

# An Efficient All-Neighbor Fuzzy Association Approach In Distributed Multisensor-Multitarget Environment

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## Abstract

This paper proposes the first all-neighbor fuzzy logic data association approach in distributed multisensor-multitarget (MSMT) tracking systems. Unlike all fuzzy logic data association algorithms, which assign only one observation to each track according to some association measure, the proposed all-neighbor fuzzy logic data association approach incorporates all observations within the gate of the predicted target state to update the state estimate using a degree of membership weighted sum of innovations. To demonstrate the feasibility, efficiency, and simplicity of the proposed approach to perform data association in multisensor-multitarget environment, it is applied to an example of a four-dimensional tracking system. The performance of the proposed approach is evaluated using Monte Carlo simulations. Its performance is also compared to the performance of the nearest neighbor standard filter (NNSF) and perfect data association. The results shows that the proposed approach is very efficient.

## 1 Introduction

The problem of associating measurements and tracking multiple-target in the presence of noise and other interferences is a significant problem in MSMT tracking systems. There are two main categories for data association in MSMT tracking systems; algorithmic and nonalgorithmic categories [4]. Algorithmic category is based on nearest neighbor (NN) and all-neighbor (AN) techniques. Nonalgorithmic (approximate) category is based on neural network and fuzzy logic techniques. In general, the computational cost in generating the optimal solutions to the data association problem is usually excessive when the number of targets and the number of measurements are large.

Fuzzy logic systems has been proven very successful in solving complicated engineering problems in many areas where conventional approaches are either very difficult or inefficiently/costly to implement. Fuzzy logic applications to multisensor-multitarget data association are very complicated in case of a dense target environment. An all-neighbor fuzzy logic data association approach is proposed to solve this problem. The proposed method is developed based on fuzzy clustering means (FCM) algorithm. The new approach is flexible and robust in that it can handle different types of information without a priori knowledge of the signal environment. The proposed approach is applied to a four-dimensional multisensor-multitarget tracking system using Monte Carlo simulations. Fuzzy system performance evaluation is presented to demonstrate the feasibility and the efficiency of the new approach. The proposed approach is proved to be simple and efficient.

## 2 Fuzzy Clustering Means Algorithm

This section introduces the FCM algorithm which is the best widely used clustering algorithm. This algorithm is developed by J. Bezdek [12], [13]. The goal of any fuzzy clustering algorithm is to classify the data into a number of clusters (groups). The clustering algorithms produce a degree (grade) of membership for each data point in each cluster. Unlike the conventional meaning of clustering, which means a partitioning of objects into disjoint clusters, the fuzzy clustering allows a data point  $x$  to have a partial degree of membership in more than one set [14], [15]. A fuzzy set  $A$  in a collection of objects  $X$  is defined as:

$$A = \{(x, u_A(x)) \mid x \in X\}, \quad (1)$$

where  $u_A(x)$  is the degree of membership function of data point  $x$  in fuzzy set  $A$ . Given a number of

data points, it is required to group (cluster) the data into clusters according to some similarity measure [3]. Let  $c$  be an integer which represents the number of clusters with  $2 \leq c \leq n$  where  $n$  is the number of data. Define  $\mathbf{U}$  as a partition matrix of elements  $u_{ij}$ ,  $i = 1, 2, \dots, c$ ,  $j = 1, 2, \dots, n$ , where  $u_{ij}$  represents the degree of membership of data point  $j$  in fuzzy cluster  $i$ , such that :

$$u_{ik} \in [0, 1], 1 \leq i \leq c, 1 \leq k \leq n, \quad (2)$$

$$\sum_{i=1}^c u_{ik} = 1 \quad \forall k, \quad (3)$$

$$0 < \sum_{k=1}^n u_{ik} < n \quad \forall i. \quad (4)$$

Define  $J_m$  as the sum of all square errors weighted by the  $m^{th}$  power of the corresponding degree of membership i.e.

