# Sixth-order switched-capacitor bandpass filter with reduced capacitor spread

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*Abstract:* **A** sixth-order bandpass SC filter based on conventional SC integrators is considered. A disadvantage of this filter is the relatively large capacitor spread (87.1) which is impractical for the monolithic integration. Subsequently the conventional integrators are replaced by gain-and-offset-compensated SC integrators, proposed by Shafeeu et al., for reducing the effects of op amp imperfections. Afterwards the capacitor spread is reduced to 23.73 by replacing one of the integrators by the VLT integrator, proposed by Nagaraj. The effect of these consecutive replacement on the amplitude response and on the output offset voltage of the filter is investigated. Simulation results are presented to confirm the analysis.

Key-Words: Circuit theory and design, Gain-and-offset-compensated switched-capacitor filters, Reduced capacitor spread IMACS/IEEE CSCC'99 Proceedings, Pages:1601-1606

### **1** Introduction

The performance of switched-capacitor (SC) circuits is limited by the characteristics of the operational amplifiers (op amps) embedded in them. Two parameters are particularly important: the settling time  $t_s$  and the voltage dc gain  $A_0$ . Frequency of operation is usually limited by the settling time which restricts the clock frequency to about 20% of the gain-bandwidth product GB of the op-amp. On the other hand, an optimum trade-off between speed and gain is a critical design aspect because they are contradictory requirements. Generally, the price to be paid for high speed is low dc gain A<sub>0</sub>. The problem is especially important for SC circuits implemented in GaAs technology where simple and fast and relatively low-gain amplifiers may be used. Amplifier gain of 60 dB is normally considered minimum for an SC circuit, whereas 40 dB is a more common figure for GaAs designs. Application of such amplifiers (A<sub>0</sub>=100) causes significant deterioration of the performance of classical SC building blocks. One of the serious problems with GaAs technology is the large offset voltages that occur. A nonzero input-referred op amp offset voltage V<sub>0S</sub>, which can be considered constant, introduces an output offset voltage which may become a significant limitation to the permissible signal swing. New integrator and gain stages with reduced sensitivity to both the offset and the finite gain of the op amps have been published.

In order to compare two different filter designs the capacitor spread is most often used as performance criterion. Large capacitance ratios (say, larger than 30) are difficult to achieve with any accuracy in integrated form [1]. To maintain an acceptable matching accuracy the smallest capacitor must not be made smaller than a certain minimum size which depends on the technology used. On the other hand, the realization of the largest capacitor will require a correspondingly large chip area. Thus it is important to keep both extremes within bounds set by the technology used. To reduce the capacitance spread of SC circuits different techniques have been Virtually proposed [2-5]. all area-efficient implementations for very large time constant (VLT) integrators generally suffer from higher sensitivity to finite amplifier gain and the offset voltages and the 1/f noise of the op amps. This occurs because all these implementations involve some form of input attenuation. In [3] a finite gain-and-offsetcompensated VLT integrator have been described making use of correlated double sampling.

In this paper a sixth-order bandpass SC filter based on conventional SC integrators is

considered. A disadvantage of this filter is the relatively large capacitor spread (87.1). Subsequently the conventional integrators are gain-and-offset-compensated replaced bv SC integrators for reducing the effects of op amp imperfections. Afterwards, the capacitor spread is reduced to 23.73 by replacing one of the integrators by the VLT integrator proposed by Nagaraj [3]. The output offset voltage  $V_{0S-out}$  of the last filter configuration, due to the input-referred dc op-amps offset voltages V<sub>0S</sub> , is cancelled by insertion of a gain-and-offset-compensated sample-hold-buffer in the filter circuit. The effect of these consecutive replacement on the amplitude response and on the output offset voltage of the filter is investigated. In the following the bandwidth of the op amps is assumed to be sufficiently large to guarantee in any case a settling time lower than the sampling period. Then, a linear frequency-independent approximation for the voltage gain of the amplifiers can be used

$$A(j\omega) = -A_0 \tag{1}$$

This assumption is adequate for the analysis of SC circuits containing fast and relatively low-gain amplifiers.

# 2 Sixth-order bandpass SC filter with conventional integrators

The circuit diagram of a sixth-order bandpass SC filter is shown in Fig.1 [1]. The circuit is scaled for maximum dynamic range and output swing. The waveform of two nonoverlapping clock signals  $\Phi_1$  and  $\Phi_2$  is sketched in Fig.2.



