Harmonic Components Detection in Stochastic Sequences Using Artificial Neural Networks

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Abstract: The problem of hidden harmonic components detection in stochastic sequences occurs quite often in practice and, first of all, in technical diagnostics and monitoring of signals of various natures. In the present paper an adaptive approach based on use of artificial neural networks is introduced. It provides a real-time detection and parameters estimation of an arbitrary number of harmonic components in a monitored stochastic sequence, as well as detection of new harmonics appearance in the signal and disappearance of the present ones (fault detection).

The solution of the problem is based on a multistage scheme, comprising of determination of the main harmonic frequency, determination of the amplitude and phase of the main harmonic, exclusion of the main harmonic from the signal; determination of the second harmonic frequency of the monitored signal (the main harmonic of the innovation signal), determination of parameters of the second harmonic, exclusion of the second harmonic from the innovation signal, and so on. When the sequence of residues becomes a white noise the process ends.

Then by means of an adaptive multimodel approach the "weight" of each harmonic in the monitored signal is estimated and a polyharmonic deterministic sequence is synthesized that best approximates the source signal.

The introduced approach is implemented by means of a multilayer artificial neural network. Neurons of the first hidden layer are tuned by means of a learning algorithm that has filtering and smoothing properties simultaneously. The smoothing parameter adjustment makes it possible to get a spectrum of procedures from Widrow-Hoff algorithm to Goodwin-Ramadge-Caines algorithm (stochastic approximation). In this layer parameters of autoregression equations with complex conjugate roots corresponding to different harmonics are estimated. In the second hidden layer parameters of individual harmonics are calculated. The output layer of the neural network consists of a single element, being in fact an elementary Adaline. It is however adjusted not by means of the traditional delta-rule, but on the basis of the proposed learning algorithm that takes into account constraints on "weights" of harmonics. These "weights" describe the influence of each harmonic on the monitored signal and provide unbiasedness of the approximating sequence.

Monitoring of changes of these "weights" provides a real-time detection of parametric and structural faults of the source signal.

The convergence and optimality of this algorithm are proved. Results of simulation confirm efficiency of the proposed approach that provides both high speed and high sensitivity to changes of the signal.

Key-Words: hidden harmonic components, stochastic sequence, real-time detection, technical diagnostics, fault detection, artificial neural networks. *IMACS/IEEE CSCC'99 Proceedings,* Pages:1421-1425

1 Introduction

The problem of hidden harmonic components detection in stochastic sequences occurs quite often in practice and, first of all, in technical diagnostics and monitoring of signals of various natures [1-4]. In the present paper an adaptive approach based on use of artificial neural networks is introduced [5]. It provides a real-time detection and parameters estimation of an arbitrary number of harmonic components (possibly of aliquant frequencies) in a monitored stochastic sequence, as well as detection of new harmonics appearance in the signal and disappearance of the present ones (fault detection).

2 Statement of the problem

Assume that the analyzed stochastic sequence can be presented in a form

$$y_k = \sum_{j=1}^{m} \left(a_j \cos \omega_j k + b_j \sin \omega_j k \right) + \xi_k , \qquad (1)$$

where m – rather great number, assigning the possible number of harmonic components; a_j, b_j – unknown parameters of separate harmonics; $0 < \omega_j = 2\pi f_j T_0 < \pi$ – unknown frequencies to be estimated; T_0 – signal quantification period; $k = 0, 1, 2, \ldots, N$ – discrete time; ξ_k – stochastic component, a white noise with zero mean and finite variance.

Let us put in correspondence to (1) a model

$$\begin{aligned} \hat{y}_{1,k} &= \hat{a}_1 \cos \hat{\omega}_1 k + \hat{b}_1 \sin \hat{\omega}_1 k = \\ &= 2 \cos \hat{\omega}_1 y_{k-1} - y_{k-2} = \hat{\beta}_1 2 y_{k-1} - y_{k-2}, \end{aligned} \qquad (2)$$

describing a monoharmonic oscillation with parameters $\hat{a}_1, \hat{b}_1, \hat{\omega}_1$. Unknown parameters of the model (2) can be estimated by means of a simple two-stage procedure. On the first stage the estimation criterion is introduced