$$J_m(\mathbf{U}, \mathbf{v}) = \sum_{k=1}^n \sum_{i=1}^c (u_{ik})^m (d_{ik})^2; \quad (5)$$

where

$$(d_{ik})^2 = \|\mathbf{x}_k - \mathbf{v}_i\|^2, \quad (6)$$

and  $\|\cdot\|$  is any inner product induced norm,  $m \in [1, \infty)$  is a weighting exponent and is called the fuzzification constant,  $\mathbf{x}_k$  is a data point and  $\mathbf{v}_i$  is a cluster center. The degrees of membership will be established by minimizing the sum of all square errors weighted by the corresponding  $m^{th}$  power of the degree of membership. The goal of the fuzzy clustering algorithm is to determine the optimum degrees of membership  $u_{ik} \forall i, k$  and the optimum fuzzy cluster centers  $\mathbf{v}_i \forall i$  such that the sum of the square errors  $J_m$  is minimum. The results are derived using Lagrange multipliers and taking derivatives. The results are given by [16]:

$$u_{ik} = \frac{1}{\left[ \sum_{j=1}^c \left( \frac{d_{ik}}{d_{jk}} \right)^{\frac{2}{m-1}} \right]} \quad \forall i, k, \quad (7)$$

$$\mathbf{v}_i = \frac{\sum_{k=1}^n (u_{ik})^m \mathbf{x}_k}{\sum_{k=1}^n (u_{ik})^m} \quad \forall i. \quad (8)$$

Solution (9) is valid for a fixed  $\mathbf{V}$  ( $\mathbf{V} = \mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_c$ ), and solution (10) is valid for a fixed  $\mathbf{U}$ . In *MSMT* tracking systems,  $c$  is the number of targets,  $n$  is the total number of received measurements,  $\mathbf{x}_k$  is s-dimensional measurement vector,  $k = 1, 2, \dots, n$ , and  $\mathbf{v}_i$  is s-dimensional predicted vector for target  $i$ ,  $i = 1, 2, \dots, c$ . The fuzzy c-means clustering algorithm referred to as the Picard algorithm. The Picard algorithm is guaranteed to converge to a local minimum [20].

### 3 Proposed All-Neighbor Fuzzy Association Approach

The probability theory deals with randomness which describes the uncertainty of occurrence. The fuzzy set theory deals with fuzziness which describes ambiguity. Unlike randomness, which determines the probability of an event to occur (it may occur or not occur), fuzziness determines the degree of membership that an event occurs (not whether it occurs). Kosko [1], addressed the similarities and the dissimilarities of probability theory and fuzzy set theory. Although both theories describe uncertainty in the interval  $[0, 1]$ , they differ conceptually and theoretically. Kosko [1] addressed the following important question: does the degree of membership to which an element belongs to fuzzy set equal the probability that the same element belongs to the same set. Toward that direction, he developed the fuzzy Subsethood theorem, which implies the Bayes theorem or equivalently, probabilities represent a special case of fuzziness. Buede [2], compared both approaches (fuzzy set and Bayes approaches) for target identification in a data fusion system. He showed that the fuzzy set theorem's results are inferior to those of probability theorem's results. This result is expected since the probability theorem utilizes a prior information about the underlying process. Buede [2] also showed that sometimes the results of both approaches are similar and sometimes they are dissimilar.

Bar-Shalom and Fortman [7], developed the Probabilistic Data Association Filter (PDAF) which updates the predicted target state using a probability weighted sum of innovations (probabilistic score). Our proposed approach associates measurements into track using a degree of membership score. Unlike many fuzzy logic data association algorithms [4], [16]-[19], which consist of four basic elements: 1) fuzzification of crisp data into fuzzy variables, 2) fuzzy knowledge base which contains IF THEN rules and fuzzy statements, 3) fuzzy inference mechanism which stimulates human decision making procedure to generate output fuzzy variables and 4) defuzzification of fuzzy variables into non-fuzzy variables (crisp data), the proposed approach performs all-neighbor data association based on the partition matrix of data (measurements) in fuzzy clusters (tracks).

Each row in the obtained partition matrix, given

by (7), is normalized such that for a given track  $i$ , the contributions of all observations must equal unity, i.e.

$$\sum_{i=1}^n u_{ik} = 1 \quad \forall k. \quad (9)$$

For a given track, the state estimate at time  $K+1$  is given by

$$\begin{aligned} \hat{\mathbf{X}}(K+1 | K+1) &= \hat{\mathbf{X}}(K+1 | K) + \\ \mathbf{W}(K+1) &\sum_{k=1}^n u_{ik}(K) \tilde{\mathbf{z}}_k(K+1), \end{aligned} \quad (10)$$

where  $\tilde{\mathbf{z}}_k(K+1)$  is the innovation due to observation  $k$  at time instant  $(K+1)$  i.e.

