Open loop time domain gradient methods of parameter and delay estimation

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Abstract:- This paper discusses the estimation of the parameters of a single input, single output (SISO) process, modelled in first order lag plus delay (FOLPD) form, using gradient methods in the open loop time domain. The paper considers the convergence of the process parameters to the model parameters. The convergence of the model delay is discussed first, when the non-delay model and process parameters are identical. The convergence of all of the model parameters is then considered, when all of the process parameters are unknown.

Keywords:- Estimation, time delay, time domain, gradient methods. *CSCC'99 Proceedings*, Pages:1301-1308

1 Introduction

Gradient methods of parameter estimation are based on updating the parameter vector (which includes the delay) by a vector that depends on information about the cost function to be minimised. The gradient algorithms normally involve expanding the cost function as a second order Taylor's expansion around the estimated parameter vector. Typical gradient algorithms are the Newton-Raphson, the Gauss-Newton and the steepest descent algorithms, which differ in their updating vectors. The choice of gradient algorithm for an application depends on the desired speed of tracking and the computational resources available. It is important that the error surface in the direction of the delay (and indeed the other parameters) should be unimodal if a gradient algorithm is to be used successfully. However, the error surface is often multimodal. In these circumstances, strategies for locating global minima may involve multiple optimisation runs, each initiated at a different starting point, with the starting points selected by sampling from a uniform distribution [1]. The global minimum is then the local minimum with the lowest cost function value among all the local minima identified.

The use of gradient algorithms to estimate the parameters of a delayed process has been discussed in full elsewhere [2]. This paper will consider further the strategy proposed by Durbin [3], in which the process is assumed to be modelled by a first order lag plus delay (FOLPD) model. The process delay variation from the model delay is approximated by a rational polynomial, and a Gauss-Newton gradient descent algorithm is used to estimate the delayed model parameters. A previous paper [4] has shown that the first order Taylor's series polynomial is the most appropriate choice of rational polynomial; this paper has also provided a proof of the convergence of the non-delay model parameters to the non-delay process parameters, when the process and model delays are equal, in the presence of uncorrelated measurement noise. Outline proofs of the convergence of the delay estimate, and of all of the parameter estimates simultaneously, will be provided in this paper; full proofs of the relevant theorems, and subsequent simulation work, are available from the author.

2 Convergence of the Model Delay

2.1 The delay as an integer multiple of the sample period

<u>Theorem 1</u>: For a first order discrete stable system of known gain and time constant, then the mean of the product of the errors (MPE) performance surface versus model delay index is unimodal, with a minimum value of the MPE occurring when the model delay index equals the process delay index, under the conditions indicated below. The delay index is the delay divided by the sample time.

(a) The delay variation is approximated by a first order Taylor's series approximation.
(b) The measurement noise is uncorrelated with the process input.

(c) The resolution on the process delay is assumed to be equal to one sample period.
(d) The error is calculated based on using a FOLPD process model; the partial derivative of the error with respect to the delay variation is calculated based on using the first order Taylor's series approximation for the delay variation.
(e) The process delay index is greater than the model delay index, as the model delay index converges.

Proof: The process difference equation, $y_2(n)$, based on using a FOLPD process model, is

$$y_2(n) = e^{-T_s/T} y_2(n-1) + K(1-e^{-T_s/T})u(n-g_p-1) + w(n) \quad (1)$$

with T_p (process time constant) $= T_m$ (model time constant) = T, K_p (process gain) = K_m (model gain) = K and process time delay, $\tau_p = g_p T_s$, T_s = sample period, $g_p =$ process delay index, u(n) = input, w(n) = measurement noise. The model difference equation, assuming that the previous process output is used in its calculation and g_m = model delay index, is

$$y_{m3}(n) = e^{-T_s/T} y_2(n-1) + K(1-e^{-T_s/T})u(n-g_m-1) \quad (2)$$

Therefore, from equations (1) and (2),

$$e_3(n) = y_2(n) - y_{m3}(n) = K(1-e^{-T_s/T})[u(n-g_p-1) - u(n-g_m-1)] + w(n) \quad (3)$$

