Linear Phase or Spectral Factor Prototype Filters to Pseudo-QMF Bank Design

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Abstract: - A new approach to design *M*-Channel pseudo-Quadrature Mirror Filter (QMF) cosine-modulated filter bank is formulated using the Spectral Factorization Approach to pseudo-QMF design. The new approach is derived fully and the overall distortion and the aliasing transfer functions are presented. Compared with Spectral Factorization Approach, the proposed technique yields filter banks with lower reconstruction errors. Several examples and measures are included in order to compare both methods.

*Key-Words:*pseudo-QMF filter bank,overall distortion transfer function,aliasing function *CSCC'99* Proc.pp..1611-1614

1 Introduction

The theory of *M*-channel maximally decimated filter bank is well established and their properties, design methods, and applications are well addressed [1]-[3]. Filterbanks based on modulated filters are well known and attractive solutions because design and implementation costs are significantly reduced.

Pseudo-QMF systems cosine modulated filter banks were introduced by Nussbaumer [4] and developed later by Rotweiler [5]. Spectral Factorization Approach to pseudo-QMF design introduced in [6] and Near-Perfect-Reconstruction pseudo-QMF filter banks developed further in [7] are the most important approaches related to our work.

In this work, we show two different forms of constructing filter banks using the scheme of modulation given in spectral factorization approach [6] with different prototype filters. On the one hand, a spectral factor of a valid 2*M*th band filter; on the other hand, a symmetric linear phase prototype filter [8].

We describe the main advantages of each approach and we analyze the three sources of error in the reconstructed output of such a filter bank: amplitude distortion, phase distortion and aliasing.

2 Obtaining the filter bank

The filter banks that we analyze in this work uses the same scheme of modulation to obtain the analysis filters $h_k[n]$ and synthesis filters $f_k[n]$. The two forms use different prototype filters, such as we explain in next subsection.

2.1 Prototype Filter Design

In spectral Factorization approach to pseudo-QMF design [6] the prototype filter is obtained as a spectral factor of a valid 2*M*th band filter [3]. One of the main features of the previous method is that initial filter can be designed by using the standard filter design techniques such as eigenfilter approach or the window-based filter design, without need of optimization. However, the main problem to design the filter bank is in this stage. If we want a prototype filter of length (*N*+1), we must design a 2*M*th band filter of length (2*N*+1), and we must obtain a spectral factor [1]. It is known that finding a spectral factor is, in general, a non-trivial problem, especially when filter has lots of zeros near to the unit circle.

Another alternative approach obtain the analysis filters $h_k[n]$ and the synthesis filters $f_k[n]$ using a Type I or Type II symmetric linear phase low pass filter as a prototype $p[n]$, without restriction in its order. The only constraint is that we must design $p[n]$ with the cutoff frequency at $\omega_c=\pi/2M$.

2.2 Obtaining the analysis and synthesis filters

Let $p[n]$ be the real coefficients prototype filter designed using the methods discussed before.

The impulse response coefficients of the analysis and synthesis filters $h_k[n]$ and $f_k[n]$ are obtained as follows:

$$h_k[n] = \begin{cases} s_k[n] & k \text{ even} \\ s_k[N-1-n] & k \text{ odd} \end{cases} \qquad (1)$$
$$f_k[n] = h_k[N-1-n]$$

where

$$s_k[n] = 2p[n]\cos\left(\left(k+\frac{1}{2}\right)\frac{\pi}{M}n + \phi_k\right)$$
$$\begin{cases} 0 \le n \le N-1 \\ 0 \le k \le M-1 \end{cases}$$

3 The Overall Distortion Transfer Function

Two sources of error in the reconstructed output, amplitude and phase distortions, can be analyzed in the overall distortion transfer function $T_0(z)$.

In order to evaluate the amplitude distortion, it is very useful to find a closed expression for $T_0(z)$, because the magnitude response $\left|T_0\left(e^{j\omega}\right)\right|$ is required to be exactly or approximately flat.