$$\tilde{\mathbf{z}}_k(K+1) = \mathbf{z}_k(K+1) - \hat{\mathbf{z}}(K+1), \quad (11)$$

and  $\mathbf{W}(K+1)$  is the standard Kalman filter gain. The state update equation is then given by

$$\hat{\mathbf{X}}(K+1 | K+1) = \hat{\mathbf{X}}(K+1 | K) + \mathbf{W}(K+1) \tilde{\mathbf{z}}(K+1), \quad (12)$$

where

$$\tilde{\mathbf{z}}(K+1) = \sum_{k=1}^n u_{ik}(K+1) \tilde{\mathbf{z}}_k(K+1), \quad (13)$$

is the sum of all weighted innovations. The weights are the normalized grades of membership function given by (9). The update covariance matrix can also derived as [7]:

$$\mathbf{P}(K+1 | K+1) = \mathbf{P}_1(K+1 | K+1) + \tilde{\mathbf{P}}(K+1), \quad (14)$$

where

$$\mathbf{P}_1(K+1 | K+1) = [\mathbf{I} - \mathbf{W}(K+1)\mathbf{H}(K+1)]\mathbf{P}(K+1 | K), \quad (15)$$

is the standard covariance update equation, and

$$\begin{aligned} \tilde{\mathbf{P}}(K+1) &= \mathbf{W}(K+1) \left[ \sum_{k=1}^n u_{ik}(K+1) \right. \\ &\quad \tilde{\mathbf{z}}_k'(K+1) - \tilde{\mathbf{z}}(K+1) \tilde{\mathbf{z}}'(K+1) \left. \right] \\ &\quad \mathbf{W}'(K+1). \end{aligned} \quad (16)$$

If there is no observation within the gate of a given track, the update state estimate will be the previous estimate, i.e.

$$\hat{\mathbf{X}}(K+1 | K+1) = \hat{\mathbf{X}}(K+1 | K). \quad (17)$$

Note that (6) is valid for any inner-product-induced norm metric and  $J_m$  can be written as

$$J_m(\mathbf{U}, \mathbf{v}, G) = \sum_{k=1}^n \sum_{i=1}^c (u_{ik})^m (d_{ik})_G^2, \quad (18)$$

where

$$(d_{ik})_G^2 = (\mathbf{x}_k - \mathbf{v}_i)^T G (\mathbf{x}_k - \mathbf{v}_i), \quad (19)$$

and  $G$  is any positive-definite matrix [12]. In this case,  $(d_{ik})_G$  is called weighted distance. The matrix  $G$  may be used as a theoretical variable for optimization [13] or can be chosen subjectively. For example, it is convenient to choose  $G$  to be related to the covariance matrix of the measurement error such that the larger the measurement error the larger is the distance (the smaller is the degree of membership) and vice versa. The fuzzification constant,  $m$  (also is called weighting exponent), plays an important rule. It reduces the influence of noise when computing the degrees of membership (7) and the cluster centers (8). The weighting exponent  $m$  reduces the influence of small  $u_{ik}$  (data are faraway from the cluster centers) compared to that of large  $u_{ik}$  (data are close to the cluster centers) [22]. As  $m$  increases, the influence becomes stronger. For more details about the rule of the weighting exponent see Windham, [15], [23].

## 4 Performance Evaluation

One example is presented to demonstrate the feasibility and the efficiency of the proposed all-neighbor fuzzy logic data association algorithm in multisensor multitarget tracking systems. The example considers the case of a two crossing targets without acceleration. The targets motion model is assumed to be :

$$\mathbf{X}(K+1) = \mathbf{F} \mathbf{X}(K), \quad (20)$$

where  $\Delta$  is the sampling interval and  $\mathbf{F}$  is the state transition matrix and is given by

$$\mathbf{F} = \begin{pmatrix} 1 & \Delta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (21)$$

The state vector  $\mathbf{X}(K)$  contains the  $x$  and  $y$  target positions and velocities, i.e.

$$\mathbf{X}(K) = \begin{pmatrix} x(K) \\ v_x(K) \\ y(K) \\ v_y(K) \end{pmatrix}. \quad (22)$$

The measurements are the  $x$  and  $y$  target positions, i.e.

$$\mathbf{z}(K) = \mathbf{H}(K) + \mathbf{w}(K) \quad (23)$$

where

$$\mathbf{H} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (24)$$

The noise sequence  $\mathbf{w}(K)$  are uncorrelated with zero mean Gaussian *pdf* and covariance matrix

$$\mathbf{R}_K = Cov(\mathbf{w}(K)) = \begin{pmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{pmatrix}, \quad (25)$$

where  $\sigma_x^2$  and  $\sigma_y^2$  are the variances of the measurements error in  $x$  and  $y$  positions. The measurement error is defined as

$$e = \sqrt{e_x^2 + e_y^2} = \sqrt{(x_{true} - \hat{x})^2 + (y_{true} - \hat{y})^2}. \quad (26)$$