The partial derivative of the error with respect to the delay variation may then be calculated by using a first order Taylor's series approximation for the delay variation. The corresponding model difference equation is (assuming the previous process output is used in its calculation)

$$y_{m2}(n) = e^{-T_s/T} y_2(n-1) - \frac{K(g_p-g_m)T_s}{T} u(n-g_m) - K(e^{-T_s/T} - 1 - \frac{(g_p-g_m)T_s}{T})u(n-g_m-1) \quad (4)$$

Therefore, from equations (1) and (4),

$$e_2(n) = y_2(n) - y_{m2}(n) = K(1-e^{-T_s/T})u(n-g_p-1) + \frac{K(g_p-g_m)T_s}{T} u(n-g_m) + K(e^{-T_s/T} - 1 - \frac{(g_p-g_m)T_s}{T})u(n-g_m-1)] + w(n) \quad (5)$$

The corresponding partial derivative is

$$\frac{\partial e_2(n)}{\partial(g_p-g_m)} = \frac{KT_s}{T}\left[u(n-g_m) - u(n-g_m-1)\right] \quad (6)$$

The update vector for updating the model delay, which depends on the product of the error $(e_3(n))$ multiplied by the partial derivative of the error with respect to the delay variation $(\partial e_2(n)/\partial(g_p-g_m))$, is then independent of g_p. The cost function that approximately corresponds to this update vector is the MPE function; this function is defined as $E[e_2(n)e_3(n)]$ in this case. The update vector that exactly corresponds to this cost function depends on $e_3(n)[\partial e_2(n)/\partial(g_p-g_m)]$ & $e_2(n)[\partial e_3(n)/\partial(g_p-g_m)]$. It is assumed that $e_3(n)[\partial e_2(n)/\partial(g_p-g_m)] \approx e_2(n)[\partial e_3(n)/\partial(g_p-g_m)]$. This is a reasonable assumption, bearing in mind that the delay variation, which is approximated by a first order Taylor's series approximation, is assumed to be small. The MPE performance surface, $E[e_2(n)e_3(n)]$, may then be calculated to be [5]

$$K^2(1-e^{-T_s/T})\frac{(g_p-g_m)T_s}{T}[r_{uu}(0) - r_{uu}(1) + r_{uu}(g_p-g_m+1)] - K^2(1-e^{-T_s/T})\frac{(g_p-g_m)T_s}{T}[r_{uu}(g_p-g_m)] + r_{ww}(0) + 2K^2(1-e^{-T_s/T})^2[r_{uu}(0) - r_{uu}(g_p-g_m)] \quad (7),$$

$r_{uu}(n)$ and $r_{ww}(n)$ being the autocorrelation functions of u(n) and w(n) respectively. Therefore,

$$E[e_2(n)e_3(n)] = r_{ww}(0) \text{ for } g_m = g_p.$$

It may be shown by comparing the sizes of the individual terms in equation (7) that $E[e_2(n)e_3(n)] > r_{ww}(0)$ for $g_p > g_m$ only [5]. Thus, the minimum value of $E[e_2(n)e_3(n)]$ occurs at $g_m = g_p$ (when g_m is restricted to be less than or equal to g_p) and the measurement noise has no effect on the estimated process delay value.

If $g_p > g_m$, then, from equation (7), the only situation that arises for which $E[e_2(n)e_3(n)] = r_{ww}(0)$ for $g_m \neq g_p$ is when the input has a flat autocorrelation function, which corresponds to a constant level input. Thus, any input change is sufficient for correct process delay index estimation, provided that the required condition on g_m is fulfilled, if the process delay index is estimated by determining the minimum of the MPE performance surface.

However, if a gradient method is used to estimate g_p, then an additional restriction that the MPE function must be unimodal for $g_p > g_m$, with a minimum MPE value occurring at $g_m = g_p$, is imposed. The unimodality of the MPE function for $g_p > g_m$ may be proved by induction; an outline of the inductive proof (provided in full in reference [5]) is as follows:

It may be proved that the MPE function at $g_m = g_p - 1$ is greater than the MPE function at $g_m = g_p$ (using equation (7)), provided that

$$2(1 - e^{-T_s/T})[r_{uu}(0) - r_{uu}(1)] + \frac{T_s}{T}[r_{uu}(0) - 2r_{uu}(1) + r_{uu}(2)] > 0 \tag{8}$$