Fig.1 Sixth-order SC bandpass filter with conventional integrators

The normalized capacitor values are:

$C_1 = 87.10$	$C_2 = 71.01$	$C_3 = 87.10$
$C_4 = 87.10$	$C_5 = 87.10$	$C_6 = 68.26$
$C_{11}=84.20$	$C_{21}=11.90$	C <sub>31</sub> =9.53
$C_{41} = 13.04$	C <sub>51</sub> =17.03	$C_{61}=18.00$
C <sub>12</sub> =72.17	C <sub>22</sub> =16.70	C <sub>32</sub> =11.61
$C_{42} = 7.82$	C <sub>52</sub> =8.73	C <sub>62</sub> =9.75
$C_{13}=1.00$	C <sub>23</sub> =22.26	$C_{33}=7.48$
$C_{43}=3.67$	C <sub>53</sub> =25.39	
•	1: 07 10	

The capacitor spread is 87.10.

The filter specifications are:

- Clock frequency  $f_C=100 \text{ kHz}$ ;
- Lower passband edge 300 Hz ;
- Upper passband edge 3400 Hz ;
- Maximum passband ripple  $|\Delta a| < 0.4 \text{ dB}$ ;
- Lower stopband edge 10 Hz ;
- Lower stopband attenuation > 20 dB;

Upper stopband edge 5000 Hz ; Upper stopband attenuation > 45 dB



Fig.2 Waveform of the clock

The ideal transfer function (Fig. 3) has a characteristic asymmetric shape owing to two finite

zeros in the upper stopband. Let us suppose that the capacitors and the switches are ideal. The op amps are also assumed to be ideal except for a finite voltage gain described by the relation (1) and a nonzero input-referred dc offset voltage  $V_{0S}$ , which is modelled as a voltage source at the noninverting input terminal.



Fig.3 Overall response of the filter

The input voltage of the filter  $V_i$  is read during the clock phase  $\Phi_2$  and the output voltage  $V_0$  is sampled in the clock phase  $\Phi_1$ . The amplitude responses of the filter in the passband for two cases are shown in Fig.4 :

- (i)  $H_{id}(f)$  ideal op amps with  $A_0 \rightarrow \infty$ ;
- (ii)  $H_c(f)$  nonideal op amps with  $A_0=100$ .



Fig.4 Passband responses of the filter

In the first case the maximum passband ripple is  $\Delta a = -0.34$  dB at frequency f=300 Hz. For nonideal op amps with A<sub>0</sub>=100 the maximum passband ripple is  $\Delta a = -2.06$  dB at 3.4 kHz. Obviously in the second case the attenuation of -0.4 dB is exceeded. In order to meet the specification of the filter it is necessary to :

(i) use op amps with higher dc gain  $A_0$ ;

(ii) investigate techniques for compensating the finite gain effect.

One approach for such a compensation is to replace the conventional (uncompensated) integrators with amplifier gain insensitive SC integrators.

The influence of the input-referred dc offset voltages  $V_{0Sq}$  (q =1÷6) is evaluated by the corresponding output voltage  $V_{00}^{1}$  in steady state for  $V_i=0$ . For  $A_0 \rightarrow \infty$  one finds

$$V_{\text{ooid}}^{1} \frac{C_{31}C_{51}}{C_{32}C_{52}} V_{\text{OS}_{1}} - \frac{C_{51}}{C_{52}} (1 + \frac{C_{31}}{C_{32}}) V_{\text{OS}_{3}} + (1 + \frac{C_{51}}{C_{52}}) V_{\text{OS}_{5}}$$
  
This gives

 $V_{ooid}^{1} = 1.601 V_{OS_{1}} - 3.552 V_{OS_{3}} + 2.951 V_{OS_{5}}$ .

It is evident that the total offset of the filter is a weighted sum of the individual offset terms contributed by different paths through which an amplifier offset travels to the output. Each weighting function is the transfer function from a particular offset source to the output through a particular path.

By computer simulation for  $A_{0q}=100$  (q =1÷6) one obtains

$$V_{00A0}^{1} = 1.545 V_{0S_{1}} + 0.0371 V_{0S_{2}} - 3.428 V_{0S_{3}} - 0.0895 V_{0S_{4}} + 2.849 V_{0S_{5}} + 0.0957 V_{0S_{6}}$$

