal recursive procedure [2, 5] for obtaining the system state estimate

$$\hat{x}_k = \sum_{i=1}^{q} m_{i,k}\hat{x}_{i,k} \qquad (3)$$

as a weighted sum of partial estimates $\hat{x}_i$, formed by a bank of operating in parallel Kalman filters. At each recurrent cycle ($k-1 \to k$) the initial conditions for the filter corresponding to the mode $M_k = i$ are computed by mixing the preceding mode-conditional estimates $\hat{x}_i$, *i=1,2,...,q*.

The order of the system model (1)-(2) is unknown. The true model parameters F_k, G_k, C_k, Q_k, R_k are also unknown but it is supposed that they belong to an uncertainty domain. The true model is approximated by *q* models from this uncertainty domain. The uncertainty domain contains *p* possible structures (models of different orders n_j, $j = 1,2,\cdots,p$). From every structure there are *s* models with different parameters. The structures with different orders are constructed in the following way: if from the full state vector $x = \left(x_1^T,\ x_2^T,\ \cdots,\ x_j^T\right)^T$ (for the structure of maximum order), the *i*-th segment x_i is dropped out, in the state vector of the respective lower order structure, the respective segment x_i is replaced by a zero vector. It means that every segment from the vector x corresponds to some structure, and the remaining segments are replaced by zeros. This is important for the overall estimate (3) formation. The estimate of the *j*-th segment of the state vector x is formed on the basis of the *j*-th segments of the vectors $\hat{x}_i$, $i = 1,2,\cdots,q$. An important condition for the applicability of the filter is to keep the correspondence between the variables in the different IMM models when the order of the state space is reduced and some variables are dropped out. The true model is determined by the highest IMM model probability. The choice of the transition probabilities matrix *P* of the IMM depends on the considered problem specificity and the initial information, if there is any. Examples satisfying the above mentioned conditions are considered below.

3 Experimental Results

The system under consideration S is linear, composed by independent subsystems S_i, $i = 1,2,\cdots,q$ connected in parallel (Fig.1), and every of these subsystems is described by a state vector x_i, $i = 1,2,\cdots,q$.

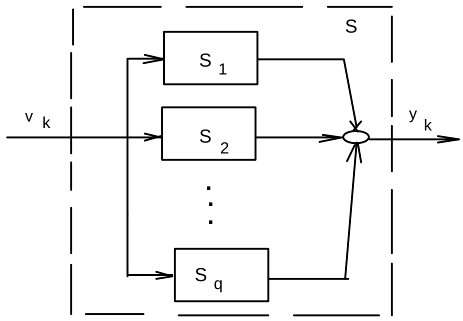

Fig. 1

For *i*=3 the general state vector of the system S is $x = \left(x_1^T,\ x_2^T,\ x_3^T\right)^T$. If some subsystems are dropped out (a situation that often arises in industrial or electronic systems), e.g. the subsystems S_2 and S_3, then the global system S is described by the vector $x = \left(x_1^T,\ 0,\ 0\right)^T$.

In all the examples below a system composed by three subsystems ($i = 3$) is considered.

The algorithm performance is evaluated by Monte Carlo experiments for 100 runs. A general measure of performance, characterizing the filter consistency, is the Normalized Estimation Error (NEE) [2] and it is presented.

Example 1. The system model (1)-(2) is stationary (with constant matrices F, G, C, Q, and R) and with unknown order. The true model of the system S is:

$$F = e^{-0.1},\ G = 1\text{-}\,e^{-0.1},\ \ C = 1,\ Q = R = 1.$$

The final state estimate is computed on the basis of three IMM models with different order (first, second and third) from the uncertainty domain. For equalizing the dimensions of the matrices and vectors, the respective elements are fulfilled with zeros:

1) $F_1 = diag\left(e^{-0.067}\ \ 0\ \ 0\right)$, $G_1 = \left(1-e^{-0.067}\ \ 0\ \ 0\right)^T$;

2) $F_2 = diag\left(e^{-0.067}\ \ e^{-0.5}\ \ 0\right)$, $G_2 = \left(1-e^{-0.067}\ \ 1-e^{-0.5}\ \ 0\right)^T$;

3) $F_3 = diag\left(e^{-0.067}\ \ e^{-0.5}\ \ e^{-1}\right)$,

$$G_3 = \left(1-e^{-0.067}\ \ \ 1-e^{-0.5}\ \ \ 1-e^{-1}\right)^T,$$

$C_i = \left(1\ 1\ 1\right)$, $Q_i = R_i = 1$, $i = 1,2,3$. Between the models the first one has structure as the structure of the true model, but its parameters are different.