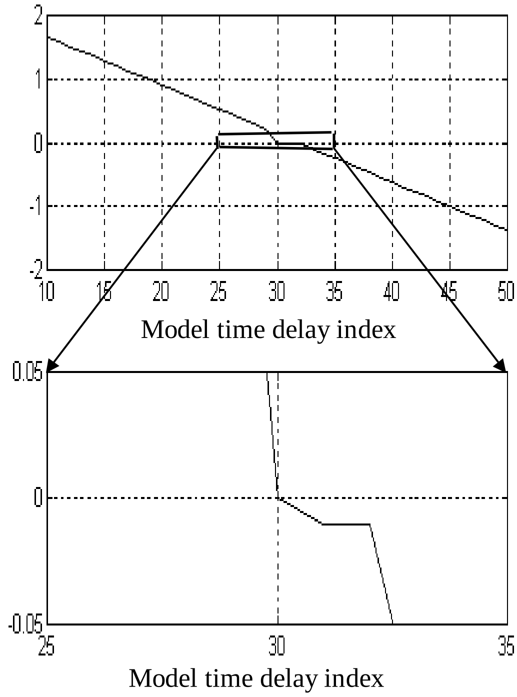
It may also be proved that the MPE function at $g_m = g_p - n - 1$ is greater than the MPE function at $g_m = g_p - n$, provided that

$$2(1 - e^{-T_s/T})[r_{uu}(n) - r_{uu}(n+1)] + \frac{T_s}{T}[r_{uu}(0) - r_{uu}(1)] + \frac{T_s}{T}[n r_{uu}(n) - (2n+1) r_{uu}(n+1) + (n+1) r_{uu}(n+2)] > 0 \tag{9}$$

Both of the conditions in equations (8) and (9) are fulfilled by many excitation signals e.g. a white noise signal.

The behaviour of the MPE function (given by equation (7)) versus model time delay index is confirmed by Figure 1, in a representative simulation result. For this simulation, $K_p = K_m = 2.0$, $T_p = T_m = 0.7$ seconds and $g_p = 30$. The normalised MPE (equal to the MPE divided by $r_{uu}(0)$) is plotted versus model time delay index, with $r_{ww}(0)$ put to zero. The excitation signal used is white noise. The plot shows that the MPE performance surface is greater than $r_{ww}(0)$ for $g_p > g_m$ only, and that when the conditions in equations (8) and (9) are fulfilled, the MPE function is unimodal for $g_p > g_m$, with a minimum MPE value occurring at $g_m = g_p$.

Fig. 1: Normalised MPE vs. model time delay index



A representative simulation result corresponding to Theorem 1 is given in Figure 2, with the time delay indices and the process minus model output plotted against sample number.

Fig. 2: Time delay index estimate

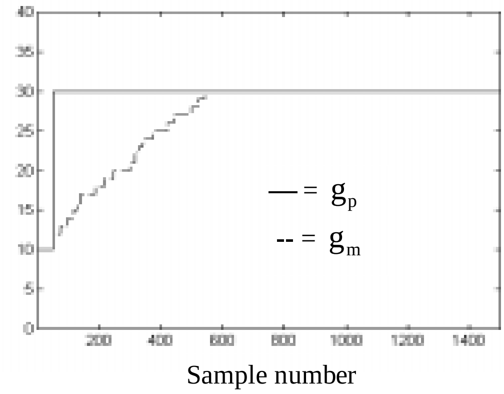
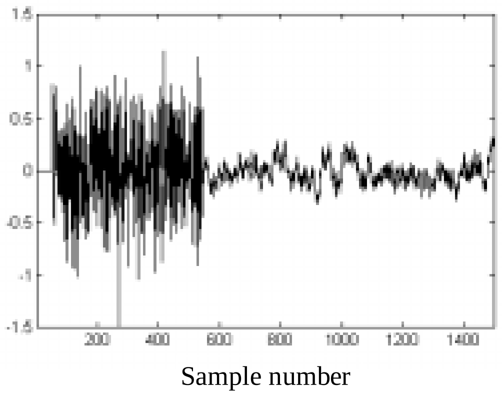