$$J_1^{\beta} = \sum_{k=3}^{N} \left(y_k + y_{k-2} - \hat{\beta}_1 2 y_{k-1} \right)^2 \qquad (3)$$

and the frequency is estimated by its minimization:

$$\begin{cases} \hat{\beta}_{1,N} = \dfrac{\sum_{k=3}^{N} \left(y_k + y_{k-2} \right) y_{k-1}}{2 \sum_{k=3}^{N} y_{k-1}^2}, \\ \hat{\omega}_{1,N} = \arccos \hat{\beta}_{1,N}, \end{cases} \qquad (4)$$

then, on the second stage, parameters $\hat{a}_1$ and $\hat{b}_1$ are estimated by minimization of the criterion

$$J_1^{\alpha} = \sum_{k=3}^{N} \left(y_k - \hat{a}_1 \cos \hat{\omega}_{1,N} k + \hat{b}_1 \sin \hat{\omega}_{1,N} k \right)^2 . \qquad (5)$$

It is straightforward to see that

$$\begin{pmatrix} \hat{a}_{1,N} \\ \hat{b}_{1,N} \end{pmatrix} = \begin{pmatrix} \sum_{k=3}^{N} \cos^2 \hat{\omega}_{1,N} k & \sum_{k=3}^{N} \cos \hat{\omega}_{1,N} k \sin \hat{\omega}_{1,N} k \\ \sum_{k=3}^{N} \cos \hat{\omega}_{1,N} k \sin \hat{\omega}_{1,N} k & \sum_{k=3}^{N} \sin^2 \hat{\omega}_{1,N} k \end{pmatrix} \times \begin{pmatrix} \sum_{k=3}^{N} \cos \hat{\omega}_{1,N} k \\ \sum_{k=3}^{N} \sin \hat{\omega}_{1,N} k \end{pmatrix} . \qquad (6)$$

3 Adaptive estimation of harmonic components parameters

With the objective of early diagnostics of monitored signal the problem of parameters estimation should be solved in real time using the exponentially weighted stochastic approximation procedure [6]. It provides a compromise between filtering and tracking capabilities of the adaptive identification process. If after N observations the approximations $\hat{a}_{1,N}, \hat{b}_{1,N}, \hat{\omega}_{1,N}$ are obtained, then with the reception of the $(N+1)$-th observation a correction according to a recursive algorithm is produced:

$$\begin{cases} \hat{\beta}_{1,N+1} = \hat{\beta}_{1,N} + r_{1,N+1}^{-1} \left(y_{N+1} + y_{N-1} - \hat{\beta}_{1,N} 2 y_N \right) 2 y_N , \\ r_{1,N+1} = \alpha r_{1,N} + 4 y_N^2 , \\ \hat{\omega}_{1,N+1} = \arccos \hat{\beta}_{1,N+1} , \\ \left(\hat{a}_{1,N+1} ; \hat{b}_{1,N+1} \right)^T = \left(\hat{a}_{1,N} ; \hat{b}_{1,N} \right)^T + R_{1,N+1}^{-1} \left(y_{N+1} - \right. \\ \left. \hat{a}_{1,N} \cos \hat{\omega}_{1,N+1} (N+1) - \hat{b}_{1,N} \sin \hat{\omega}_{1,N+1} (N+1) \right) \times \\ \times \left(\cos \hat{\omega}_{1,N+1} (N+1) ; \sin \hat{\omega}_{1,N+1} (N+1) \right)^T , \\ R_{1,N+1} = \alpha R_{1,N} + 1, \end{cases} \qquad (7)$$

where $0 \le \alpha \le 1$ – smoothing parameter, determining a compromise between filtering and tracking capabilities of the algorithm.

It is clear to see that first two relations in (7) describe the process the frequency parameter $\hat{\beta}_1$ estimation which under $a = 1$ becomes a Goodwin-Ramadge-Caines algorithm [7], and under $a = 0$ – a widespread in adaptive systems theory Kaczmarz algorithm [8, 9]. The last two relations in (7) provide approximation of the parameters $\hat{a}_1$ and $\hat{b}_1$ and under $a = 1$ coincide with the Kiefer-Wolfowitz stochastic approximation algorithm [10], and under $a = 0$ – with a unit step gradient search algorithm.