In our example, we assume that  $X$  and  $Y$  positions (measurements) are taken every 0.1 second, with sensor uncertainties  $\sigma_{x1}=\sigma_{y1}=150$  meters and  $\sigma_{x2}=\sigma_{y2}=200$  meters. The initial targets states are given by

$$\mathbf{X}_{t1}(0) = \begin{pmatrix} 6000 \text{ m} \\ 158.9 \text{ m/sec} \\ 6000 \text{ m} \\ 3.3 \text{ m/sec} \end{pmatrix}, \quad (27)$$

$$\mathbf{X}_{t2}(0) = \begin{pmatrix} 6000 \text{ m} \\ 158.9 \text{ m/sec} \\ 6050 \text{ m} \\ -3.3 \text{ m/sec} \end{pmatrix}. \quad (28)$$

In case of NNSF, the nearest observation, that maximizes the likelihood function of the residual error associated with the selection, is selected to update the target's track. Maximization of the likelihood function is equivalent to minimize the following distance [21]:

$$d^2 = d_{ij}^2 + \ln|S_i|, \quad (29)$$

where  $d_{ij}^2$  is the weighted sum of innovation, i.e.

$$d_{ij}^2 = [\mathbf{z}_j - \hat{\mathbf{z}}_i]' \mathbf{S}_i^{-1} [\mathbf{z}_j - \hat{\mathbf{z}}_i], \quad (30)$$

and  $S_i$  is the residual covariance matrix and is given by :

$$\mathbf{S}_i = \mathbf{H} \mathbf{P} \mathbf{H}' + \mathbf{R}. \quad (31)$$

The measurements initializations are obtained from the first two measurements. We process 150 measurements (15 seconds of data) over a 50 runs Monte Carlo simulations. Tracking results are shown in Fig. 1 - Fig. 4. Fig. 1 shows the actual targets trajectories. Fig. 2 shows the actual targets trajectories along with the measurements trajectories. Fig. 3 shows the actual targets trajectories along with the mean Kalman

Filter (KF) track ( estimated positions) obtained using the proposed fuzzy approach. Fig. 4 shows the mean measurement error for both targets. Fig. 5 and 6 show the results of tracking using the NNSF. Fig. 7 and 8 show the results in case of perfect data association. The results show that the proposed approach successfully tracks both targets and its performance is superior to that of the NNSF.

## 5 Conclusion

The problem of data association in MSMT tracking systems has been considered. An all-neighbor fuzzy logic data association approach has been proposed. The proposed approach incorporates all the observations within the gate of the predicted target state to update the state estimate using a degree of membership weighted sum of innovations. Performace evaluation has been evaluated using Monte Carlo simulations. It has been shown that the proposed approach has better performance over a comparable NNSF approach and is proved to be efficient with respect to perfect data association. This is the first all-neighbor data association approach in multisensor multitarget tracking systems. Of course the proposed approach is not the optimal approach. The results are promising and shows the feasibility and the efficiency of using fuzzy logic approach in all-neighbor data association techniques.

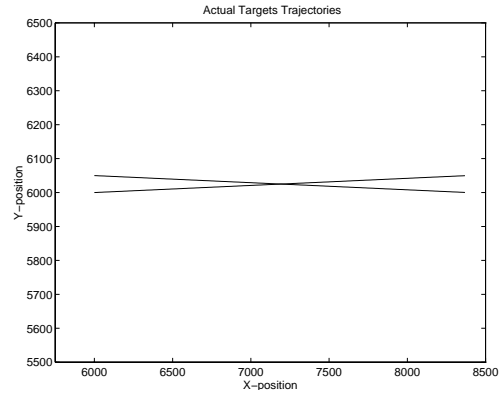


Figure 1: Actual Targets Trajectories

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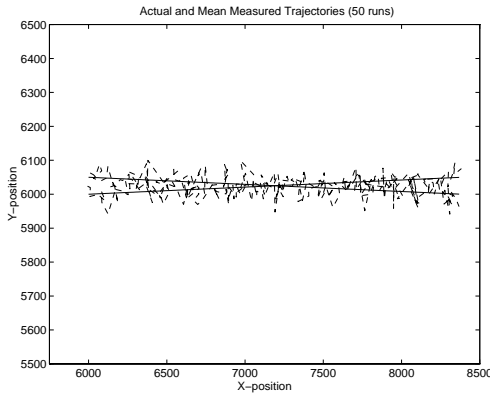