Fig. 3: $e_3(n)$



At the beginning, the starting values of the process and model time delay index were both equalised; a step change was then made to the process time delay index value. In the simulation, the update for the model time delay is a fractional multiple of the sample period; when the addition of these updates exceeds the value of the sample period (in either the positive or negative direction), then an appropriate adjustment is made in the model time delay index, with the update for the model time delay reset to zero. The process and model gain and time constant parameters were put equal to 2.0 and 0.7 seconds, respectively (i.e. the simulation conditions correspond to the conditions taken to calculate the MSE curves in Figure 1). The Levenberg-Marquardt gradient algorithm [6] was used to update the model time delay index; the sample time was defined equal to 0.1 seconds. Coloured measurement noise, generated by low-pass filtering a white noise signal, was added. The model time delay index was limited in variation to one sample period per iteration (which is a form of filtering on the time delay index value); such filtering was found to be desirable in simulation. Good convergence to the process time delay index is seen for $g_p > g_m$. Other supplementary simulation results show no convergence to the process time delay index when $g_p < g_m$. This verifies Theorem 1. The error, $e_3(n)$, in Figure 3 is non-zero due to the presence of the coloured measurement noise.

It is also possible to construct a block diagram representation of the gradient method to update the model delay index [5].

2.2 The delay as a real multiple of the sample period

Theorem 1 dealt with the estimation of delays that are integer multiples of the sample period. For the estimation of delays that are real multiples of the sample period (and assuming $T_p = T_m = T,\ K_p = K_m = K$), the FOLPD process difference equation is [5]:

$$y_3(n) = e^{-T_s/T} y_3(n-1) + K(1 - e^{g_b T_s/T}) u(n - g_p) + K(e^{g_b T_s/T} - e^{-T_s/T}) u(n - g_p - 1) + w(n) \quad (10)$$

with g_b = process delay minus the process delay index. The corresponding model difference equation (assuming the previous process output is used in its calculation) is

$$y_{m4}(n) = e^{-T_s/T} y_3(n-1) + K(1 - e^{g_a T_s/T}) u(n - g_m) + K(e^{g_a T_s/T} - e^{-T_s/T}) u(n - g_m - 1) \quad (11)$$

with g_a = model delay minus model delay index. The model difference equation for calculating the partial derivative of the error with respect to the delay variation (and assuming that the previous process output is used in its calculation) is

$$y_{m5}(n) = e^{-T_s/T} y_3(n-1) - \frac{K(g_p - g_m + g_b - g_a)T_s}{T} u(n - g_m) - K[e^{-T_s/T} - 1 - \frac{(g_p - g_m + g_b - g_a)T_s}{T}] u(n - g_m - 1) \quad (12)$$

This equation may be deduced from equation (4). Therefore, from equations (10) and (11),

$$e_4(n) = y_3(n) - y_{m4}(n) = K(1 - e^{g_b T_s/T}) u(n - g_p) + K(e^{g_b T_s/T} - e^{-T_s/T}) u(n - g_p - 1) - K(1 - e^{g_a T_s/T}) u(n - g_m) - K(e^{g_a T_s/T} - e^{-T_s/T}) u(n - g_m - 1) + w(n) \quad (13)$$

and, from equations (10) and (12),

$$e_5(n) = y_3(n) - y_{m5}(n) = K(1 - e^{g_b T_s/T}) u(n - g_p) + K(e^{g_b T_s/T} - e^{-T_s/T}) u(n - g_p - 1) + \frac{KT_s(g_p - g_m + g_b - g_a)}{T} u(n - g_m) + K[e^{-T_s/T} - 1 - \frac{T_s(g_p - g_m + g_b - g_a)}{T}] u(n - g_m - 1) + w(n) \quad (14)$$

The MPE performance surface, $E[e_4(n)e_5(n)]$, may then be calculated to be [5]