Then let us introduce an innovations sequence $\tilde{y}_{1,k}$ by excluding the first harmonic from the source signal

$$\tilde{y}_{1,k} = y_k - \hat{y}_{1,k} \tag{8}$$

and let us put in correspondence to it a monoharmonic model.

$$\begin{aligned}\hat{y}_{2,k} &= \hat{a}_2 \cos\hat{\omega}_2 k + \hat{b}_2 \sin\hat{\alpha}_2 k = \\ &= 2\cos\hat{\omega}_2 \tilde{y}_{1,k-1} - \tilde{y}_{1,k-2} = \hat{\beta}_2 2\tilde{y}_{1,k-1} - \tilde{y}_{1,k-2}.\end{aligned} \tag{9}$$

Minimizing the estimation criteria

$$J_2^{\beta} = \sum_{k=3}^{N} \left(\tilde{y}_{1,k} + \tilde{y}_{1,k-2} - \hat{\beta}_2 2\tilde{y}_{1,k-1}\right)^2 \tag{10}$$

and

$$J_2^{\alpha} = \sum_{k=3}^{N} \left(\tilde{y}_{1,k} - \hat{a}_2 \cos\hat{\omega}_{2,N} k + \hat{b}_2 \sin\hat{\alpha}_{2,N} k\right)^2 \tag{11}$$

with a recurrent procedure

$$\begin{cases}\hat{\beta}_{2,N+1} = \hat{\beta}_{2,N} + r_{2,N+1}^{-1}\left(\tilde{y}_{1,N+1} + \tilde{y}_{1,N-1} - \right. \\ \left. - \hat{\beta}_{2,N} 2\tilde{y}_{1,N}\right) 2\tilde{y}_{1,N}, \\ r_{2,N+1} = \alpha r_{2,N} + 4\tilde{y}_{1,N}^2, \\ \hat{\alpha}_{2,N+1} = \arccos \hat{\beta}_{2,N+1}, \\ \left(\hat{a}_{2,N+1}; \hat{b}_{2,N+1}\right)^T = \left(\hat{a}_{2,N}; \hat{b}_{2,N}\right)^T + R_{2,N+1}^{-1}\left(\tilde{y}_{1,N+1} - \right. \\ \left. \hat{a}_{2,N} \cos\hat{\alpha}_{2,N+1}(N+1) - \hat{b}_{2,N} \sin\hat{\omega}_{2,N+1}(N+1)\right) \times \\ \times \left(\cos\hat{\omega}_{2,N+1}(N+1); \sin\hat{\omega}_{2,N+1}(N+1)\right)^T, \\ R_{2,N+1} = \alpha R_{2,N} + 1,\end{cases} \tag{12}$$

the second harmonic signal is obtained

$$\begin{aligned}\hat{y}_{2,N+1} &= \hat{a}_{2,N+1} \cos\hat{\alpha}_{2,N+1}(N+1) + \\ &+ \hat{b}_{2,N+1} \sin\hat{\alpha}_{2,N+1}(N+1)\end{aligned} \tag{13}$$

Then introducing an innovation

$$\tilde{y}_{2,k} = \tilde{y}_{1,k} - \hat{y}_{2,k}, \tag{14}$$

the third harmonic parameters approximations $\hat{\beta}_3, \hat{\alpha}_3, \hat{a}_3, \hat{b}_3$ can be obtained, and so on.

And, at last, for the m-th harmonic

$$\begin{aligned}\hat{y}_{m,k} &= \hat{a}_m \cos\hat{\omega}_m k + \hat{b}_m \sin\hat{\omega}_m k = \\ &= 2\cos\hat{\alpha}_m \tilde{y}_{m-1,k-1} - \tilde{y}_{m-1,k-2} = \\ &= \hat{\beta}_2 2\tilde{y}_{m-1,k-1} - \tilde{y}_{m-1,k-2}.\end{aligned} \tag{15}$$