The transition probabilities matrix and the initial model probability vector are

$$P = \begin{pmatrix} 0.98 & 0.01 & 0.01 \\ 0.01 & 0.98 & 0.01 \\ 0.01 & 0.01 & 0.98 \end{pmatrix}, \; m(0) = \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix}. \qquad (4)$$

The initial model probabilities in $m(0)$ are chosen equal, because the three models are equally probable. The average model (mode) IMM probabilities from Monte Carlo simulations are given in Fig.2. It is seen that the probability of the first model is the greatest, whereas the other two are considerably smaller. On the basis of them an inference can be drawn that the first order model is the true one.

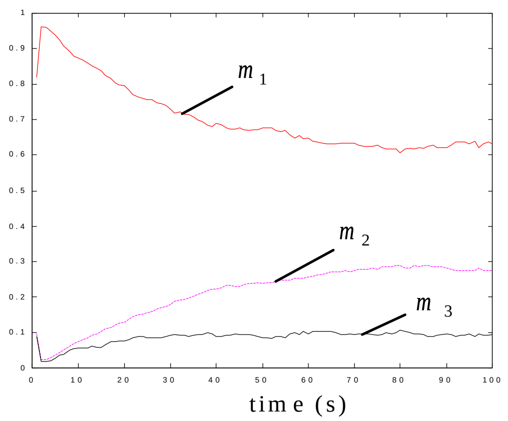

Fig. 2 Model probabilities

The NEE is given in Fig. 3 and it demonstrates that the IMM state estimate is consistent.

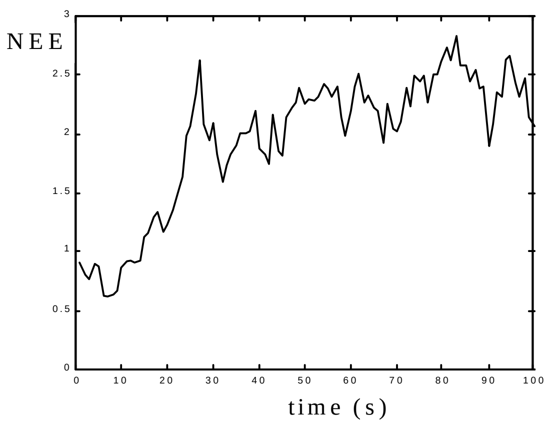

Fig. 3 Normalized Estimation Error

Example 2. The IMM performance is investigated when the true unknown system model is

$$F = diag\left\{e^{-0.1} \;\; e^{-0.5} \;\; e^{-1}\right\},$$

$$G = \left(1-e^{-0.1} \;\; 1-e^{-0.5} \;\; 1-e^{-1}\right)^T, \; C = \left(1 \;\; 1 \;\; 1\right).$$

The general IMM models are:

1) $F_1 = diag\left(e^{-0.067} \;\; 0 \;\; 0\right)$, $G_1 = \left(1-e^{-0.067} \;\; 0 \;\; 0\right)^T$;

2) $F_2 = diag\left(e^{-0.067} \;\; e^{-0.33} \;\; 0\right)$,

$G_2 = \left(1-e^{-0.067} \;\; 1-e^{-0.33} \;\; 0\right)^T$;

3) $F_3 = diag\left(e^{-0.067} \;\; e^{-0.33} \;\; e^{-1}\right)$,

$G_3 = \left(1-e^{-0.067} \;\; 1-e^{-0.33} \;\; 1-e^{-1}\right)^T$,

$C_i = \left(1 \;\; 1 \;\; 1\right)$ and $Q_i = R_i = 1$. The third order model coincides with the true order model, but its parameters are different.The transition probabilities matrix and the initial model probability vector have the form (4). The IMM model probabilities and the NEE are given in Figs.4 and 5.