$$K^2[(1 - e^{g_b T_s/T})^2 + (e^{g_b T_s/T} - e^{-T_s/T})^2] r_{uu}(0)$$
$$+ K^2(1 - e^{-T_s/T})(e^{g_a T_s/T} - e^{-T_s/T}) r_{uu}(0)$$
$$- K^2(1 + e^{-T_s/T} - 2e^{g_a T_s/T}) \frac{T_s}{T}(g_p - g_m + g_b - g_a) r_{uu}(0)$$
$$+ 2K^2(1 - e^{g_b T_s/T})(e^{g_b T_s/T} - e^{-T_s/T}) r_{uu}(1)$$
$$+ K^2(1 - e^{-T_s/T})(1 - e^{g_a T_s/T}) r_{uu}(1)$$
$$- K^2(2e^{g_a T_s/T} - 1 - e^{-T_s/T}) \frac{T_s}{T}(g_p - g_m + g_b - g_a)] r_{uu}(1)$$
$$+ K^2 \frac{(g_p - g_m + g_b - g_a)T_s}{T}(1 - 2e^{g_b T_s/T} + e^{-T_s/T}) r_{uu}\left(g_p - g_m\right)$$
$$- K^2(1 - e^{g_b T_s/T})(1 - e^{g_a T_s/T}) r_{uu}\left(g_p - g_m\right)$$
$$- K^2(e^{g_b T_s/T} - e^{-T_s/T})(1 - 2e^{-T_s/T} + e^{g_a T_s/T}) r_{uu}(g_p - g_m)$$
$$- K^2(1 - e^{g_b T_s/T}) \frac{(g_p - g_m + g_b - g_a)T_s}{T} r_{uu}(g_p - g_m - 1)$$
$$+ K^2(1 - e^{g_b T_s/T})(1 - 2e^{-T_s/T} + e^{g_a T_s/T}) r_{uu}(g_p - g_m - 1) +$$

$$K^2(e^{g_b T_s/T} - e^{-T_s/T})\frac{(g_p - g_m + g_b - g_a)T_s}{T} r_{uu}(g_p - g_m + 1)$$

$$-K^2(e^{g_b T_s/T} - e^{-T_s/T})(1 - e^{g_a T_s/T}) r_{uu}(g_p - g_m + 1) + r_{ww}(0) \quad (15)$$

Now, using equation (15), it may be shown that $E[e_4(n)e_5(n)] = r_{ww}(0)$ if $g_p = g_m$ and $g_b = g_a$ [5].

Simulation results confirm the true multimodal nature of the MPE function versus model delay when the delay is a real multiple of the sample period. The estimation of the real value of the process time delay, using the approach, is impossible using gradient methods.

3 Convergence of the full parameter set

3.1 The delay as an integer multiple of the sample period with white noise input

Theorem 2: For a first order discrete stable system of unknown parameters, the MPE performance surface versus model delay index is minimised when the model delay index equals the process delay index, under the following conditions:

(a) The delay variation is approximated by a first order Taylor's series approximation.
(b) The measurement noise is uncorrelated with the process input and output.
(c) The resolution on the process delay is assumed to be equal to one sample period.
(d) The error is calculated based on using a FOLPD process model; the partial derivative of the error with respect to the delay variation is calculated based on the first order Taylor's series approximation for the delay variation.
(e) The conditions provided in the theorem are observed on the model parameters.
(f) The input to the model and the process is assumed to be a white noise signal.

Proof: The process difference equation, $y_1(n)$, is

$$y_1(n) = e^{-T_s/T_p} y_1(n-1) + K_p(1 - e^{-T_s/T_p})u(n - g_p - 1) + w(n) \quad (16)$$

The model difference equation, $y_{m1}(n)$, is

$$y_{m1}(n) = e^{-T_s/T_m} y_1(n-1) + K_m(1 - e^{-T_s/T_m})u(n - g_m - 1) \quad (17)$$

The partial derivative of the error with respect to the delay variation may then be calculated by using a first order Taylor's series approximation for the delay variation. The corresponding model difference equation is

$$y_{m8}(n) = e^{-T_s/T_m} y_1(n-1) - \frac{K_m(g_p - g_m)T_s}{T_m} u(n - g_m)$$
$$-K_m(e^{-T_s/T_m} - 1 - \frac{(g_p - g_m)T_s}{T})u(n - g_m - 1) \quad (18)$$

Error, $e_1(n) = y_1(n) - y_{m1}(n) =$
$$(e^{-T_s/T_p} - e^{-T_s/T_m})y_1(n-1)$$
$$+K_p(1 - e^{-T_s/T_p})u(n - g_p - 1)$$
$$-K_m(1 - e^{-T_s/T_m})u(n - g_m - 1) + w(n) \quad (19)$$

Error, $e_8(n) = y_1(n) - y_{m8}(n) =$
$$\frac{K_m(g_p - g_m)T}{T_m} u(n - g_m)$$
$$+K_m(e^{-T_s/T_m} - 1 - \frac{(g_p - g_m)T_s}{T_m})u(n - g_m - 1) + w(n)$$
$$+K_p(1 - e^{-T_s/T_p})u(n - g_p - 1) + (e^{-T_s/T_p} - e^{-T_s/T_m})y(n-1) \quad (20)$$