In general, the overall distortion transfer function can be obtained as

$$T_0(z) = \mathbf{f}(z)\cdot \mathbf{H}_{\mathbf{m}}^{1}(z) = \sum_{k=0}^{M-1} F_k(z) H_k(z)$$

where $\mathbf{f}(z)$ is the synthesis bank vector and $\mathbf{H}_{\mathbf{m}}^{k}(z)$ denotes the k-th column for the modulation or alias component matrix for the analysis bank [3].

In the spectral factorization approach, the overall transfer function $T_0(z)$ can be expressed as

$$T_0(z) = \frac{z^{-(N-1)}}{M}\cdot c + \frac{z^{-(N-1)}}{M}\cdot\left(P_1(z)+P_2(z)\right)$$

where c is a constraint and the magnitude responses of $P_1(z)$ and $P_2(z)$ are only significant in certain frequency regions [6].

If the prototype filter $P(z)$ is a symmetric linear-phase filter [8], the distortion transfer function can be expressed by (2).

Using expression (2), we can evaluate in a detailed way the magnitude response $\left|T_0\left(e^{j\omega}\right)\right|$ and the influence of the initial and final phase factors ϕ_0 and ϕ_{M-1} in this magnitude response, and therefore, in amplitude distortion.

$$T_0(z) = \sum_{k=0}^{2M-1} P^2\left(zW_{2M}^{(k+1/2)}\right)\cdot W_{2M}^{(k+1/2)(N-1)} +$$
$$P\left(zW_{2M}^{1/2}\right)\cdot P\left(zW_{2M}^{-1/2}\right)\cdot 2\cdot\cos\left(\frac{1}{2}\cdot\frac{\pi}{M}\cdot(N-1)+2\phi_0\right)+$$
$$P\left(zW_{2M}^{(M-1/2)}\right)\cdot P\left(zW_{2M}^{-(M-1/2)}\right)\cdot 2\cdot\cos\left(\left(M-1+\frac{1}{2}\right)\cdot\frac{\pi}{M}\cdot(N-1)+2\phi_{M-1}\right)$$
(2)

Figures 1 and 2 show the magnitude response of $T_0(z)$ for eight-channel pseudo-QMF cosine modulated filter banks designed in different manners. The prototype filters are *n65fe* in Figure 1 and *hny8c* in Figure 2 (see Table 1).

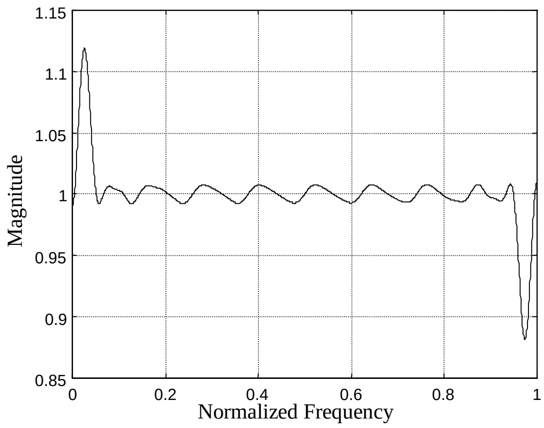

Fig. 1: magnitude response of $T_0\left(e^{j\omega}\right)$ (spectral factor prototype).

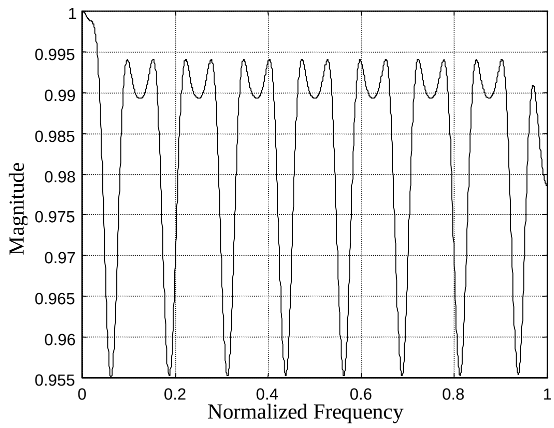

Fig. 2: magnitude response of $T_0\left(e^{j\omega}\right)$ (linear-phase prototype).