Figure 2: Actual and Mean Measured Targets Trajectories

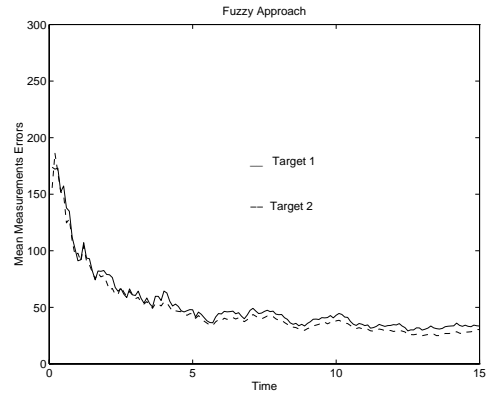


Figure 4: Mean Measurements Error: Fuzzy Approach

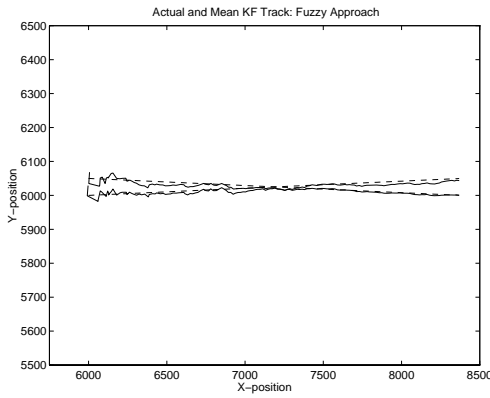


Figure 3: Actual and Mean Estimated Track: Fuzzy Approach

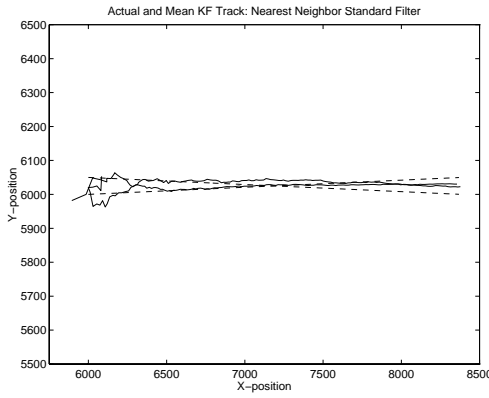


Figure 5: Actual and Mean Estimated Track: NNSF

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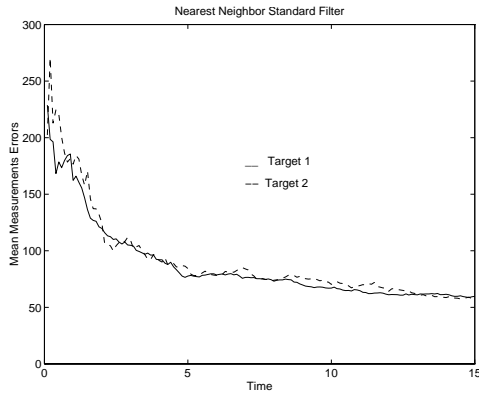


Figure 6: Mean Measurements Error: NNSF

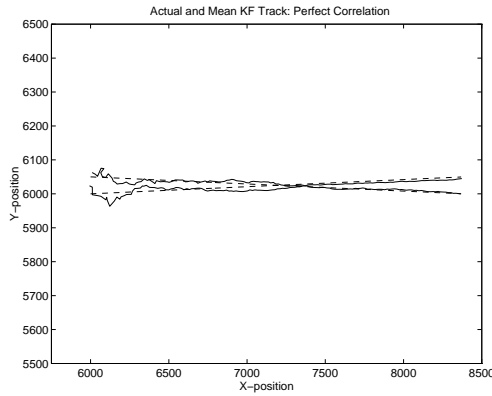


Figure 7: Actual and Mean Estimated Track: Perfect Association

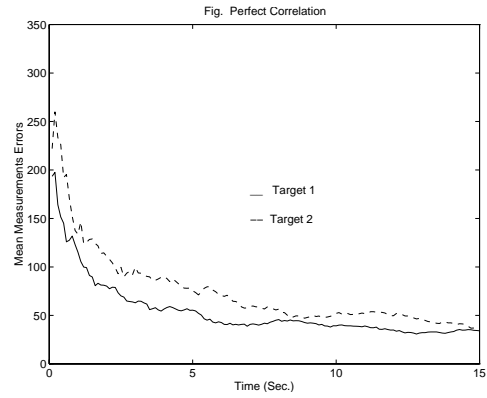


Figure 8: Mean Measurements Error: Perfect Association

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