Electromagnetic Dyadic Green's Function of an Implantable Medical Device Model for Numerical EMC Investigation

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Abstract: - Modern wireless telecommunication devices (GSM Mobile system) can interfere with implantable medical devices/prostheses and cause possible malfunction. Also the performance of an antenna is significantly altered by the presence of conducting medical devices/prostheses. The principle objective of this paper is to outline a general expression of dyadic Green's function (DGF) for the problem of electromagnetic radiation from a source of excitation in the presence of a finite length of perfectly conducting circular cylinder of any size as well as of resonant length, which is valid everywhere, including the source region. The whole structure is assumed to be uniform along the propagation direction. The DGFs are obtained by employing the method of scattering superposition.

Key-Words: - Electromagnetic, Circular Cylinder, Implants, Antenna, Dipole, Dyadic Green's function.

1 Introduction

Although electromagnetic scattering by a finite cylinder is a well known cannonical problem, published work does not include the effects of arbitrary placed source point. The derivations presented here are motivated by the need to understand the behaviour of antennas near to or embedded in living tissue. The Eigen-function expansion (EFE) of DGFs in electromagnetic theory provide a systematic means of constructing and interpreting these dyadics. The pioneering work by Tai [1] has set the stage for most of what has been achieved over the last two and a half decades. The expansion of DGFs in terms of the Hansen [2] vector wave functions must be carried out carefully in order to ensure that one is dealing with a complete expansion.

This paper is organized as follows. The complete set of cylindrical vector wave functions are introduced in section 2. This material is included here in order to explicitly define notation and to call attention to a few points in connection with these expansions.

In section 3 we begin to formulate the problem for a finite circular cylinder and in subsection 3.1, we set out with the case, in which we construct the DGF, $\overline{\overline{G}}_{e1}(\overline{R}, \overline{R}')$, in terms that constitute the continuous Eigen-function expansion (EFE) in which the Eigen-functions are guided in preferred r and z-coordinate directions, using the procedures described in Tai [3] or Collin [4]. This expansion also contains an explicit dyadic delta function term which is required for completeness at the source point. It is considered as a correction to the general solenoidal EFE which is valid outside the source point.

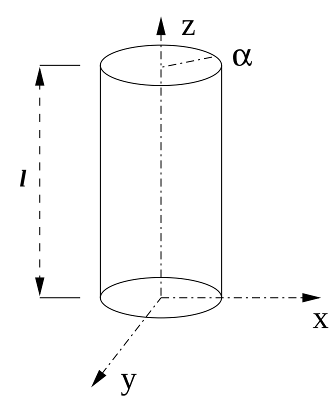

Fig. 1. Diagram of a Finite Circular Cylinder

The procedure required to derive the complete EFE of the scattering DGF for the finite circular cylinder, in terms of only the solenoidal Eigen-functions is shown to be a simple and straight-forward general expression and is summarized in section 4. The DGF for a finite conducting cylinder, $\overline{\overline{G}}_{E1}(\overline{R}, \overline{R}')$ can be constructed from the principle of the superposition, where it satisfies the boundary conditions.

Magnetic type DGF discussed in section 5, can be found by invoking duality or once the electric field is obtained the magnetic field is derivable by taking the curl of the electric field, and vice versa.

Conclusions are then presented in section 7 summarizing the important points contained in this work and finally a short bibliography is provided for further research.

2 Vector Wave Functions for a Circular Cylinder of Finite Length

The cylindrical vector wave functions are the building blocks of the EFE of various kinds of DGF. They are denoted by $\overline{L}_{\substack{ee\\oo}n\lambda}$, $\overline{P}_{\substack{ee\\oo}n\lambda}$ and $\overline{Q}_{\substack{ee\\oo}n\lambda}$, that are solutions of the homogeneous vector Helmholtz equation. The generating or Eigen-functions, which are solutions of the cylindrical scalar wave equation $\nabla^2\Psi + k_\lambda^2\Psi{=}0$, with the differential equation in the cylindrical coordinate system

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial \Psi}{\partial r}) + \frac{1}{r^2}\frac{\partial^2\Psi}{\partial\phi^2} + \frac{\partial^2\Psi}{\partial\phi^2} + \frac{\partial^2\Psi}{\partial z^2} + K^2\Psi = 0 \tag{1}$$

with K, the separation constant and k_λ being an undetermined wave number. Implementation of the method of separation of variables in this system finally results the generating function [5] in the form

$$\Psi_{\substack{ee\\oo}n}(h) = j_n(\lambda r)^{\cos}_{\sin} n\phi^{\cos}_{\sin} hz, \tag{2}$$