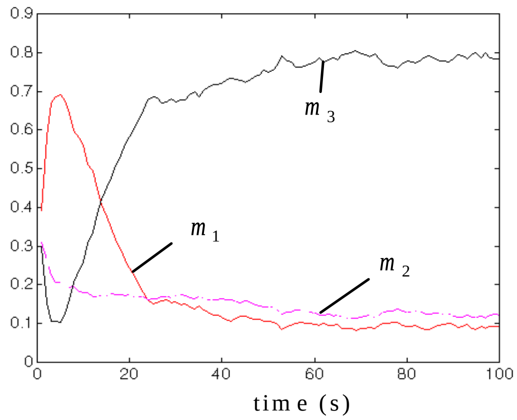

Fig. 4 Model probabilities

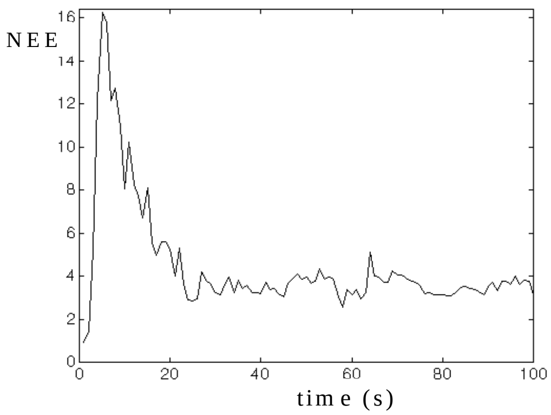

Fig. 5 Model probabilities

Because the first IMM model structure does not coincide with the true model structure and at the beginning the greatest transition probability is given to the first model, the estimator needs some period of time for finding the true structure (the probability m_3 corresponds to it).

Example 3. The system (1)-(2) is characterized by changeable structure in the course of the time (structural nonstationarity)

$$S = \begin{cases} [S_1, \;\; S_2, \;\; S_3], & for \quad 0 \le k \le 150, \\ [S_1, \;\; S_2], & for \quad 150 < k \le 300, \\ [S_1], & for \quad 300 < k \le 500. \end{cases} \qquad (5)$$

The models of the subsystems S_1, S_2 and S_3 have the form

$$S_1: F_1 = e^{-0.1}, G_1 = 1\text{-}e^{-0.1}, C_1 = 1, Q_1 = R_1 = 1, \quad (6)$$

$$S_2: F_2 = e^{-0.5}, G_2 = 1\text{-}e^{-0.5}, C_2 = 1, Q_2 = R_2 = 1, \quad (7)$$

$$S_3: F_3 = e^{-1}, G_3 = 1\text{-}e^{-1}, C_3 = 1, \quad Q_3 = R_3 = 1. \quad (8)$$

The IMM models are:

1) $F_1 = diag\{e^{-0.1} \quad e^{-0.5} \quad e^{-1}\}$,

$G_1 = \left(1-e^{-0.1} \quad 1-e^{-0.5} \quad 1-e^{-1}\right)^T$,

2) $F_2 = diag\{e^{-0.1} \quad e^{-0.5} \quad 0\}$,

$G_2 = \left(1-e^{-0.1} \quad 1-e^{-0.5} \quad 0\right)^T$,

3) $F_3 = diag\{e^{-0.1} \quad 0 \quad 0\}$, $G_3 = \left(1-e^{-0.1} \quad 0 \quad 0\right)^T$,

$C_i = \left(1 \;\; 1 \;\; 1\right)$, $Q_i = R_i = 1$, and $diag\{.\}$ denotes a diagonal matrix. The structure and the parameters of every IMM model coincide with the structure and the parameters of the true system models in the different time intervals.

The transition probabilities matrix has the form as in (4) and the initial model probabilities vector is $m(0) = \left(0.98 \quad 0.01 \quad 0.01\right)^T$. The average model probabilities computed by the IMM filter are given in Fig. 6 and the NEE - in Fig. 7.

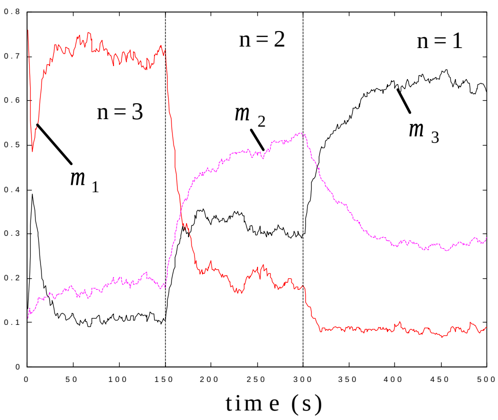

Fig. 6 Model probabilities

A second experiment is performed when the system true structure is changed according to (5), but only the structure of every model coincides with the structure of the true models of the subsystems, whereas the parameters are different. The IMM models matrices chosen from the uncertainty domain are :