Regarding phase distortion, the above choice (expression (1)) for the synthesis filters is essential because it ensures the linearity in the phase response of $T_0(z)$.

4 The Aliasing Function

Aliasing is the third error that we can find in the reconstructed signal of a filter bank and it is introduced because real filters have nonzero transition bandwidth and stopband gain. When the phase factors ϕ_k are chosen appropriately and the stopband attenuation is sufficiently large, the effect of aliasing may not be serious and we can consider the system "approximately" alias-free. Anyhow, aliasing contributions in the output signal can be measured through the aliasing functions:

$$T_\ell(z) = \mathbf{f}(z) \cdot \mathbf{H}_\mathbf{m}^{\ell+1}(z) = \sum_{k=0}^{M-1} F_k(z) H_k\left(zW_M^\ell\right)$$
$$\ell = 1, \cdots, M-1$$

In spectral factorization approach there is not a general formula for $T_\ell(z)$. However, it is demonstrated that, if the prototype order is a multiple of the number of channels *M*, i. e., *(N-1)=mM*, where *m* is a positive integer, it is simplified the derivation of the aliasing constraint, and all the significant aliasing terms are canceled when ϕ_k are chosen such that

$$\phi_{k+1} = \pm(2i+1)\frac{\pi}{2} - \phi_k \qquad 0 \le k \le M-2$$

where i is an integer. The last expression is satisfactory and valid as long as the scheme of modulation given by expression (1) is used.

When the prototype is a linear-phase filter, the aliasing functions are

$$T_\ell(z) = \sum_{k=0}^{M-1}\Big(t_k P\left(zW_{2M}^{(k+1/2)}\right)P\left(zW_{2M}^{-(k+1/2-2\ell)}\right) + t_k^* P\left(zW_{2M}^{-(k+1/2)}\right)P\left(zW_{2M}^{(k+1/2+2\ell)}\right)\Big) \qquad (3)$$

where

$$t_k = \begin{cases} a_k^{*2} W_{2M}^{(k+1/2)(N-1)} & k \text{ par} \\ a_k^{\,2} W_{2M}^{-(k+1/2)(N-1)} & k \text{ impar} \end{cases}$$

Expression (3) is useful in order to minimize aliasing in the output signal.

From functions $T_\ell(z)$, a new aliasing function $T_{al}\left(e^{j\omega}\right)$ is defined [3] as

$$T_{al}(z) = \left(\sum_{\ell=1}^{M-1}\left|\frac{1}{M}\mathbf{f}(z)\mathbf{H}_\mathbf{m}^{\ell+1}(z)\right|^2\right)^{\frac{1}{2}} = \sqrt{\sum_{\ell=1}^{M-1}\left|\frac{1}{M}T_\ell\left(e^{j\omega}\right)\right|^2} \qquad (4)$$

Figures 3 and 4 show the aliasing functions $T_{al}\left(e^{j\omega}\right)$ when the prototype filters are *n65fe* and *hny8c* (see Table I).

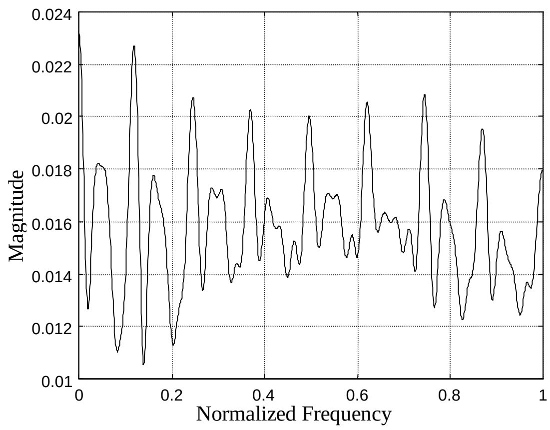

Fig. 3: magnitude response of $T_{al}\left(e^{j\omega}\right)$ *(spectral factor prototype).*