The MPE performance surface, $E[e_1(n)e_8(n)]$, may then be calculated to be [5]:

$$(e^{-T_s/T_p} - e^{-T_s/T_m})^2 r_{y_1 y_1}(0) + K_p^2(1 - e^{-T_s/T_p})^2 r_{uu}(0)$$
$$+K_m^2(1 - e^{-T_s/T_m})(1 - e^{-T_s/T_m} + \frac{(g_p - g_m)T_s}{T_m}) r_{uu}(0)$$
$$+K_p K_m(1 - e^{-T_s/T_p})\frac{(g_p - g_m)T_s}{T_m} r_{uu}(g_p - g_m + 1)$$
$$-2K_p K_m(1 - e^{-T_s/T_p})(1 - e^{-T_s/T_m}) r_{uu}(g_p - g_m)$$
$$-2K_p K_m(1 - e^{-T_s/T_p})\frac{(g_p - g_m)T_s}{T_m} r_{uu}(g_p - g_m)$$
$$+K_m(e^{-T_s/T_p} - e^{-T_s/T_m})\frac{(g_p - g_m)T_s}{T_m}[r_{uy_1}(g_m - 1) - r_{uy_1}(g_m)]$$
$$-2K_m(e^{-T_s/T_p} - e^{-T_s/T_m})(1 - e^{-T_s/T_m}) r_{uy_1}(g_m) + r_{ww}(0)$$
$$-K_m^2(1 - e^{-T_s/T_m})\frac{(g_p - g_m)T_s}{T_m} r_{uu}(1) \quad (21)$$

White noise excitation: $r_{uu}(k) = r_{uu}(0)$, $k = 0$ and $r_{uu}(k) = 0$ otherwise;
$r_{uy_1}(g_p + n) = (e^{-T_s/T_p})^{n-1} K_p(1 - e^{-T_s/T_p}) r_{uu}(0)$, $n \geq 1$, $r_{uy_1}(g_p + n) = 0$ otherwise [5].

At $g_m = g_p$, the value of the MPE equals (using equation (21)):

$$\mathrm{MPE_{opt}} = (e^{-T_s/T_p} - e^{-T_s/T_m})^2 r_{y_1 y_1}(0) + [K_p(1 - e^{-T_s/T_p}) - K_m(1 - e^{-T_s/T_m})]^2 r_{uu}(0) + r_{ww}(0) \quad (22)$$

By comparing the amplitudes of the individual terms in equations (21) and (22), it may be shown that $E[e_1(n)e_8(n)] > \mathrm{MPE_{opt}}$ for

(a) $g_p > g_m$ (for all values of process and model parameters) and

(b) $g_p < g_m$, provided $K_p/K_m \ge (g_m - g_p)/2$ and $T_m \ge T_p$ [5].

The conditions in (b) are sufficient, rather than necessary conditions.

However, if a gradient method is used to determine g_p, then, as before, the MPE function must be unimodal with a minimum MPE value occurring at $g_m = g_p$. The conditions for unimodality may be proved by induction [5]; these conditions are:

(a) $g_p > g_m$: Conditions for unimodality are fulfilled for all process and model parameters.

(b) $g_p < g_m$: The MPE function at $g_m = g_p + 1$ is greater than the MPE function at $g_m = g_p$, provided that the following sufficient conditions are obeyed:

$$(1 - T_s/T_m)K_p(1 - e^{-T_s/T_p}) > K_m(1 - e^{-T_s/T_m})$$

and

$$T_m > T_p . \quad (23)$$

The nature of the MPE function means that for a full inductive proof, it is necessary to prove that the MPE function at $g_m = g_p + 2$ is greater than the MPE function at $g_m = g_p + 1$ (this is because the MPE function in equation (21) depends on $r_{uu}(g_p - g_m + 1)$). A necessary condition for this to be true is [5]:

$$\frac{T_s}{T_m}[K_p(1 - e^{-T_s/T_p}) - K_m(1 - e^{-T_s/T_m})] > (e^{-T_s/T_m} - e^{-T_s/T_p})K_p(1 - e^{-T_s/T_p}) \left[2(1 - e^{-T_s/T_m} - \frac{T_s}{T_m})(1 - e^{-T_s/T_p}) - \frac{T_s}{T_m}\right] \quad (24)$$