1) $F_1 = diag\{e^{-0.067} \quad e^{-0.5} \quad e^{-1}\}$,

$G_1 = \left(1-e^{-0.067} \quad 1-e^{-0.5} \quad 1-e^{-1}\right)^T$,

2) $F_2 = diag\{e^{-0.067} \quad e^{-0.5} \quad 0\}$,

$G_2 = \left(1-e^{-0.067} \quad 1-e^{-0.5} \quad 0\right)^T$,

3) $F_3 = diag\{e^{-0.067} \quad 0 \quad 0\}$, $G_3 = \left(1-e^{-0.067} \quad 0 \quad 0\right)^T$.

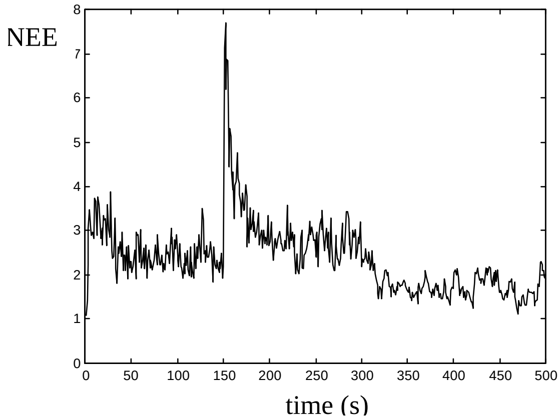

Fig. 7 Normalized Estimation Error

The average IMM model probabilities are shown in Fig. 8 and the NEE - in Fig. 9.

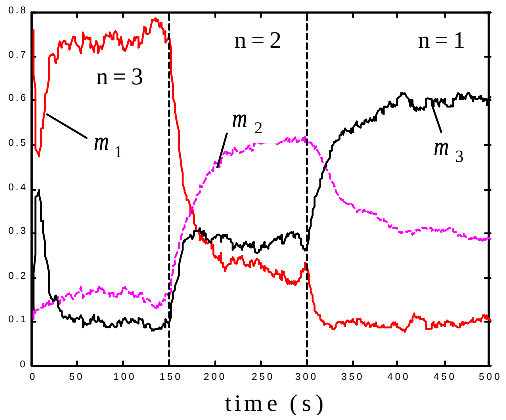

Fig. 8 Model probabilities

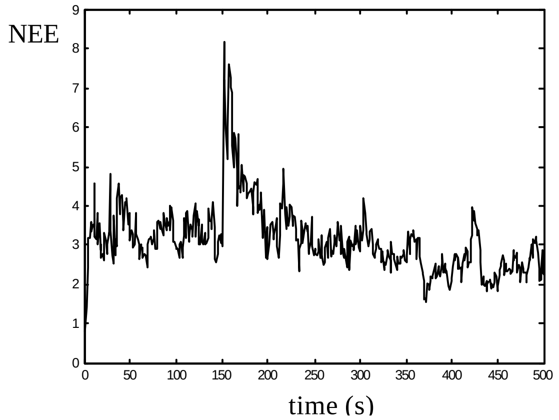

Fig. 9 Normalized Estimation Error

The results from the two tests are similar. In both cases at the beginning ($k \le 150$) the first probability has the highest value, latter - the second one, and finally ($k \ge 300$) - the third model probability. These changes correspond to the changes in the model structure: at the beginning the model order is $n = 3$, after that $n = 2$, and for $k \ge 300$, $n = 1$. It can be seen from the comparison of the plots in Figs. 7 and 9 that the NEE for the models with accurate parameters is smaller than the NEE for the models with inaccurate ones.

Example 4. The considered system (1)-(2) is multivariable $(v_k \in \Re^3,\ \ y_k \in R^3,\ \ w_k \in \Re^3)$ with the following structure (unknown)

$$S=\begin{cases}[S_1\ \ S_2\ \ S_3], & for\quad 0\le k<100\\ [S_1\ \ S_2], & for\quad 100\le k<150\\ [S_2], & for\quad 150\le k<200\\ [S_2\ \ S_3], & for\quad 200\le k<250\\ [S_1\ \ S_2\ \ S_3], & for\quad 250\le k<300\end{cases}$$

where S_1, S_2 and S_3 have the form (6), (7) and (8) respectively.