### **3** SC filter with amplifier gain insensitive SC integrators

Fig.5 shows the circuit diagram of a sixth-order bandpass SC filter with reduced sensitivity to op amps dc gain A<sub>0</sub> and offset voltage V<sub>0S</sub>. This filter is synthesized by the replacement of the conventional integrators (Fig.1) with amplifier gain insensitive SC integrators proposed by Shafeeu et al. [6]. In addition to the clock phases  $\Phi_1$  and  $\Phi_2$ , the new integrators require two nonoverlapping clocks,  $\Phi_e$ and  $\Phi_o$ , as shown in Fig.2. During the subintervals  $\Phi_{2o}=1$  (nT<sub>c</sub> - 0.25T<sub>c</sub><sup>-</sup>  $\leq$  t  $\leq$  nT<sub>c</sub><sup>-</sup>) of the clock phase  $\Phi_2$ , and also during the subintervals  $\Phi_{1o}=1$ (nT<sub>c</sub> + 0.25T<sub>c</sub><sup>+</sup>  $\leq$  t  $\leq$  nT<sub>c</sub> + 0.5T<sub>c</sub><sup>-</sup>) of the clock phase  $\Phi_1$ , the potential at node 1' is given by :

$$V_{1'}^{2o}(t) = -\frac{1}{A_{01}} V_{o1}^{2o}(nT_{c} - 0.25T_{c}^{-}) + \frac{1}{A_{01}} V_{o1}^{2e}(nT_{c} - 0.5T_{c}^{-}),$$

$$V_{1'}^{1o}(t) = -\frac{1}{A_{01}} V_{o1}^{1o}(nT_{c} + 0.25T_{c}^{+}) + \frac{1}{A_{01}} V_{o1}^{1e}(nT_{c}^{+}).$$
(2)



Fig.5 Sixth-order bandpass SC filter with Shafeeu-91 SC integrators

During the subintervals  $\Phi_0=1$  of  $\Phi_1$  and  $\Phi_2$  the node 1' behaves as a perfect (super) virtual ground; this node is the summing point of the equivalent amplifier. The potential of the node 1' is very close to zero because  $V_{o1}^{2o} \approx V_{o1}^{2e}$  and  $V_{o1}^{1o} \approx V_{o1}^{1e}$ . This can be viewed as an enhancement of the effective dc gain of OA<sub>1</sub>. Similarly one can explain this finite gain "predict and correct" compensation technique for op amps OA<sub>2</sub>-OA<sub>6</sub>.

In order to determine the transfer function of the filter the even ( $\Phi_e$ ) and the odd ( $\Phi_o$ ) subintervals of the two clock phases  $\Phi_2$  and  $\Phi_1$  are subsequently considered. At the beginning of the subintervals  $\Phi_{2e}=1$  of  $\Phi_2$  and  $\Phi_{1e}=1$  of  $\Phi_1$  the charge-conservation law is applied to the op amps inverting input terminals. At the beginning of the subintervals  $\Phi_{2o}=1$  of  $\Phi_2$ , and  $\Phi_{1o}=1$  of  $\Phi_1$  this law is applied to the inverting input terminals 1+6, as well to the nodes 1'+6'. Taking into account the relation (1) one arrives at a system of 24 difference equations. Taking the z-transform of the obtained system for  $V_{0s}=0$  one arrives in matrix form

$$\left[A(z)\right]_{24\times 24} \left[V_0(z)\right]_{24\times 1} = \left[B(z)\right]_{24\times 1} V_i^{2e}(z)$$
(3)

In (3),  $z = \exp(j2\pi f/f_C)$  is substituted. For each value of current frequency f a linear system with complex coefficients of 24 th-order is obtained. This system can be solved by a computer for

$$H_{s}(f) = \frac{V_{o6}^{o}(f)}{V_{i}^{2e}(f)} .$$
 (4)

The values of holding capacitors  $Ch_q$  (q=1÷6) are

not crucial and can generally be set equal to one unit (the smallest) capacitor giving a very small amplitude error [6]. In this case  $Ch_q=C_{13}=1$  are chosen.

The magnitude response of the investigated filter  $H_{s}(f)$  for op amps with  $A_{0}=100$  is shown in Fig. 7. The maximum passband ripple is  $\Delta a = -0.35$  dB at frequency f=300 Hz. The curve  $H_{s}(f)$  is compared with the ideal response  $H_{id}(f)$  (Fig.7). The output voltage  $V_{oo}^{10}$  in steady state is :

(i) 
$$V_{oo id}^{10} = 0$$
  
 $V_{oo Ao}^{10} = 0.0523V_{OS1} - 0.014V_{OS2} - 0.0567V_{OS3} + 0.0035V_{OS4} + 0.0454V_{OS5} - 0.0008V_{OS6}$   
for A<sub>0</sub>=100.