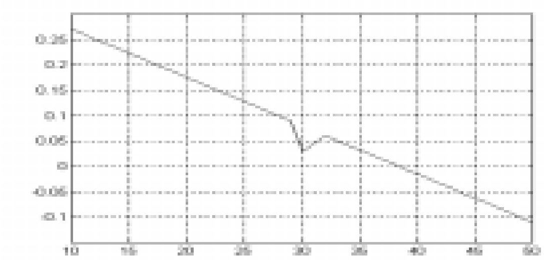
Similarly, it may be proved, that the MPE function at $g_m = g_p + n + 1$ is greater than that at $g_m = g_p + n$, provided

$$\frac{K_m(1 - e^{-T_s/T_m})\frac{T_s}{T_m}}{K_p(1 - e^{-T_s/T_p})e^{-(n-2)T_s/T_p}(e^{-T_s/T_p} - e^{-T_s/T_m})} < \frac{T_s}{T_m}[(n+1)e^{-2T_s/T_p} - (2n+1)e^{-T_s/T_p} + n] + 2(1 - e^{-T_s/T_m})(1 - e^{-T_s/T_p})e^{-T_s/T_p} \quad (25)$$

This is a necessary condition.

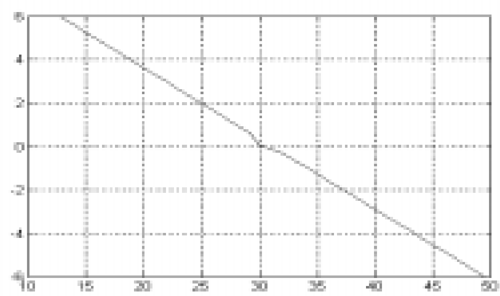
The theorem indicates that if K_p and T_p are unknown, then convergence of the model time delay index to the process time delay index may only be completely guaranteed if the value of the model time delay index is always less than or equal to the process time delay index. The behaviour of the MPE function (given by equation (21)) versus time delay index is confirmed, in representative simulation results, by Figures 4 and 5. In Figure 4, $K_p = 2.0$, $K_m = 1.0$, $T_p = 0.7$ s and $T_m = 1.0$ s so that the conditions given in equations (23) and (24) (but not (25)) are fulfilled; in Figure 5, $K_p = 2.0$, $K_m = 3.0$, $T_p = 0.7$ s and $T_m = 0.5$ s, so that none of the conditions in equations (23), (24) or (25) are fulfilled. The normalised MPE (equal to the MPE divided by $r_{uu}(0)$) is plotted versus time delay index in both cases, with $r_{ww}(0)$ put to zero and $g_p = 30$. The excitation signal in both cases is a white noise signal. The results are as expected from the theorem.

Fig. 4: Normalised MPE vs. time delay index



Model time delay index

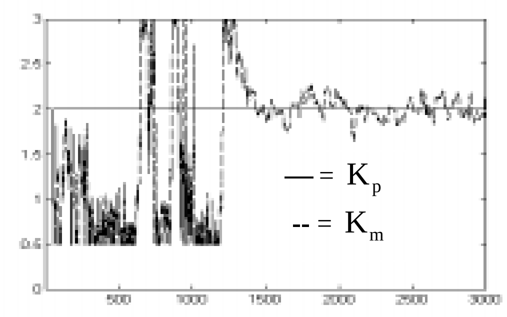
Fig. 5: Normalised MPE vs. time delay index



Model time delay index

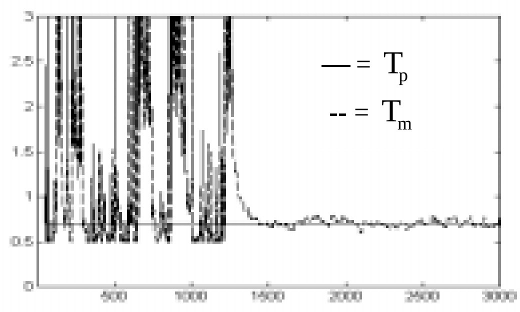
A representative simulation result corresponding to Theorem 2 is given in Figures 6 to 8, with the parameters plotted against sample number. It was also found necessary to limit the variation of the non-time delay model parameters; for the simulations taken, $0.5 < K_m < 3.0$ and $0.5s < T_m < 3.0$ s were the limits. The normalised MPE curve corresponding to these simulation results is given by Figure 4.