Different subsystems are working in the course of the time - one of them are switched on, another are switched off. The parameters of the subsystems are accurate. The IMM is using seven models - corresponding to all the possible combinations of working subsystems.

The IMM models matrices are:

1) $F_1 = diag\{e^{-0.1}\quad e^{-0.}\quad e^{-1}\}$,

$G_1 = diag\{1-e^{-0.1}\quad 1-e^{-0.5}\quad 1-e^{-1}\}^T$;

2) $F_2 = diag\{e^{-0.1}\quad e^{-0.5}\quad 0\}$,

$G_2 = diag\{1-e^{-0.1}\quad 1-e^{-0.5}\quad 0\}^T$;

3) $F_3 = diag\{0\quad e^{-0.5}\quad e^{-1}\}$,

$G_3 = (0\quad 1-e^{-0.5}\quad 1-e^{-1})^T$;

4) $F_4 = diag\{e^{-0.1}\quad 0\quad e^{-1}\}$,

$G_4 = diag\{1-e^{-0.1}\quad 0\quad 1-e^{-1}\}^T$;

5) $F_5 = diag\{e^{-0.1}\ \ 0\ \ 0\}$, $G_5 = diag\{1-e^{-0.1}\ \ 0\ \ 0\}^T$;

6) $F_6 = diag\{0\ \ e^{-0.5}\ \ 0\}$, $G_6 = diag\{0\ \ 1-e^{-0.5}\ \ 0\}^T$;

7) $F_7 = diag\{0\ \ 0\ \ e^{-1}\}$, $G_7 = diag\{0\ \ 0\ \ 1-e^{-1}\}^T$,

$C_i = I$, $Q_i = I_3$, $R_i = 0.01 I_3$, $i=\overline{1,7}$, where I is the identity matrix. The IMM transition probabilities matrix and the initial model probabilities vector are:

$$P=\begin{pmatrix}0.94&0.01&0.01&0.01&0.01&0.01&0.01\\0.01&0.94&0.01&0.01&0.01&0.01&0.01\\0.01&0.01&0.94&0.01&0.01&0.01&0.01\\0.01&0.01&0.01&0.94&0.01&0.01&0.01\\0.01&0.01&0.01&0.01&0.94&0.01&0.01\\0.01&0.01&0.01&0.01&0.01&0.94&0.01\\0.01&0.01&0.01&0.01&0.01&0.01&0.01\end{pmatrix},$$

$$m(0)=(1/7\quad 1/7\quad 1/7\quad 1/7\quad 1/7\quad 1/7\quad 1/7)^T.$$

The model probabilities and NEE plots are given in Figs. 10 and 11. Because the IMM models correspond to all the combinations of working subsystems, it is possible by the model probabilities to determine the model order and to identify the active subsystems in every moment.

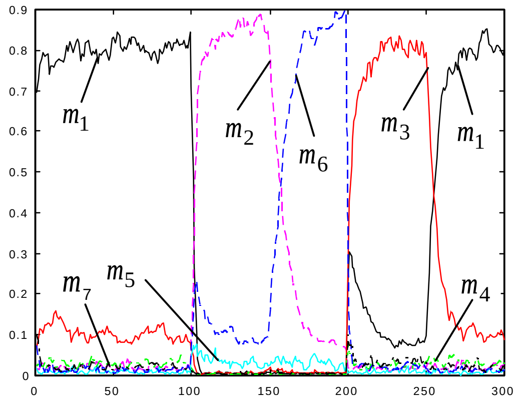

Fig. 10 Model probabilities

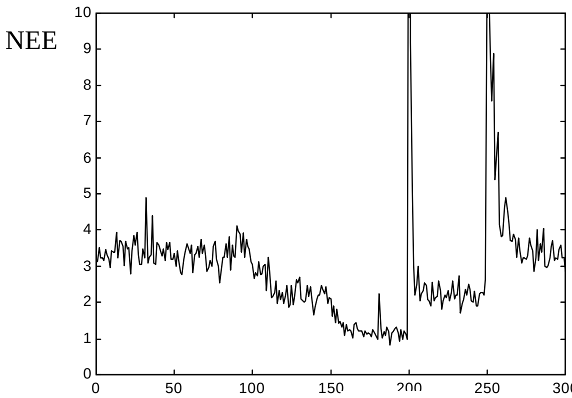

Fig. 11 Normalized Estimation Error

Another experiment is made by IMM models corresponding to all the possible combinations of subsystems but with inaccurate parameters