where

$$\tilde{y}_{m-1,k} = \tilde{y}_{m-2,k} - \hat{y}_{m-1,k}, \tag{16}$$

a recurrent procedure can be written

$$\begin{cases}\hat{\beta}_{m,N+1} = \hat{\beta}_{m,N} + r_{m,N+1}^{-1}\left(\tilde{y}_{m-1,N+1} + \tilde{y}_{m-1,N-1} - \right. \\ \left. - \hat{\beta}_{m,N} 2\tilde{y}_{m-1,N}\right) 2\tilde{y}_{m-1,N}, \\ r_{m,N+1} = \alpha r_{m,N} + 4\tilde{y}_{m-1,N}^2, \\ \hat{\omega}_{m,N+1} = \arccos \hat{\beta}_{m,N+1}, \\ \left(\hat{a}_{m,N+1}; \hat{b}_{m,N+1}\right)^T = \left(\hat{a}_{m,N}; \hat{b}_{m,N}\right)^T + R_{m,N+1}^{-1}\left(\tilde{y}_{m-1,N+1} - \right. \\ \left. \hat{a}_{m,N} \cos\hat{\alpha}_{m,N+1}(N+1) - \hat{b}_{m,N} \sin\hat{\omega}_{m,N+1}(N+1)\right) \times \\ \times \left(\cos\hat{\alpha}_{m,N+1}(N+1); \sin\hat{\alpha}_{m,N+1}(N+1)\right)^T, \\ R_{m,N+1} = \alpha R_{m,N} + 1.\end{cases} \tag{17}$$

The obtained parameters approximations $\hat{\alpha}_j, \hat{a}_j, \hat{b}_j$ of separate harmonics form the corresponding vectors $\hat{\alpha}, \hat{a}, \hat{b}$.

4 Adaptive approximation of multimodel sequence parameters

Let us form a multimodel filtered sequence based on the obtained m harmonics [11]

$$\hat{\hat{y}}_k = \sum_{j=1}^{m} c_j \hat{y}_{j,k} = c^T \hat{y}_k, \tag{18}$$

where $c = (c_1, c_2, \ldots c_m)^T$ – weights vector, describing a “contribution” of each harmonic to the sequence y_k, $\hat{y}_k = \left(\hat{y}_{1,k}, \hat{y}_{2,k}, \ldots, \hat{y}_{m,k}\right)^T$ – $(m\times 1)$-vector, comprising of the obtained harmonics. To provide unbiasedness of $\hat{y}_k$ an additional constraint on weights c_j is introduced

$$\sum_{j=1}^{m} c_j = E^T c = 1, \tag{19}$$

where $E = (1, 1, \ldots 1)^T$ – $(m\times 1)$-vector.

Unknown weights c_j can be found with Lagrange undetermined multipliers method. Introducing $(N\times 1)$ observations vector $Y_N = (y_1, y_2, \ldots y_N)^T$, $(N\times m)$ model signals matrix $\hat{Y}_N = \left(\hat{y}_1, \hat{y}_2, \ldots, \hat{y}_N\right)^T$, error signals

$$\begin{cases}v_{1,k} = y_k - \hat{y}_{1,k} = \tilde{y}_{1,k}, \\ v_{2,k} = y_k - \hat{y}_{2,k}, \\ \vdots \\ v_{m,k} = y_k - \hat{y}_{m,k}, \\ v_k = y_k - \hat{\hat{y}}_k = y_k - c^T \hat{y}_k = c^T E y_k - c^T \hat{y}_k = \\ c^T\left(E y_k - \hat{y}_k\right) = c^T V_k\end{cases} \tag{20}$$

and Lagrangian

$$L(c,\lambda)=\sum_{k=3}^{N}c^TV_kV_k^Tc+\lambda\left(c^TE-1\right)= \\ =c^TW_Nc+\lambda\left(c^TE-1\right) \quad (21)$$

where λ – Lagrange undetermined multiplier, $W_N=\sum_{k=3}^{N}V_kV_k^T$,

and then solving the Kuhn-Tucker equations system [12]

$$\begin{cases}\nabla_c L(c,\lambda)=2W_nc+\lambda E=0,\\ \partial L(c,\lambda)/\partial\lambda=c^TE-1=0,\end{cases} \quad (22)$$

the desired result is obtained in a form

$$\begin{cases}c_N=W_N^{-1}E\left(E^TW_N^{-1}E\right)^{-1},\\ \lambda_N=-2E^TW_N^{-1}E,\\ L(c_N,\lambda_N)=\left(E^TW_N^{-1}E\right)^{-1}\end{cases} \quad (23)$$