Here subscripts "e" stands for even and "o" is odd character of the generating functions. $h = \frac{q\pi}{l}$ are the eigenvalues in the z-direction with $q = 0, 1, 2, ...$ and l is the length of cylinder. $j_n(\lambda r)$ identifies the cylindrical Bessel functions of the order n to represent both out-going and in-coming waves. λ is the continuous eigen-value. Cylindrical vector wave functions are akin to the Debye potentials.

$$\overline{L}_{\substack{ee\\oo}n\lambda}(h) = \nabla\Psi_{\substack{ee\\oo}n}, \tag{3}$$

$$\overline{P}_{\substack{ee\\oo}n\lambda}(h) = \nabla\times[\Psi_{\substack{ee\\oo}n}\hat{z}], \tag{4}$$

$$\overline{Q}_{\substack{ee\\oo}n\lambda}(h) = \frac{1}{k_\lambda}\nabla\times\nabla\times[\Psi_{\substack{ee\\oo}n}\hat{z}]. \tag{5}$$

Where $\hat{z}$ is the piloting vector.

The complete expressions for the solenoidal (rotational or transverse) functions are

$$\overline{P}_{\substack{ee\\oo}n\lambda}(h) = \begin{Bmatrix} \mp\frac{n}{r}j_n(\lambda r)^{\sin}_{\cos} n\phi^{\cos}_{\sin} hz\hat{r} \\ -(\frac{\partial j_n(\lambda r)}{\partial r})^{\cos}_{\sin} n\phi^{\cos}_{\sin} hz\hat{\phi} \\ 0 \end{Bmatrix} \tag{6}$$

$$\overline{Q}_{\substack{ee\\oo}n\lambda}(h) = \begin{Bmatrix} \mp h[\frac{\partial j_n(\lambda r)}{\partial r}]^{\cos}_{\sin} n\phi^{\sin}_{\cos} hz\hat{r} \\ \frac{hn}{r}[j_n(\lambda r)]^{\sin}_{\cos} n\phi^{\sin}_{\cos} hz\hat{\phi} \\ \lambda^2[j_n(\lambda r)]^{\cos}_{\sin} n\phi^{\cos}_{\sin} hz\hat{z} \end{Bmatrix} \frac{1}{k_\lambda} \tag{7}$$

And the complete expressions for the non-solenoidal (irrotational or lamellar) functions are

$$\overline{L}_{\substack{ee\\oo}n\lambda}(h) = \begin{Bmatrix} \frac{\partial}{\partial r}[j_n(\lambda r)]^{\cos}_{\sin} n\phi^{\cos}_{\sin} hz\hat{r} \\ \mp\frac{n}{r}[j_n(\lambda r)]^{\sin}_{\cos} n\phi^{\cos}_{\sin} hz\hat{\phi} \\ \mp h[j_n(\lambda r)]^{\cos}_{\sin} n\phi^{\sin}_{\cos} hz\hat{z} \end{Bmatrix} \tag{8}$$

where $k_\lambda^2 = \lambda^2 + h^2$ and in these vector wave functions one should be careful with the sign of the elements in the matrices when cross-multiplying the terms from "e" to "o" and vice-versa e.g. "sin sin" always remains negative while "cos cos" positive. Also "$-$ cos sin" and "$-$ sin cos" in second elements of matrices in $\overline{P}_{eo}$ and $\overline{P}_{oe}$ respectively. In $\overline{L}_{eo}$ and $\overline{L}_{oe}$ both "cos sin" and "sin cos" are positive in the first element of their respective matrix. For $\overline{Q}_{eo}$, "$-$ sin cos" in second element of matrix, while "$+$ cos sin" in the third element. For $\overline{Q}_{oe}$, "$-$ cos sin" and "$+$ sin cos" in the elements 2 and 3 respectively. "$\mp$" applies the negative to the top line while positive to the bottom line.

Note that in the set of cylindrical vector wave functions only $\overline{P}_{\substack{ee\\oo}n\lambda}$ do not possess the z component. The $\hat{r}$, $\hat{\phi}$ and $\hat{z}$ are the cylindrical unit vectors. These functions are defined in the entire space, corresponding to $0 \le r \le \infty$, $0 \le \phi \le 2\pi$ and $0 < z < l$.