Fig. 6: Gain estimate



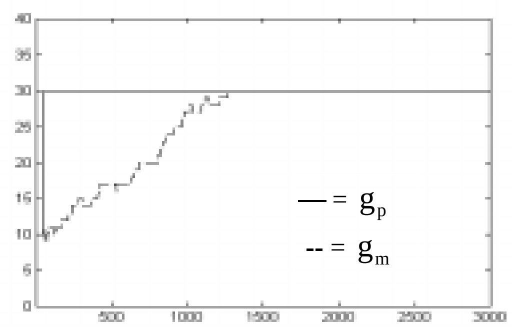
Sample number

Fig. 7: Time constant estimate



Sample number

Fig. 8: Time delay index estimate



Sample number

These results conform with Theorem 2.

Other simulation results show the convergence of the model parameters to the process parameters when $g_p > g_m$. It may also be shown, using analysis similar to that performed in Section 2.2, that the MPE function determined when the delay is a real multiple of the sample period is also multimodal with respect to delay [5].

3.2 The delay as an integer multiple of the sample period with square wave input

Theorem 3: For a first order discrete stable system of unknown parameters, the MPE performance surface versus model delay index is unimodal, with a minimum value of the MPE occurring when the model delay index equals the process delay index, under the following conditions:

(a) The delay variation is approximated by a first order Taylor's series approximation.
(b) The measurement noise is uncorrelated with the process input and output.
(c) The resolution on the process delay is assumed to be equal to one sample period.
(d) The error is calculated based on using a FOLPD process model; the partial derivative of the error, with respect to the delay variation, is calculated based on the first order Taylor's series approximation for the delay variation.
(e) The conditions provided in the theorem are observed on the model parameters.
(f) The excitation signal input is a square wave with a half period greater than the maximum possible process delay.

Proof: The MPE performance surface is given by equation (21). By determining the cross-correlation terms for a square wave input [5], it may be shown that

$$E[e_1(n)e_8(n)] > MPE_{opt} \text{ (= MPE when } g_p = g_m\text{)}$$

when

(a) $g_p > g_m$ and
(b) $g_p < g_m$, provided $K_p \geq K_m$ and $T_m \geq T_p$ [5].

The conditions in (b) are sufficient, rather than necessary conditions. Necessary and sufficient conditions for cost function unimodality may be determined by induction, as in Theorem 2 [5].

As before, simulation results confirm the results of the theorem [5]. It may also be shown, using analysis similar to that performed in Section 2.2, that the MPE function determined when the delay is a real multiple of the sample period is also multimodal with respect to delay [5].

4 Conclusions

A number of theorems have been developed to analytically describe the conditions under which the model parameters may converge to the process parameters. The corresponding cost functions may be unimodal when $g_p > g_m$ only. Some simulations show that unimodality can exist for all delay index values [5]; however, various conditions must be observed on the process and model parameters to achieve this result, which are impossible to evaluate prior to the implementation (as the process parameters are generally unknown). In addition, the inability of the relevant proposed methods to estimate delays that are real multiples of the sample period is disappointing. Both of these features are difficult to reconcile with a practical application. The requirement that in some cases the excitation signal to the process should be of white noise form is another difficulty, as such a signal is not realisable in practice; however, other excitation signals may also be used, as described in the theorems. On a positive note, the fact that unimodality does exist on the cost function for some conditions, when the delay is unknown *a priori*, provides some encouragement. One possibility may be to filter the data before identification, as this may increase the range of delay over which the cost function is unimodal, though the speed of convergence of any gradient algorithm used tends to be reduced [7]. In addition, if the process delay index may be estimated accurately, an estimate of a process delay that is a real multiple of the sample period could be determined by fitting an appropriate curve to a plot of the cost function (calculated, perhaps, in simulation) versus model delay index. The main difficulty with the use of the gradient algorithm, as implemented, is the estimation of the time delay term. One avenue of future work that may be fruitful would be to estimate the delay using an alternative (non-gradient) approach, and estimate the non-delay parameters using the gradient approach.

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