and in this connection the following can be proved [11]

$$\sum_{k=3}^{N}v_k^2\le\sum_{k=3}^{N}v_{j,k}^2 \quad (24)$$

for any $j = 1, 2, \ldots, m$. Introducing a matrix $P_N=W_N^{-1}$ and using the lemma of matrix inversion, a real-time estimation of weights c_j can be organized

$$\begin{cases}P_{N+1}=P_N-\dfrac{P_NV_{N+1}V_{N+1}^TP_N}{1+V_{N+1}^TP_NV_{N+1}},\\ c_{N+1}=\dfrac{P_{N+1}E}{E^TP_{N+1}E}.\end{cases} \quad (25)$$

5 Architecture of artificial neural network for harmonic components detection

With large m (m should always be less than $0.5N$) numerical implementation of this approach meets significant difficulties. To overcome them the power of parallel calculations of neural technologies is used.

Figure 1 shows the scheme of a neural spectrum analyzer. It is a three-layer structure implementing the introduced approach.

The first layer consists of elementary neurons of the same type. Each of them contains two back shift elements z^{-1}, two adders $\sum$, and one tuned element $\hat{\beta}_j$. The first neuron of the first layer AN_1^{β} is fed with the signal y_{N+1}, and on the back shift elements signals y_N and y_{N-1} are formed. This neuron forms the model of the first harmonic (2). The synaptic weight $\hat{\beta}_1$ is tuned by the first two relations of the procedure (7). Outputs of the neuron AN_1^{β} are signals $\tilde{y}_{1,N+1}$, $\hat{\beta}_{1,N+1}$, and $\hat{y}_{1,N+1}$. The signal $\tilde{y}_{1,N+1}$ is fed to the input of the second neuron AN_2^{β}. It acts the same way and forms on its outputs signals $\tilde{y}_{2,N+1}$, $\hat{\beta}_{2,N+1}$, $\hat{y}_{2,N+1}$. And, at last, the last neuron of the first layer AN_m^{β} forms signals $\tilde{y}_{m,N+1}$, $\hat{\beta}_{m,N+1}$, $\hat{y}_{m,N+1}$. Thus in the first layer of the neural spectrum analyzer signals $\tilde{y}_{1,N+1}$, $\hat{\beta}_{1,N+1}$, $\hat{y}_{1,N+1}$, $\tilde{y}_{2,N+1}$, $\hat{\beta}_{2,N+1}$, $\hat{y}_{2,N+1}$, …, $\tilde{y}_{m,N+1}$, $\hat{\beta}_{m,N+1}$, $\hat{y}_{m,N+1}$ are calculated in a parallel manner.

These signals are fed to the hidden layer that consists of m similar neurons AN_j^{ω}. Each neuron contains three nonlinear activation functions: $\psi_1=\arccos\hat{\beta}$, $\psi_2=\cos\hat{\omega}k$, $\psi_3=\sin\hat{\omega}k$; two adders and two synaptic weights that are tuned by the last two relations of (17). Each AN_j^{ω} first using ψ_1 calculates $\hat{\omega}_{j,N+1}=\arccos\hat{\beta}_{j,N+1}$, and then using ψ_2 and ψ_3 – $\cos\hat{\omega}_{j,N+1}(N+1)$ and $\sin\hat{\omega}_{j,N+1}(N+1)$ (these elements also receive a signal from an iteration counter – on the scheme ↑). Then with the learning procedure weights $\hat{a}_{j,N+1}$, $\hat{b}_{j,N+1}$ are calculated that describe each of the harmonics being separated.

The output layer of the neural network consists of a single element AN_3, which is in fact an elementary McCulloch-Pitts neuron. It is however tuned not by traditional procedures of Widrow-Hoff algorithm type, but using the algorithm (25) that takes into consideration constraints on the weights (19). These weights define “contribution” of each harmonic $\hat{y}_{j,k}$ into the analyzed process y_k, and their changes allow a real-time detection of parametrical and structural faults of the input signal.