The volume integral of the product of the cylindrical vector wave functions is clearly zero if $n \ne n'$ and $h \ne h'$ because of the orthogonal property of the $\cos n\phi$ and $\sin n\phi$ functions and the Fourier integral relation. The derivation of the orthogonal properties of these vector wave functions are very similar to those for infinite circular cylinder discussed by Tai [3] and Collin [4].

3 Formulation of the Problem

Consider a cylinder (fig. 1) of radius "α" concentric along z-axis with length "l" is illuminated by an electromagnetic wave. An electromagnetic field is induced in the system and an electromagnetic wave is scattered by the system.

A time dependence $e^{j\omega t}$ is assumed and suppressed throughout.

3.1 DGF for a finite Length Cylinder of Circular Cross-Section

Because the dyadic $\nabla\times[\overline{\overline{I}}\delta_e(\overline{R}-\overline{R}')]$ is solenoidal, it can be expanded in terms of solenoidal vector wave functions; $\overline{P}_{\substack{e e\\ o o}n\lambda}$ and $\overline{Q}_{\substack{e e\\ o o}n\lambda}$ defined previously.

Applying the method of (G_m) and according to the Ohm-Rayleigh procedure, an EFE for the source function $\nabla\times[\overline{\overline{I}}\delta_e(\overline{R}-\overline{R}')]$ using the solenoidal vector wave functions can be

$$\nabla\times[\overline{\overline{I}}\delta_e(\overline{R}-\overline{R}')] = \int_0^\infty d\lambda\int_0^l dh\sum_{n=0}^\infty \cdot\begin{bmatrix}\overline{Q}_{\substack{e o\\ o}n\lambda}(h)\overline{A}_{\substack{e o\\ o}n\lambda}(h)\\ \overline{P}_{\substack{e e\\ o}n\lambda}(h)\overline{B}_{\substack{e e\\ o}n\lambda}(h)\end{bmatrix}, \tag{9}$$

where λ and h are continuous eigen-values and $\overline{A}_{\substack{e o\\ o}n\lambda}(h)$ and $\overline{B}_{\substack{e e\\ o}n\lambda}(h)$ are two unknown vector functions to be determined. This is a three-dimensional problem with a dyadic singular function, therefore the above equation can be treated as the Fourier transform and the Fourier-Bessel transform or the Hankel transform of $\nabla\times[\overline{\overline{I}}\delta_e(\overline{R}-\overline{R}')]$. By taking the anterior scalar product of the above equation with $\overline{Q}_{\substack{e o\\ o}n'\lambda'}(h')$ and integrating the resultant equation through the entire space and as a result of the orthogonal relationships and repeating the same routine with the $\overline{P}_{\substack{e e\\ o}n'\lambda'}(h')$ we can obtain the EFE, where we have preserved the Fourier integration. The plane of discontinuity for the magnetic DGF is located at $r = r'$. The expression for $(\overline{\overline{G}}_{e1})$ for a finite cylinder of radius "α" concentric with the z-axis can now be written in the form

$$\overline{\overline{G}}_{e1}(\overline{R},\overline{R}') = -\frac{\hat{r}\hat{r}}{k^2}\delta_e(\overline{R}-\overline{R}') + \int_0^l dh\sum_{n=0}^\infty C_\lambda \cdot\begin{cases}\begin{Bmatrix}[\overline{P}^{(1)}_{\substack{e o\\ o}\eta_o}(h;\eta_o)\overline{P}'_{\substack{e o\\ o}\eta_o}(h;\eta_o)]\\ [\overline{Q}^{(1)}_{\substack{e e\\ o}\eta_o}(h;\eta_o)\overline{Q}'_{\substack{e e\\ o}\eta_o}(h;\eta_o)]\end{Bmatrix}, & r>r',\\[2ex] \begin{Bmatrix}[\overline{P}_{\substack{e o\\ o}\eta_o}(h;\eta_o)\overline{P}'^{(1)}_{\substack{e o\\ o}\eta_o}(h;\eta_o)]\\ [\overline{Q}_{\substack{e e\\ o}\eta_o}(h;\eta_o)\overline{Q}'^{(1)}_{\substack{e e\\ o}\eta_o}(h;\eta_o)]\end{Bmatrix}, & r<r'.\end{cases} \tag{10}$$

where

$$C_\lambda = \frac{i(2-\delta_o^n)}{2l\eta_o^2} \tag{11}$$

Coefficient C_λ depends on the value of δ_o^n which is the Kronecker delta functions defined with respect to n, when

$$\delta_o^n = \begin{cases}1, & \text{if } n = o\\ 0, & \text{if } n \neq o\end{cases} \tag{12}$$