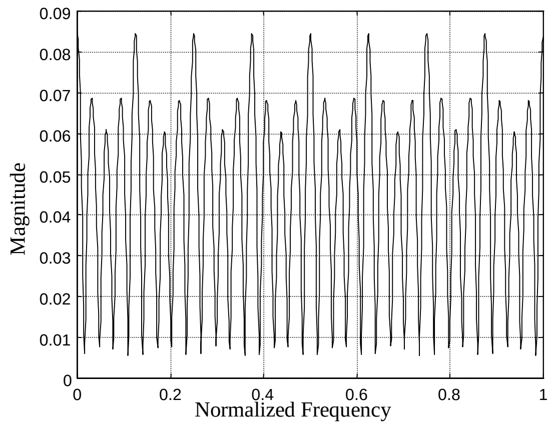

Fig. 4: magnitude response of $T_{al}\left(e^{j\omega}\right)$ *(linear-phase prototype).*

5 Examples of pseudo-QMF banks

We can use two measures in order to establish the filter bank quality [3], [9]. First, maximum peak to peak ripple R_{pp} over all ω (expression (5)) is an effective measure of amplitude distortion because it is the worst possible amplitude distortion. $\left|T_0\left(e^{j\omega}\right)\right|_{MAX}$ and $\left|T_0\left(e^{j\omega}\right)\right|_{MIN}$ indicate the maximum and the minimum values respectively of the magnitude response of the overall distortion transfer function.

$$R_{pp} = 20\log\left(E_{pp}\right) = 20\log\left(\left|T_0\left(e^{j\omega}\right)\right|_{MAX} - \left|T_0\left(e^{j\omega}\right)\right|_{MIN}\right) \tag{5}$$

Second, the quantity E_a is defined as the maximum value of the aliasing function $T_{al}\left(e^{j\omega}\right)$ and it is useful in order to measure aliasing distortion because it is the worst possible *peak aliasing distortion*.

Moreover, in order to measure the differences between the reconstructed signal $\hat{x}[n]$ and the original signal $x[n]$ we have used the percent root-mean-square difference (PRD) [2] and the signal noise ratio (SNR)

$$SNR = 10\log\left(\frac{\sum\left(x[n]\right)^2}{\sum\left(\hat{x}[n] - x[n]\right)^2}\right)$$

As an example, we have designed *8*-channels pseudo-QMF cosine-modulated filter banks using five prototype filters $p[n]$. Several sound signals (marimba.wav, moonrive.wav, quest.wav and morning.wav), of which energy is distributed over all ω, have been applied as input signals to the filter banks. We have measure the quantities given before: PRD, SNR, R_{pp} and E_a.

Table 1 shows the standard filter design techniques used to obtain the prototype filters, and the method used to obtain the filter banks: spectral factorization or linear-phase prototype approaches. Due to space limitations, Table 2 only shows measures in the filter banks when the input signal is marimba.

In all our experiments, the results have been more satisfactory when we use the linear-phase prototype approach.

6 Conclusion

A new approach to design pseudo-QMF cosine modulated filter banks has been introduced, and the expressions of the overall distortion and the aliasing transfer functions have been shown. Using these expressions, it is possible to reduce the amplitude distortion and aliasing. We have used the same scheme of modulation proposed in [6] but different prototype filters: a spectral factor of a valid 2*M*th band filter, and a symmetric linear phase filter. A comparative analysis has been done, and several quantities in filter banks and output signals have been measured. The results have always been more favorable when the prototype is a symmetric linear phase filter.

Table 1: prototype filter characteristics

Filter	Order	Design Technique	Approach
hny8c	64	Eigenfilter	Linear-Phase
hr8c	64	Parks-McClellan	Linear-Phase
n65fe	64	Eigenfilter	Spectral Factor
k65fe	64	Kaiser $\beta = 7.865$	Spectral Factor
Pm65fe	64	Parks-McClellan	Spectral Factor

Table 2: comparison between several pseudo-QMF filter banks

Filter	PRD	SNR	R_{pp}	E_a
hny8c	4.9096	26.1790	0.3994	-21.431
hr8c	7.4426	22.5655	0.6846	-17.7179
n65fe	7.3633	22.6585	2.0713	-32.6639
k65fe	7.2936	22.7412	11.806	-22.383
Pm65fe	7.0617	23.0218	2.4639	-35.8334

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