1) $F_1 = diag\{e^{-0.067}\quad e^{-0.33}\quad e^{-1.25}\}$,

$G_1 = diag\{1-e^{-0.067}\quad 1-e^{-0.33}\quad 1-e^{-1.25}\}^T$;

2) $F_2 = diag\{e^{-0067}\quad e^{-0.33}\quad 0\}$,

$G_2 = diag\{1-e^{-0.067}\quad 1-e^{-0.33}\quad 0\}^T$;

3) $F_3 = diag\{0\quad e^{-0.33}\quad e^{-1.25}\}$,

$G_3 = (0\quad 1-e^{-0.33}\quad 1-e^{-1.25})^T$;

4) $F_4 = diag\{e^{-0.067}\quad 0\quad e^{-1.25}\}$,

$G_4 = diag\{1-e^{-0.067}\quad 0\quad 1-e^{-1.25}\}^T$;

5) $F_5 = diag\{e^{-0.067}\quad 0\quad 0\}$, $G_5 = diag\{1-e^{-0.067}\quad 0\quad 0\}^T$;

6) $F_6 = diag\{0\quad e^{-0.33}\quad 0\}$, $G_6 = diag\{0\quad 1-e^{-0.33}\quad 0\}^T$;

$$7)\ F_7 = diag\{0 \quad 0 \quad e^{-1.25}\},\ G_7 = diag\{0 \quad 0 \quad 1-e^{-1.25}\}^T,$$

$$C_i = I,\ Q_i = I_3,\ R_i = 0.01 I_3,\ i = \overline{1,7}.$$

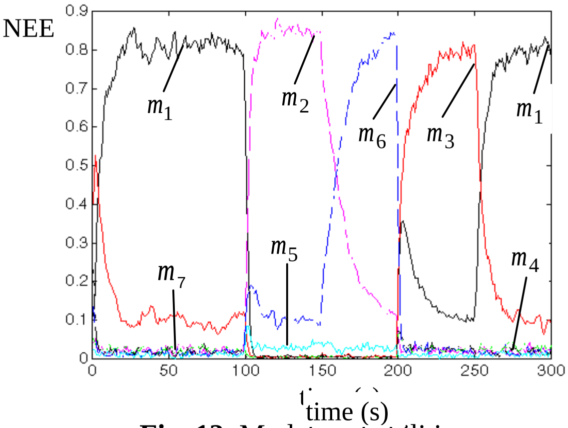

Fig. 12 Model probabilities

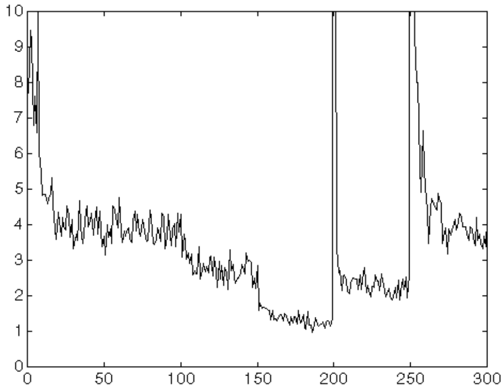

Fig. 13 Normalized Estimation Error

The model probabilities and the NEE are presented in Figs. 12 and 13. In spite of the fact that only the structures of the models are true, whereas the parameters do not coincide with their accurate values, the IMM estimator is finding the true model structure in the different periodes of time. The NEE is bigger then the NEE for the case with accurate parameters. It should also be emphasized that in all examples, the state estimates are characterized by a very good consistency (obvious from the NEE plots).

4 Conclusions

The Interacting Multiple Model filter has been applied for state estimation in the presence of structural uncertainty - unknown or changing dimension of the system state space and its performance is evaluated by Monte Carlo simulations. The restrictions, concerning the IMM application to state estimation under structural uncertainty are formulated. The most important one is to keep the correspondence between the variables in the different IMM models. The highest IMM model probability is an indicator for the true model order and it can be used for structural identification. In comparison to the standard methods for structural identification, where the model order selection is an iterative process, the IMM estimator directly provides the model order. Results from test examples with stationary systems and systems with structural nonstationary (changeable structure in the course of the time) demonstrate the filter efficiency. The scalar and multivariable cases are investigated.

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