Here $\hat{r}\hat{r}$ is a dyad (dyadic product of the unit vectors) and $\delta(\overline{R}-\overline{R}')$ is weighted Dirac delta function in three dimensions. This is included explicitly as a correction to the general solenoidal EFE which is valid outside the source point. The dyadic delta function term at the source point in cylindrical coordinates

$$\delta(\overline{R}-\overline{R}') = \frac{1}{r'}\delta(\overline{r}-\overline{r}')\delta(\overline{\phi}-\overline{\phi}')\delta(\overline{z}-\overline{z}') \tag{13}$$

Comparing the DGFs for a finite cylinder developed here with those presented by other authors e.g. Tai [3] for an infinite cylinder, one can notice that they are similar in mathematical form but different in the calculations of Ps and Qs and the limits of integration for a finite cylinder.

4 Scattering DGF for a Finite Conducting Cylinder of Circular Cross-Section

When a perfectly conducting cylinder of the same size as above is illuminated by an electromagnetic wave, the scattered terms can be written in the form

$$\overline{\overline{G}}_{es}(\overline{R},\overline{R}') = \int_0^l dh\sum_{n=0}^\infty C_\lambda \cdot\begin{bmatrix}\alpha_{\substack{e o\\ o}\eta}\overline{P}^{(1)}_{\substack{e o\\ o}\eta}(h;\eta)\overline{P}'^{(1)}_{\substack{e o\\ o}\eta}(h;\eta)\\ \beta_{\substack{e e\\ o}\eta}\overline{Q}^{(1)}_{\substack{e e\\ o}\eta}(h;\eta)\overline{Q}'^{(1)}_{\substack{e e\\ o}\eta}(h;\eta)\end{bmatrix}. \tag{14}$$

Applying the principle of scattering superposition, we obtain

$$\overline{\overline{G}}_{E1}(\overline{R},\overline{R}') = \overline{\overline{G}}_{e1}(\overline{R},\overline{R}') + \overline{\overline{G}}_{es}(\overline{R},\overline{R}') \tag{15}$$

Where we consider the function for a finite circular cylinder in a region $0 \le r \le \infty$. After applying the boundary condition one can determine the unknown coefficients. In order to satisfy the boundary condition at interface $r = \alpha$,

$$\hat{r}\times\left[\overline{P}_{\substack{e o\\ o}\eta}(h;\eta)\overline{P}'^{(1)}_{\substack{e o\\ o}\eta}(h;\eta) + \alpha_{\substack{e o\\ o}\eta}\overline{P}^{(1)}_{\substack{e o\\ o}\eta}(h;\eta)\overline{P}'^{(1)}_{\substack{e o\\ o}\eta}(h;\eta)\right]_{r=\alpha} \tag{16}$$

$$\hat{r}\times\left[\overline{Q}_{\substack{e e\\ o}\eta}(h;\eta)\overline{Q}'^{(1)}_{\substack{e e\\ o}\eta}(h;\eta) + \beta_{\substack{e e\\ o}\eta}\overline{Q}^{(1)}_{\substack{e e\\ o}\eta}(h;\eta)\overline{Q}'^{(1)}_{\substack{e e\\ o}\eta}(h;\eta)\right]_{r=\alpha} \tag{17}$$

$$\hat{r}\times\left[\overline{P}_{\substack{e o\\ o}\eta}(h;\eta) + \alpha_{\substack{e o\\ o}\eta}\overline{P}^{(1)}_{\substack{e o\\ o}\eta}(h;\eta)\right]_{r=\alpha} = 0 \tag{18}$$

$$\hat{r}\times\left[\overline{Q}_{\substack{e e\\ o}\eta}(h;\eta) + \beta_{\substack{e e\\ o}\eta}\overline{Q}^{(1)}_{\substack{e e\\ o}\eta}(h;\eta)\right]_{r=\alpha} = 0 \tag{19}$$

substituting for $\overline{P}_{\substack{e o\\ o}\eta}(h;\eta)$ and $\overline{P}^{(1)}_{\substack{e o\\ o}\eta}(h;\eta)$

$$\overline{P}_{\substack{e o\\ o}\eta}(h;\eta) = \nabla\times[j_n(\eta r)^{\cos}_{\sin} n\phi\sin hz\hat{z}], \tag{20}$$

$$\overline{P}^{(1)}_{\substack{e o\\ o}\eta}(h;\eta) = \nabla\times[H_n^{(1)}(\eta r)^{\cos}_{\sin} n\phi\sin hz\hat{z}], \tag{21}$$

in equation (18) produces $\alpha_{\substack{e \\ o}\,\substack{o \\ }\eta} = -\frac{[\partial j_n(\eta\alpha)]/\partial(\eta\alpha)}{[\partial H_n^{(1)}(\eta\alpha)]/\partial(\eta\alpha)}$.