# 4 Sixth-order bandpass SC filter with reduced capacitor spread

The capacitor spread of the filter can be reduced by replacing the left-most integrator in Fig.5 by the one proposed by Nagaraj for the realization of large time constants (Fig.5 in [3]). The circuit diagram of the resulting filter is shown in Fig.6 (without the elements enclosed in broken line).

The capacitor values  $C_{13} = C_1$  calculated from the design equation

$$\frac{C_{13}^{2}}{C_{1}(C_{1}+C_{13})} = \frac{C_{13}}{C_{1}} = \frac{1}{87.10}$$
(5)



Fig.6 Sixth-order bandpass SC filter with reduced capacitor spread

are  $C_{13}=C_1=9.8461222$ . Hence, the capacitor spread is reduced to 87.10/3.67 = 23.73. The value of the holding capacitor Ch<sub>1</sub> is not critical and can generally be set equal to one unit capacitance [3]. Here Ch<sub>q</sub>=C<sub>43</sub>=C<sub>min</sub>=3.67 are chosen.

The magnitude response of the filter  $H_a(f)$  in the passband for nonideal op amps with  $A_0=100$  is shown in Fig.7. The maximum passband ripple is  $\Delta a = -0.3$  dB at frequency f=1.4 kHz.



Fig.7 Passband responses of the filters from Fig.5 and Fig.6

The output voltage  $V_{00}^{10}$  in steady state is :

(i) 
$$V_{oo_{oo_{id}}}^{10} = -2.1032 V_{OS_2} + 0.5231 V_{OS_4} - 0.0212 V_{OS_6}$$
  
 $V_{oo_{AO}}^{10} = 0.3006 V_{OS_1} - 2.0718 V_{OS_2} - 0.0568 V_{OS_3} + 0.5048 V_{OS_4} + 0.0453 V_{OS_5} - 0.0207 V_{OS_6}$ 

The influence of the offset voltage  $V_{\rm OS_2}$  can be reduced by the insertion a gain-and-offset-compensated sample-and-hold buffer [7], enclosed in broken line (Fig.6). The following expression for the output voltage  $Vo_7^{2e}(nT_C-0.5T_C^-)$  of the op amps  $OA_7$  at the beginning of the subinterval  $\Phi_{2e}=1$   $(nT_C-0.5T_C^-\leq t\leq nT_C-0.25T_C^+)$  can be found :  $Vo_7^{2e}(nT_C-0.5T_C^-)=a\,Vo_2^{1o}(nT_C-0.75T_C^-)+b\,V_{087}$  where

$$a = \frac{A_{07}(A_{07} + 2)}{(1 + A_{07})^2} , \quad b = \frac{A_{07}}{(1 + A_{07})^2}$$

For  $A_{07}$ =100 one finds

$$Vo_7^{2e}(nT_C - 0.5T_C^-) =$$

 $= 0.99990197 \, Vo_2^{10} (nT_c - 0.75T_c^-) + 0.0098 \, V_{0S_7} .$ 

The only nonidealities of the circuits components (op amps, switches and capacitors) that are taken into account are the finite dc gain  $A_0$  and the offset voltage. Thus the values of the capacitances  $Ch_7$  and  $C_H$  do not affect the transfer function of the filter. A systematic and detailed analysis requires considering other limiting factors of the op amps such as : non- zero output resistance, finite input resistance, parasitic capacitances. It is also necessary to have in mind the stray capacitances of capacitors, on-and off-resistances of switches. In this case the values of  $Ch_7$  and  $C_H$  depend on the values of the corresponding time constants and the clock period  $T_C=1/f_C$ .

The magnitude response of the filter with the sample-and-hold buffer  $H_b(f)$  in the passband for nonideal op amps with  $A_0=100$  is shown in Fig.7. The maximum passband ripple is  $\Delta a = -0.347$  dB at frequency f=1.4 kHz. The curve  $H_b(f)$  practically converges in the scale chosen to the response  $H_s(f)$ .

The output voltage  $V_{oo}^{1o}$  in steady state is :

(i) 
$$V_{oo id}^{10} = 0$$
  
 $V_{oo Ao}^{10} = 0.2959 V_{OS_1} + 0.00012 V_{OS_2} - 0.00567 V_{OS_3} - 0.000017 V_{OS_4} + 0.0453 V_{OS_5} - 0.00066 V_{OS_6} + 1.2.10^{-7} V_{OS_7}$ 

#### **5** Conclusion

A 6th-order SC bandpass filter with reduced capacitance spread has been presented. Gain-and-offset-compensation technique was applied to improve the performance.

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