

# On the Errors of Impedance Measurement Relays used in Distance Protections for Electrical Lines

CRISTEA PAL, <sup>a</sup> TEFAN DUMBRAVĂ

Department of Automatic Control and Industrial Informatics

“Gh. Asachi” Technical University of Iași

53 A D. Mangeron Blvd., RO–6600, Iași, Romania

**Abstract:** In this paper, some considerations concerning the errors induced by impedance measuring elements in case of distance protection are presented. The real position form of the actuating characteristic is determined and compared to the ideal situation defined in case of using low sensitivity threshold elements.

**Key-words:** Distance protection, Function generators, Impedance *CSCC'99 Proceedings*: - Pages 2001-2003

## 1 Introduction

The shape design of the actuating characteristic of the impedance measuring elements requires computations in which current  $\underline{I}$  and voltage  $\underline{U}$  in the fault loop are implied. In the specific case of protections based on absolute values comparison the principle scheme is presented in Fig. 1.

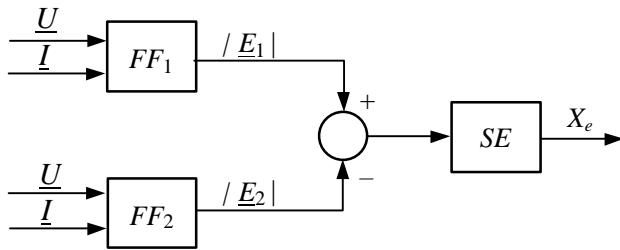


Fig. 1. The principle protection scheme based on absolute values comparison

It was denoted as  $FF_1$  and  $FF_2$  the function generators, and  $SE$  the sensible element. In general, the values  $|\underline{E}_1|$  and  $|\underline{E}_2|$  are described by:

$$|\underline{E}_1| = k_n |\underline{K}_{1U} \underline{U} + \underline{K}_{1I} \underline{I}|^n \quad (1)$$

$$|\underline{E}_2| = k_n |\underline{K}_{2U} \underline{U} + \underline{K}_{2I} \underline{I}|^n \quad (2)$$

where:  $\underline{K}_{1U}$ ,  $\underline{K}_{2U}$ ,  $\underline{K}_{1I}$ ,  $\underline{K}_{2I}$ , depend on the type and connection of the voltage and current transformers and  $n$  and  $k_n$  represent the degree and function generator constant, respectively. The actuating condition of the protection is emphasized by the following inequality:

$$\Delta E = K_d |\underline{E}_1| - K_d |\underline{E}_2| \geq U_0 \quad (3)$$

where  $K_d$  is a constant that affects the computed modulus and comparison circuit, and  $U_0$  the sensibility threshold of the measuring element. In many research papers [1], [2], [3], the fulfillment of the condition (3) is considered for the ideal case  $U_0 = 0$ . This assumes the usage of sensible elements with very low threshold. In practice,  $U_0$  can be decreased, but it is always different from zero and that implies errors both in shape and position between the real and ideal actuating characteristics, in  $(R, jX)$  plane. Some considerations concerning these errors for the specific cases of linear ( $n = 1$ ) and quadratic ( $n = 2$ ) function generators usage are further presented.

## 2 The insensibility influence on the actuating characteristic shapes of the impedance measuring element

### 2.1 The case of a linear function generator

The actuating condition (3) of protection in case of  $n = 1$ ,  $k_n = 1$ , becomes

$$|\underline{Z} - \underline{Z}_1| - K_1 |\underline{Z} - \underline{Z}_2| \geq K_{3L} \quad (4)$$

where:

$$\begin{aligned} \frac{\underline{U}}{\underline{I}} = \underline{Z}, \quad \frac{\underline{K}_{1U}}{\underline{K}_{1I}} = -\underline{Z}_1, \quad \frac{\underline{K}_{2U}}{\underline{K}_{2I}} = -\underline{Z}_2, \\ \frac{\underline{K}_{2U}}{\underline{K}_{1U}} = K_1, \quad \frac{U_0}{K_d K_{1U} I} = K_{3L}; \end{aligned} \quad (5)$$

For the minimal impedance protection the condition  $K_1 > 1$  is added. The following variable transformation is considered in (4):

$$\underline{Z}_a = |\underline{Z} - \underline{Z}_1|, \quad \underline{a} = \text{Arg}(\underline{Z} - \underline{Z}_1) \quad (6)$$