Similarly inserting for $\overline{Q}_{\substack{e \\ o}e\,\eta}(h;\eta)$ and $\overline{Q}^{(1)}_{\substack{e \\ o}e\,\eta}(h;\eta)$

$$\overline{Q}_{\substack{e \\ o}e\,\eta}(h;\eta) = \frac{1}{k}\nabla\times\nabla\times[j_n(\eta r)^{\cos}_{\sin} n\phi \cos hz\hat{z}], \qquad (22)$$

$$\overline{Q}^{(1)}_{\substack{e \\ o}e\,\eta}(h;\eta) = \frac{1}{k}\nabla\times\nabla\times[H_n^{(1)}(\eta r)^{\cos}_{\sin} n\phi \cos hz\hat{z}], \qquad (23)$$

in equation (19) produces $\beta_{\substack{e \\ o}e\,\eta} = -\frac{[j_n(\eta\alpha)]}{[H_n^{(1)}(\eta\alpha)]}$.

5 Magnetic DGF in the Antenna-Prosthesis Configuration

The principle of duality states that once the electric DGF is obtained, the magnetic DGF is derivable by interchanging the field functions $\overline{P}_{\substack{ee \\ oo}} \to k\overline{Q}_{\substack{ee \\ oo}}$ and $\overline{Q}_{\substack{ee \\ oo}} \to k\overline{P}_{\substack{ee \\ oo}}$ and omitting the singularity term contribution and vice versa.

On the other hand the corresponding total magnetic DGF at any point in the system can be calculated from $\nabla\times\overline{\overline{G}}_e = \overline{\overline{G}}_m$, bearing in mind the discontinuous nature of magnetic DGF across a point source at $R = R'$ and the Ampere-Maxwell equation relating $\overline{\overline{G}}_e$ and $\overline{\overline{G}}_m$ in the dyadic form i.e.: $\nabla\times\overline{\overline{G}}_m = \overline{\overline{I}}\delta_e(\overline{R}-\overline{R}') + k^2\overline{\overline{G}}_e$.

6 Electric and Magnetic Field at any Point in the Configuration

The use of DGF technique allows us to determine the expansion of the electric and magnetic fields in a cylinder/antenna configuration in a direct and elegant manner.

For any current source with current density function $\overline{J}(\overline{R}')$ located outside the cylinder, the electric or magnetic field radiated by such a dipole can be calculated using the formulae,

$$\overline{E}(\overline{R}) = i\omega\mu_o \iiint_V \overline{\overline{G}}_{E1}(\overline{R},\overline{R}')\cdot\overline{J}(\overline{R}')dV' \qquad (24)$$

$$\overline{H}(\overline{R}) = i\omega\varepsilon_o \iiint_V \overline{\overline{G}}_{M1}(\overline{R},\overline{R}')\cdot\overline{J}(\overline{R}')dV'. \qquad (25)$$

These signify the computation of the E and H-fields in the structure, which states the superposition of the incident field $\overline{E}_i(\overline{R})$ or $\overline{H}_i(\overline{R})$ and the scattered field $\overline{E}_s(\overline{R})$ or $\overline{H}_s(\overline{R})$ is given by

$$\overline{E}(\overline{R}) = \overline{E}_i(\overline{R}) + \overline{E}_s(\overline{R}) \qquad (26)$$

$$\overline{H}(\overline{R}) = \overline{H}_i(\overline{R}) + \overline{H}_s(\overline{R}). \qquad (27)$$

7 Concluding Remarks

General expressions have been derived in simple form for the finite conducting circular cylinder (medical devices/prostheses) of any size as well as of very small radius (resonant length). The DGFs are obtained by employing the EFE and the method of scattering superposition.

The results of this paper could be useful for a further analysis of the problem as a thin wire or an implant such as heart pace-maker embedded in the body and biotelemetry transmitters for medical applications and could easily be expanded so as to handle any scatterer having finite radius and length.

They can also be applied to problems of optical fibers and waveguides for the investigation of inhomogeneities or obstacles inside them or by considering the cylinder as an excitation or scatterer. They can also be of use in the study and design of antennas of high frequency.

The usefulness of the present technique obviously requires comparison with numerical and experimental results. it is envisaged that a later publication will address this aspect of the problem in more detail.

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