6 Conclusion

Thus, using algorithms (17), (25), it is possible to analyze in real time a spectral composition of the monitored signal, determine parameters of harmonics that form it, a contribution of each harmonic into the signal, and also detect emerging faults by monitoring changes of components of weights vector c. Implementation of the proposed approach using artificial neural networks provides high performance and ease of schematic design of the spectrum analyzer.

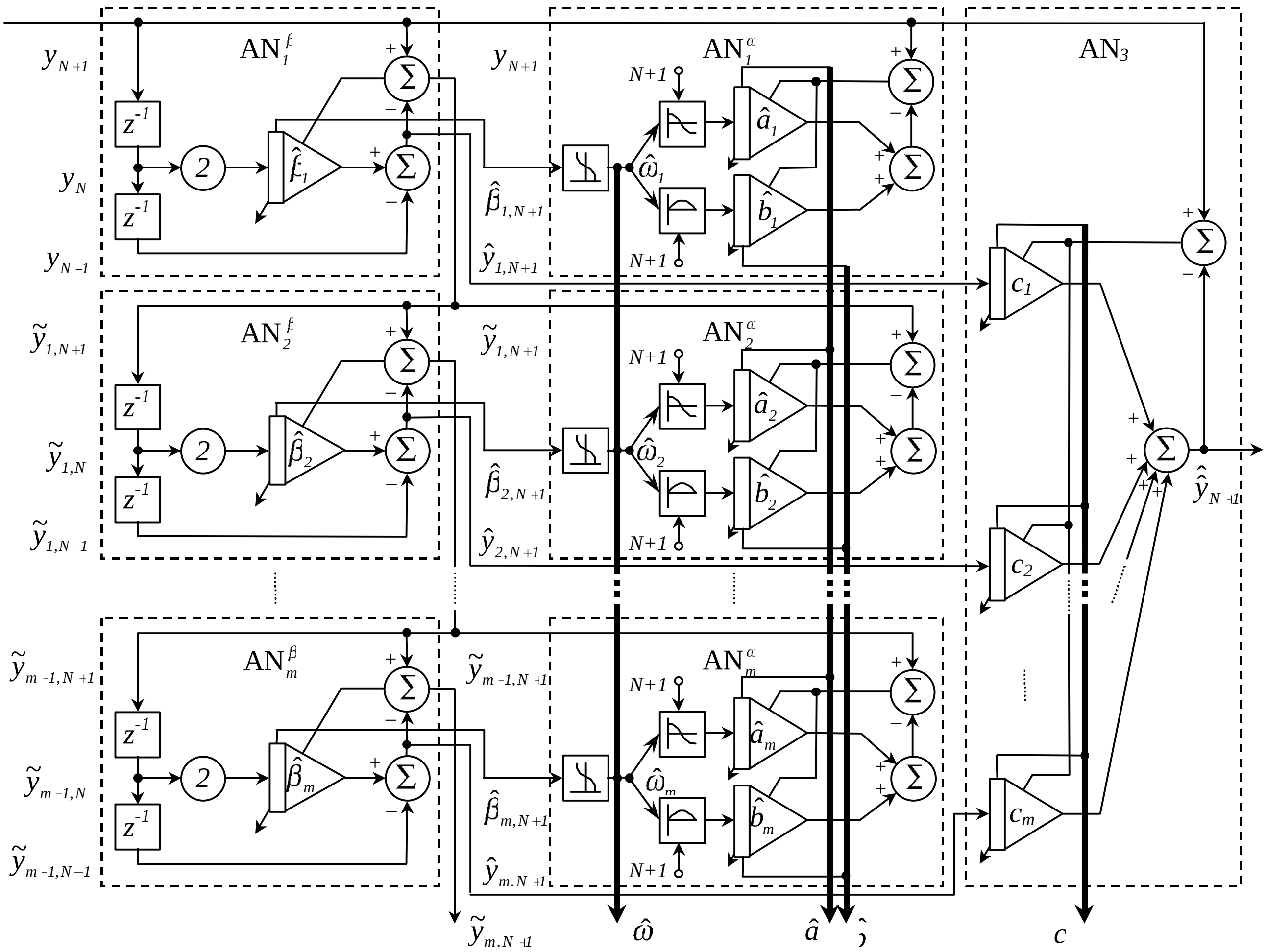

Figure 1 – Neural spectrum analyzer

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