Denoting as:

$$|\underline{Z}_1 - \underline{Z}_2| = a, \quad \frac{Z_a}{|\underline{Z}_1 - \underline{Z}_2|} = v, \quad (7)$$

$$\frac{U_0}{K_d K_{1U} I |\underline{Z}_1 - \underline{Z}_2|} = K_{2L}$$

the coordinate system can be changed from  $(R, jX)$  into the adimensional system  $(R'/a, jX'/a)$ . Thus, the actuating characteristic equation becomes:

$$(1 - K_1^2)v^2 + 2(K_1^2 \cos a - K_{2L})v + K_{2L}^2 - K_1^2 = 0 \quad (8)$$

An ideal actuating characteristic in shape of a circle, with radius  $w_i$  and abscissa of the center  $U_i$  given by the following:

$$w_i = \frac{K_1}{K_1^2 - 1}, \quad U_i = \frac{K_1^2}{K_1^2 - 1} \quad (9)$$

is obtained. For that,  $K_{2L} = 0$  ( $U_0 = 0$ ) and  $K_1 = 1$ , can be defined.

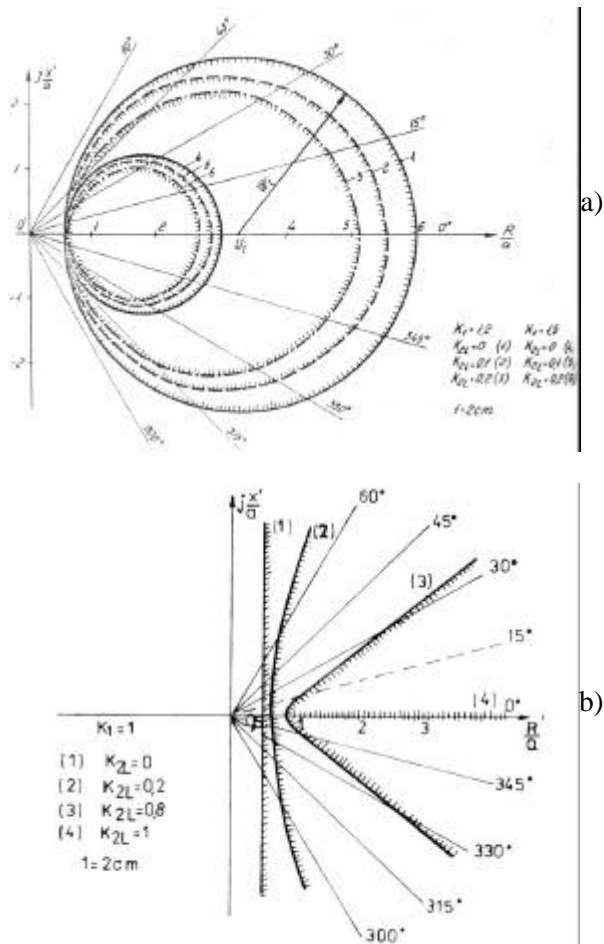


Fig. 2. The actuating characteristics in case of linear function generators: a)  $K_1 = 1,2$  and  $1,5$ ; b)  $K_1 = 1$

The ideal characteristic defined above is used as a term of comparison for the evaluation of real characteristic deviations. In Fig. 2a-2b, families of actuating characteristic for different values of  $K_1$  and  $K_{2L}$  parameters, are presented. From Fig. 2a-2b, a change of the actuating characteristic of impedance relay by an increase of  $U_0$  and evaluated by means of  $K_{2L}$ , results.

Thus, the circular characteristics obtained for  $K_1 > 1$  and  $K_{2L} = 0$  (the curves 1 and 4 in Fig. 2a) become closed curves, different from circles for  $K_{2L} \neq 0$ , (curves 2, 3 and 5, 6 in Fig. 2a). The characteristic of the resistance relay obtained for  $K_1 = 1$  and  $K_{2L} \neq 0$ , (the curve 1 in Fig. 2b), changes into a hyperbola type characteristic, (the curves 2, 3 in Fig. 2b). In case that  $K_{2L} = K_1 = 1$ , a degenerated characteristic with the actuating zone reduced to a line, is obtained, (curve 4 in Fig. 2b).

## 2.2 The case of quadratic function generator

The condition (3), for the case  $n = 2$  and  $k_n = K_M$  has the following expression:

$$|\underline{Z} - \underline{Z}_1|^2 - K_1^2 |\underline{Z} - \underline{Z}_2|^2 \geq K_{3P}^2 \quad (10)$$

where:  $\underline{Z}_1, \underline{Z}_2, K_1$  and  $\underline{Z}$  are given by (5) and

$$K_{3P}^2 = \frac{U_0}{K_d K_M K_{1U}^2 I^2} \quad (11)$$

Making a change of variables, similar to that in paragraph 2.1, using (7) and denoting as:

$$K_{2P}^2 = \frac{K_{3P}^2}{a^2} = \frac{U_0}{K_d K_M K_{1U}^2 I^2 |\underline{Z}_1 - \underline{Z}_2|^2} \quad (12)$$

the equation of the actuating characteristic in the new, adimensional system becomes:

$$(1 - K_1^2)v^2 + 2vK_1^2 \cos a - K_{2P}^2 - K_1^2 = 0 \quad (13)$$

where  $a$  and  $v$  are given by (6) and (7). For  $K_1 \neq 1$ , in the new axes system, the equation (13) represents a family of circles, no matter the constant value of  $K_{2P}$ . In the specific case  $K_1 > 1$ ,  $K_{1U} \neq 0$  the circles have the center situated on the real positive axis, with abscissa and radius given by the following relations:

$$U = \frac{K_1^2}{K_1^2 - 1}, \quad w = \frac{\sqrt{K_1^2 - K_{2P}^2 (K_1^2 - 1)}}{(K_1^2 - 1)} \quad (14)$$

In Fig. 3, the actuating characteristics described in (13) for the same values of  $K_1$  as in Fig. 2 case and  $K_{2P} = K_{2L}$ , are presented. The characteristics are placed inside the ideal characteristics  $K_{2P} = 0$ , and

practically, are identical to this. Moreover, in the same conditions referred to the value of constant  $K_1$  and for  $K_{2P} = K_{2L}$  the characteristic deformation when using the quadratic functions is lower than in case of the linear functions. For  $K_1 = 1$  and  $K_{2P} = K_{2L}$  the actuating characteristics becomes lines with displacement from the ideal line ( $K_{2P} = 0$ ), and  $R'/a = 0.5$  (see curve 1, Fig. 4). When using the quadratic functions, the displacement of the characteristics is lower than in case of the linear functions, too. Thus, the distance protection errors are lower.

### 3 Conclusions

In the presence of unavoidable  $U_0$  insensibility of the measuring element used in distance protection schemes, differences in shape and position between the actual and ideal actuating characteristics exist. The insensibility of the measuring element has a reduced influence in case of using quadratic functions by comparison with linear functions. When using quadratic functions only their position is modified.

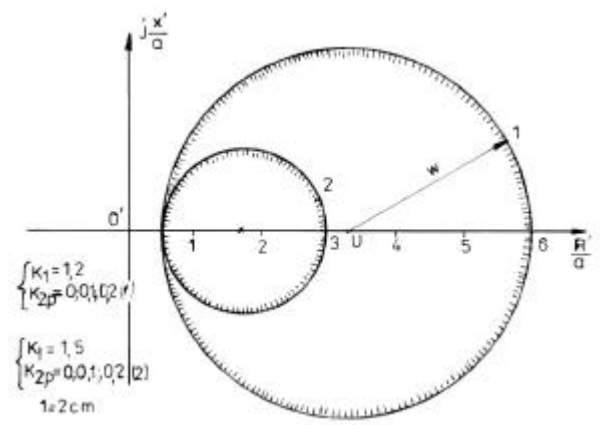


Fig. 3. The actuating characteristics in case of quadratic functions generators for  $K_1 = 1, 2$  and  $1, 5$  and  $K_{2L} = K_{2P}$

A qualitative and quantitative evaluation of the impedance measurement errors could be obtained by

introducing an error index that would take into account the actual impedance measured value,  $v$  and  $a$ , and the actual position in the plane ( $R, jX$ ) or ( $R'/a, jX'/a$ ) in rapport an ideal work characteristic. The study of the extended  $n$ -degree function case is only theoretical as long as functions with a bigger than two degree are not used in practice.

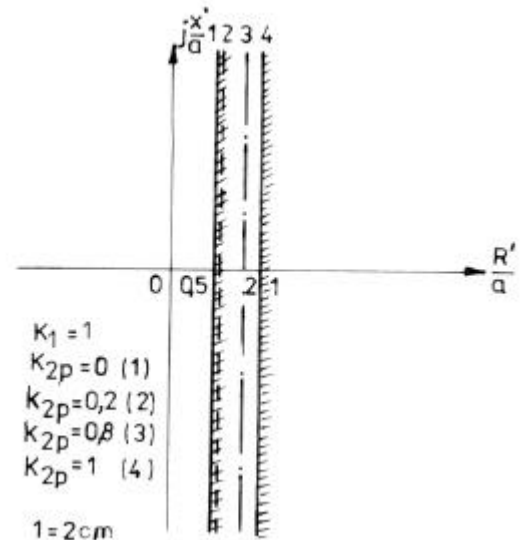


Fig. 4. The actuating characteristics in case of quadratic functions generators for  $K_1 = 1$  and  $K_{2L} = K_{2P}$

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